Conformal and CR Geometry (24w5260)

Jeffrey S. Case (Penn State University),
Azahara DelaTorre Pedraza (Sapienza Università di Roma),
Pengfei Guan (McGill University),
Andrea Malchiodi (Sculoa Normale Superiore),
Yi Wang (Johns Hopkins University)

December 1–6, 2024

Conformal and CR geometry are closely related subjects in which one studies manifolds with a well-defined notion of angles but not of lengths of tangent and contact tangent vectors, respectively. Indeed, these geometries are closely related through constructions of Fefferman and Fefferman–Graham, and are among the simplest examples of parabolic geometries. There has been substantial progress in both fields, though not always through parallel developments. The purpose of this workshop was to bring together people working in both areas to appraise the current status of development and to encourage the cross-fertilization of ideas. Given the number of important new developments discovered by young researchers, this workshop provided an excellent opportunity to spotlight their work and introduce them to leading experts in both areas.

1 Overview of the Field

A *conformal manifold* is a pair $(M^n, \mathfrak{c})$ of a smooth n-manifold and a conformal class $\mathfrak{c}$; i.e. an equivalence class of Riemannian metrics whose metrics $g, \hat{g} \in \mathfrak{c}$ are related by $\hat{g} = e^{2u} g$ for some $u \in C^\infty(M)$. A (strictly pseudoconvex) *CR manifold* is a pair $(M^{2n+1}, T^{1,0})$ of a smooth $(2n+1)$-manifold and a distribution $T^{1,0} \subset TM \otimes \mathbb{C}$ of rank n which satisfies $T^{1,0} \cap \overline{T^{0,1}} = \{0\}$, is integrable in the sense that $[Z, W] \in \Gamma(T^{1,0})$ whenever $Z, W \in \Gamma(T^{1,0})$, and is such that $H := Re(T^{1,0} \oplus \overline{T^{0,1}})$ is a (real) contact distribution. Any two contact forms $\theta, \hat{\theta}$ for H are related by $\hat{\theta} = e^u \theta$ for some $u \in C^\infty(M)$, and the Levi form $\mathcal{L}_\theta(Z, W) := -i\, d\theta(Z, \bar{W})$ is a positive definite hermitian metric on $T^{1,0}$ which satisfies $\mathcal{L}_{\hat{\theta}} = e^u \mathcal{L}_\theta$.

The ambient spaces of Fefferman–Graham and Fefferman, which are formally Ricci-flat manifolds determined by the underlying conformal or CR manifold, provide a powerful method to construct conformal and CR invariants. Indeed, an old result of Bailey, Eastwood, and Graham classifies *scalar* conformal and CR invariants in terms of the scalar Riemannian and Kähler invariants of the corresponding ambient space. These ambient spaces also determine formally Einstein manifolds called *Poincaré manifolds*; i.e. compact $(N+1)$-manifolds-with-boundary X together with a complete metric g_+ in their interior which formally solve $\mathrm{Ric}_{g_+} + N g_+ = 0$ and for which $r^2 g_+$ determines the underlying conformal or CR structure for any defining function r of ∂X (with some additional conditions in the CR case). An old result of Cheng and Yau showed that one can always produce *exact* solutions to the Einstein equation $\mathrm{Ric}_{g_+} + N g_+ = 0$ when one starts with a CR manifold. However, neither existence nor uniqueness of exact solutions—called *Poincaré–Einstein manifolds*—when starting with a conformal manifold is true, and it remains an important open problem to understand the moduli space of Poincaré–Einstein manifolds with prescribed boundary.

A *scalar integral invariant* in conformal or CR geometry is an invariant which can be written as the integral of a scalar Riemannian or pseudohermitian invariant, respectively. Unlike their local counterparts, such integral invariants are not classified. In the conformal case, an old result of Alexakis proves that any scalar integral conformal invariant is a linear combination of the total Q-curvature, the integral of a scalar conformal invariant, and a divergence. Missing still is a description of the set of scalar conformal invariants which are divergences. In the CR case, there is not even an analogue of Alexakis' result. However, there is a scalar pseudohermitian invariant, the *Q'-curvature*, whose integral is a CR invariant. A distinguishing property of the Q- and Q'-curvatures is that they have a linear transformation law under change of metric or contact form on a conformal or CR manifold, respectively, making the problems of prescribing these curvatures analogous to the problem of prescribing the Gauss curvature of a surface or, when moving outside of the realm of scalar integral invariants, of the Yamabe problem of prescribing the scalar curvature of a manifold of dimension $n \geq 3$ within a conformal class. Fundamental issues, such as the compactness of the set of metrics in a conformal class with constant Q-curvature, remain open.

The Q- and Q'-curvatures, and more generally the conformally covariant operators which control their behavior under change of metric or contact form, have many constructions. One of the more useful constructions is via scattering theory, where they arise as the obstructions of smooth solutions to the generalized eigenvalue problem $\Delta_{g_+} u + s(n-s)u = 0$ with given "Dirichlet data" at infinity in a Poincaré manifold. More generally, the "Dirichlet-to-Neumann" map for this problem, also known as the scattering operator, allows one to define nonlocal conformally covariant operators on conformal and CR manifolds with leading order term a fractional power of the Laplacian (these depend on the Poincaré manifold in general). Chang–González showed that the resulting operators on Euclidean space are exactly fractional powers of the Laplacian, and that in this setting the scattering construction coincides with the highly influential construction of Caffarelli–Silvestre. This led to the CR analogue of the latter construction by Frank–González–Monticelli–Tan and curved analogues thereof by others.

One is also interested in studying the geometry of (distinguished) submanifolds of conformal and CR manifolds. For example, the distinguished curves on conformal and CR manifolds are called *conformal geodesics* and *chains*, respectively, and the Willmore Conjecture (resolved by Marques–Neves) can be understood as the question of finding the immersed orientable surface of genus $g \geq 1$ of minimal energy into the standard conformal three-sphere. In conformal geometry, there has been a lot of recent activity on the construction of conformal invariants of submanifolds, and similar activity is starting to be carried out in the CR case. However, classification of local or global invariants remains a fundamental open problem.

The problem of prescribing Q-curvatures in a conformal class is often reduced to a semi-linear PDE. One is also interested in prescribing other curvature invariants, such as the σ_k-curvatures, which reduce to quasi-linear or fully nonlinear PDEs. In the fully nonlinear setting, it is a difficult and very active research problem to derive *a priori* estimates. The difficulties faced here are similar to the difficulties faced in studying fully nonlinear problems in other settings, such as the k-Hessian equation or extrinsic curvature prescription problems in $\mathbb{R}^n$.

2 Recent Developments and Open Problems

In the organizational phase, our workshop was loosely organized around the five topics named in the previous section:

1. Existence and uniqueness of Poincaré–Einstein metrics
2. Q- and Q'-curvatures
3. Scattering theory and nonlocal operators
4. Geometric problems on submanifolds
5. Fully nonlinear equations in conformal geometry and their apriori estimates

Below we report on recent developments in these areas, both as reported in talks during our workshop and as achieved by other participants of our workshop (as discussed in Section **??**, we prioritized in-person talks, which meant that certain important advances were not represented in talks).

2.1 Existence and uniqueness of Poincaré–Einstein metrics

A fundamental open problem in conformal geometry is to understand the moduli space of Poincaré–Einstein metrics with specified conformal boundary. Chang, Ge and their collaborators have made great strides on studying this problem in dimension four. For example, Chang–Ge–Jin–Qing proved a compactness theorem for Poincaré–Einstein four-manifolds under a smallness assumption on the L^2-norm of the Weyl tensor and compactness assumptions on the conformal infinity. As an application, they proved the uniqueness of the Graham–Lee metrics extending perturbations of the three-sphere.

The significance of the L^2-norm of the Weyl tensor is that it and the Euler characteristic (a topological invariant) span the set of renormalized curvature integrals (analogues of integral conformal invariants) in the sense of Albin. In higher dimensions the space of such invariants is of much higher dimension; by work of Chang–Qing–Yang, it essentially coincides with the dimension of the space of scalar conformal invariants whose integrals are conformally invariant. A recent breakthrough of Case–Khaitan–Lin–Tyrrell–Yuan gives an explicit algorithm for computing these invariants, and shows that actually the dimension of the space of conformal invariants is not so large as expected. More precisely, they gave a systematic construction of a large family of scalar conformal invariants which are divergences, bringing new attention to the open question associated to Alexakis' result mentioned above. Gover–Latini–Waldron–Zhang, using an argument similar to that of Chang–Qing–Yang, computed the renormalized energy of a Yang–Mills connection on a Poincaré–Einstein six-manifold, raising the possibility of also understanding the moduli space of such connections on more general vector bundles.

The existence question is also open and relevant in other Einstein-type situations. For example, Bamler classified the four-dimensional singularity models for the Ricci flow, motivating the desire to construct expanding gradient Ricci solitons which are asymptotic to a given four-dimensional Riemannian cone. Bamler–Chen developed a degree theory which is capable of producing such solitons in the case when the link of the cone is diffeomorphic to S^3 and the cone has nonnegative scalar curvature.

2.2 Q- and Q'-curvatures

A fundamental open problem in CR geometry is to classify integral scalar invariants of pseudo-Einstein manifolds. The total Q'-curvature and the integrals of scalar CR invariants are examples. More recently, Marugame and Takeuchi constructed, generalizing five-dimensional examples of Case–Gover, families of so-called $\mathcal{I}'$-curvatures that are not scalar CR invariants but whose integrals are invariants of pseudo-Einstein manifolds. It remains unclear if these three classes, together with total divergences, are the only integral scalar invariants.

On CR three-manifolds, the CR Paneitz operator is a CR invariant operator with many applications: its kernel is infinite-dimensional and contains the CR pluriharmonic functions, and it controls the CR Q-curvature and, equivalently, the logarithmic singularity of the Szegö kernel. Takeuchi has recently developed a very good understanding of the spectrum of this operator. On compact embeddable CR three-manifolds, the operator is nonnegative and its kernel is exactly the space of CR pluriharmonic functions. However, this need not be true in the noncompact setting; for example, on the Rossi spheres, it has infinitely many nonnegative eigenvalues counted without multiplicity.

In conformal geometry, there remains much to understand about the moduli space of metrics of constant Q-curvature; i.e. of positive smooth solutions of the semilinear PDE

$$P_{2k}u = \lambda u^{\frac{n+2k}{n-2k}} \tag{2.1}$$

on a compact n-manifold, where $2k < n$ and $P_{2k} \equiv (-\Delta)^k$ modulo lower-order terms. In the case $k = 1$, Brendle–Marques showed that the set of volume-normalized solutions to (2.1) need not be compact when $n \geq 25$; Khuri–Marques–Schoen showed that this dimensional assumption is sharp. In the case $k = 2$, Wei–Zhao showed that compactness also fails in dimensions $n \geq 25$. Very recently, Gong–Kim–Wei showed that this dimensional assumption is also sharp.

The question of the moduli space of solutions to (2.1) is also of great importance on manifolds with prescribed singularities; this generalizes the Loewner–Nirenberg problem of finding complete metrics of constant scalar curvature on $S^n \setminus \Sigma$ for some prescribed singular set. Very recently, Caju–Ratzkin–Santos gave a very detailed study of the moduli space of solutiosn to (2.1) on a finitely-punctured sphere when $k = 2$, generalizing work of Kusner–Mazzeo–Pollack and Mazzeo–Pollack–Uhlenbeck.

Cherrier and Escobar introduced the $k = 1$ analogue of (2.1) on manifolds with boundary, where now a suitable conformally invariant boundary condition involving the mean curvature of the boundary is also imposed. Most cases of this boundary version of the Yamabe problem were resolved by Mayer–Ndiaye. The Kazdan–Warner-type problem of prescribing, as functions, the scalar curvature of the interior and the mean curvature of the boundary is much less developed. Recent progress on this problem and its four-dimensional analogue has been obtained by Cruz Blazquez and Cruz Blazques–Delatorre.

Techniques used to study (2.1) also strongly influence the study of other semilinear PDEs. For example, Agostiniani–Bernardini–Borghini–Mazzieri recently used conformal techniques to characterize the rotationally symmetric solutions of $\Delta u = -n$ on ring-shaped domains in $\mathbb{R}^n$ under the additional assumption that the $\mathcal{H}^{n-1}$-measure of the set of maximum points is positive.

There is also growing interest in studying quasi-linear analogues of (2.1), with the focus so far on second-order equations whose leading-order term is given by the p-Laplacian. Recent work of Liu–Ma–Qing–Zhang has identified the relevant conformally invariant equation and studied its geometric applications. Most notably, their work has introduced new techniques from potential theory to conformal geometry, leading to a powerful new method to study the singular sets of these equations.

2.3 Scattering theory and nonlocal operators

As discussed in the Section 1, a choice of Poincaré–Einstein manifold (X^{n+1}, g_+) determines, via scattering theory, conformally covariant nonlocal operators on its conformal boundary $(M^n, [g])$. It also determines conformal invariants on submanifolds $\Sigma \subset M$ by solving the asymptotic Plateau problem—that is, by finding minimal submanifolds of X whose asymptotic boundary is Σ. A fundamental problem is the inverse scattering problem: To what extent do these invariants, viewed as conformal invariants of $(M, [g])$, determine the Poincaré–Einstein manifold (X^{n+1}, g_+).

Significant progress on the inverse scattering problem via renormalized invariants has been made recently. Graham–Guillarmou–Stepanov–Uhlmann showed that the renormalized lengths of g_+-geodesics asymptotic to the boundary determine g_+ near the boundary. Marx-Kuo proved the analogous statement for the renormalized areas of g_+-minimal surfaces.

2.4 Geometric problems on submanifolds in conformal and CR geometry

It remains an important open problem to obtain a systematic understanding of the invariants of submanifolds of conformal and CR manifolds, and hence to better understand their distinguished submanifolds. Progress in this direction has been made in multiple directions.

One way to try to construct global invariants of a submanifold Σ of a conformal manifold $(M, [g])$ is via the renormalized area of a singular Yamabe (or Loewner–Nirenberg) metric; i.e. via complete metric $\hat{g}$ of constant scalar curvature in the conformal class to the complement of Σ. Previous work of Gover–Waldron, building on the existence, uniqueness, and regularity of $\hat{g}$ obtained by many others, indicates that the renormalized volume of $\hat{g}$ is not a global invariant. This is because the *local* expansion of the volume element has no "nice" parity properties. Surprisingly, Kushtagi–McKeown recently showed that the renormalized volume *is* in fact invariant when $\dim \Sigma$ is odd, and thus produces interesting new examples of global conformal invariants of submanifolds in this dimensional parity. These invariants are new even in the case of knot embeddings $S^1 \hookrightarrow S^3$.

Another way to construct invariants of a submanifold $\Sigma \subset M$ is to look for a solution to the asymptotic Plateau problem of finding a minimal extension of Σ into a Poincaré–Einstein extension of M. For various applications, one wants to relax the assumptions on the extension of M. Recent work of Curry–Nandi considered the case of the boundary behavior of geodesics (i.e. solutions of the asymptotic Plateau problem with boundary a point) in a conformally compact but not necessarily asymptotically hyperbolic, establishing regularity results in the spirit of those for geodesics in asymptotically hyperbolic manifolds.

There are also distinguished submanifolds in conformal and CR geometry. For curves in conformal manifolds, these are known as conformal geodesics; they are described by a third-order ODE. For curves in CR manifolds, these include chains and horizontal circles; they are described by a fourth-order ODE. Recent work of Herfray–Fine and Matsumoto realizes these distinguished curves as boundaries of proper harmonic maps from the hyperbolic plane into a Poincaré–Einstein manifold (for conformal circles) or complex hyperbolic space (for horizontal circles in the standard CR sphere).

2.5 Fully nonlinear equations in conformal geometry and their a priori estimates

The study of fully nonlinear equations in conformal geometry has mostly centered around the σ_k-curvatures introduced by Viaclovsky, though there is growing interest in other such equations, such as those proposed by Chang–Fang–Graham involving the renormalized volume coefficients. A key difficulty is always to deduce C^2-estimates. This difficulty is shared with fully nonlinear problems in other settings, and has lead to a productive synergy between different fields.

A key property of the σ_2-curvature, which is not shared with the σ_k-curvatures for $k \geq 3$, is that it is variational. This property allowed Ge–Wang, Guan–Wang, and Sheng–Trudinger–Wang to solve the 2-Yamabe problem for the σ_2-curvature; i.e. to construct metrics, by minimizing in the positive cone Γ_2^+, for which $\sigma_2 = 1$. Recent work of Ge–Wang shows that one can in fact do this by minimizing in Γ_1^+. This result is analogous to previous work of Ge–Wang on the equation $\frac{\sigma_2}{\sigma_1} = 1$ by minimization in Γ_1^+.

More generally, one wonders whether there are smooth solutions to the quotient equation $\frac{\sigma_k}{\sigma_\ell}$ in a suitable elliptic cone. Interior regularity of solutions of the corresponding Hessian equation $\frac{\sigma_k(D^2u)}{\sigma_\ell(D^2u)} = f$ on $\mathbb{R}^\ltimes$ is likewise an important issue. Lu recently proved interior C^2-regularity in the case $k = n$ and $\ell \geq n-2$, but exhibited examples without interior C^2-estimates when $k = n$ but $\ell < n-3$.

The minimization problem above can be formulated as a sharp fully nonlinear Sobolev inequality. A natural question, inspired by work of Bianchi–Egnell, is whether this Sobolev inequality is stable. Frank–Peteranderl proved such a stability result in $\mathbb{R}^n$, representing the first time a stability result has been proven for a fully nonlinear Sobolev inequality. A key difficulty here is the lack of a concentration compactness principle for fully nonlinear equations. Interestingly, this was overcome using the work of Ge–Wang on the quotient equation σ_2/σ_1.

Given the many applications of the Loewner–Nirenberg problem, it is interesting to pose also the k-Loewner–Nirenberg problem; i.e. to ask for complete metrics of constant σ_k-curvature with prescribed singular set. Work of Li–Nguyen established the existence of Lipschitz viscosity solutions in some situations, and showed that interior C^1-regularity cannot be expected in general. Duncan–Nguyen were able to significantly generalize their work, with the surprising observation of a simple sufficient condition which guarantees interior regularity.

One route towards the construction of metrics with constant σ_k-curvature is via a fully nonlinear analogue of the Yamabe flow. To use this flow, one must understand its self-similar solutions. Espinal–Saez, generalizing work of Daskalopoulos–Sesum from the case $k = 1$, recently carried out a detailed analysis of the conformally flat self-similar solutions in $\mathbb{R}^n$. In particular, they characterized when solutions exist, showed that solutions are rotationally symmetric, and described the asymptotic behavior of solutions.

In the context of hypersurfaces in Euclidean or Minkowski space, the σ_k-curvature flow is also an important tool. A notable recent advance by Wang–Xiao establishes long-time existence for this flow starting from a spacelike hypersurface satisfying certain conditions in Minkowski space; moreover, they established that the flow converges to a self-expander after a suitable rescaling.

3 Presentation Highlights

The research presentations in our workshop consisted of 20 talks, each lasting 50 minutes, with an additional 10 minutes for questions. We prioritized in-person presentations to encourage interaction between the speakers and other participants during meals and breaks. Despite this, and due to the great importance of the results, we asked Rupert Frank, who was unable to attend in-person due to last-minute circumstances, to present his talk online. We were ultimately happy with this decision, as it led to numerous questions and discussions involving both in-person and online participants. We include some comments about and potential improvements to the hybrid format in Section 8.

As mentioned, there were 20 talks in our workshop, the first two of which were introductory. Section 2 discusses the significance of the results of these talks in context. Here we name the talks in the order given:

- **Jie Qing** (UC Santa Cruz) gave an introductory talk on conformal geometry which discussed Q-curvature, its quasi-linear analogue involving the p-Laplacian, and their applications to locally conformally flat manifolds.
- **Kengo Hirachi** (University of Tokyo) gave an introductory lecture on CR geometry which discussed the Q'- and $\mathcal{I}'$-curvatures.
- **Siyuan Lu** (McMaster University) spoke about interior C^2-estimates for Hessian quotient equations.
- **Jonah Duncan** (Johns Hopkins University) spoke about recent progress on the k-Loewner–Nirenberg problem.
- **Yueh-Ju Lin** (Wichita State University) spoke about renormalized curvature integrals on Poincaré–Einstein manifolds.
- **Andrew Waldron** (University of California, Davis) spoke about the renormalized Yang–Mills energy.
- **Rupert Frank** (University of Munich) spoke about the stability of the σ_2-Sobolev inequality on the sphere.
- **Sergio Cruz Blazquez** (Universidad de Granada) spoke about the Kazdan–Warner problem on manifolds with boundary.
- **Ling Xiao** (University of Connecticut) spoke about the σ_k-curvature flow in Minkowski space.
- **Jesse Ratzkin** (Universität Würzburg) spoke about the moduli space of constant Q-curvature metrics on punctured spheres.
- **Eric Chen** (UC Berkeley) spoke about expanding Ricci solitons asymptotic to cones.
- **Stephen McKeown** (University of Texas at Dallas) spoke about volume renormalization of singular Yamabe metrics.
- **Jared Marx-Kuo** (Rice University) spoke about determining the metric from minimal surfaces in asymptotically hyperbolic spaces.
- **Yoshihiko Matsumoto** (Osaka University) spoke about the relation between proper harmonic maps and horizontal circles in the CR sphere.
- **Sean Curry** (Oklahoma State University) spoke about boundary behavior of geodesics in conformally compact manifolds.
- **Juncheng Wei** (Chinese University of Hong Kong) spoke about compactness for the Q-curvature problem.
- **Chiara Bernardini** (Università degli Studi di Trento) spoke about Serrin-type results for ring-shaped domains in $\mathbb{R}^n$ via a conformal splitting technique.
- **Yuya Takeuchi** (University of Tsukuba) spoke about spectral analysis of the CR Paneitz operator.
- **Mariel Saéz** (Pontificia Universidad Catolica de Chile) spoke about the existence and classification of k-Yamabe gradient solitons.
- **Guofang Wang** (Freiburg University) spoke about the σ_k-Yamabe problem and Sobolev inequalities.

We would like to emphasize both the quantity and the quality of the talks by a diverse group of speakers. Half of the speakers were postdocs or young researchers, there were a significant number of talks by female speakers, and our speakers represented numerous countries and continents. We would like to highlight not only the quality of the research presented, but also the ability of all participants to communicate clearly and passionately, which contributed significantly to the success of our workshop.

4 Empowering Young Researchers

Our workshop successfully highlighted and encouraged the active participation of young researchers, creating a vibrant platform for them to present their recent research findings. With a dedicated focus on nurturing early-career talent, our workshop featured 13 (out of 20) talks exclusively for young scientists, allowing them to showcase their innovative ideas to a diverse audience of peers and senior experts. Many participants expressed their appreciation for the opportunity to gain visibility for their work and to receive valuable feedback from senior professionals in their fields. Some of the discussions and conversations continued during lunchtime and extended into the working group sessions (see Section 5). It also enabled young researchers to forge meaningful connections and explore potential collaborations. Additionally, the two panels included discussions of career development and funding opportunities, providing them with practical tools to navigate their academic and professional journeys (see Sections 6 and 7 for further discussion). The enthusiasm and engagement of young attendees was evident, as they actively participated in discussions and demonstrated their keen interest in advancing knowledge. The workshop thus played a crucial role in boosting their confidence, equipping them with essential skills, and inspiring them to pursue impactful research. The overwhelming positive response highlights the importance of continued investment in platforms that empower the next generation of scholars.

The 13 speakers mentioned above and their career status are:

- Siyuan Lu — Associate Professor
- Yoshihiko Matsumoto — Associate Professor
- Ling Xiao — Associate Professor
- Jesse Ratzkin — Lecturer
- Eric Chen — Assistant Professor
- Sean Curry — Assistant Professor
- Yueh-Ju Lin — Assistant Professor
- Stephen McKeown — Assistant Professor
- Yuya Takeuchi — Assistant Professor
- Chiara Bernardini — Postdoc
- Sergio Cruz Blazquez — Postdoc
- Jonah Duncan — Postdoc
- Jared Marx-Kuo — Postdoc

5 Scientific Progress Made

In addition to scheduled talks, our workshop included two one-hour periods dedicated to establishing working groups and two panels aimed at supporting young researchers and underrepresented minorities. Here we discuss the working groups; the next two sections discuss each of the panels.

Working groups met during the last speaking slot on Tuesday and Thursday afternoon, though there were also many informal meetings happening during coffee and meal breaks, and after dinner. We did not carefully track which participants joined which groups, or how many groups there were, but we did make full use of all available space to the point that multiple groups were meeting in the main seminar room. We are aware of groups that met to discuss ongoing research projects and also groups that formed to discuss new projects arising from questions raised during the talks. Some of the more senior experts circulated between groups to answer questions from younger participants regarding their research topics.

6 Panel discussions to address career problems for young people

The first panel in our workshop, held on Monday evening, was a panel discussion aimed at addressing career challenges faced by young researchers. The panel included established senior researchers representing a diverse range of locations and experiences. Young attendees benefited significantly from the discussion on job applications, which covered strategies for crafting compelling CVs and cover letters, as well as preparing for interviews. The panel explored the review process of a job applicant's file from the department's perspective, focusing on key considerations and evaluation criteria. They discussed how factors such as qualifications, experience, and alignment with departmental objectives influence decision-making.

The panel also discussed the unique characteristics of math departments across various types of institutions, ranging from liberal arts colleges to research-oriented universities. They explored how the mission and priorities of each type of institution shape their hiring objectives. It was noted that these differences significantly impact how candidates should prepare their applications and approach interviews. Tailoring application materials and interview responses to align with the institution's specific mission and expectations is essential for success.

The panel discussed the comparison between job markets in Europe and North America, including insights into regional differences in opportunities, work culture, and hiring practices. Panelists emphasized the importance of networking and mentorship, encouraging young researchers to actively seek connections that could open doors to new opportunities. Attendees appreciated the chance to ask panelists direct questions and gain personalized advice tailored to their specific goals. The vibrant Q&A session fostered a collaborative atmosphere where concerns were addressed openly and constructively.

The panel also shared their experiences with the two-body problem and the challenges of achieving work-life balance in academic careers. They discussed the difficulties faced by dual-career couples in securing positions within the same geographic area, often requiring significant compromises or creative solutions. Female researchers in particular, highlighted the unique obstacles they face in balancing family responsibilities with the demands of their careers, including managing childcare, meeting research deadlines (tenure clock issues), and navigating societal expectations. For those with young children, the panel emphasized how travel for conferences and research can add another layer of complexity, requiring careful planning and support networks. These candid discussions underscored the need for institutional policies that support work-life balance, such as flexible schedules, parental leave, and dual-career hiring programs, to foster a more inclusive and equitable academic environment.

In addition to job market insights, the panel delved deeply into the topic of funding applications, with a particular focus on research and teaching/education experiences. Panelists shared expert advice on crafting competitive proposals, including writing effective statements of purpose and addressing the broader impacts criterion. Good examples of outreach programs and educational components were also discussed, inspiring young researchers to integrate community engagement and public education into their projects. Attendees learned how to design initiatives that could enhance their proposals while contributing to societal advancement. Panelists highlighted the importance of persistence and collaboration in securing funding, reassuring participants that challenges are a normal part of the process. The panel also talked about funding requirements at different universities and career stages. These discussions equipped young researchers with actionable strategies and boosted their confidence in applying for grants. Many attendees expressed that the panel not only clarified uncertainties but also motivated them to approach their career and funding goals with renewed determination.

7 Panel discussions on the importance of EDI in mathematical education and strategies in making a change

The second panel in our workshop, held on Tuesday evening, was a panel discussion on the importance of Equity, Diversity, and Inclusion (EDI) in mathematical education across colleges, emphasizing how fostering inclusive environments can improve both teaching and learning experiences. The opportunities and challenges that different types of institutions face in implementing EDI initiatives were discussed, explored strategies for securing resources and support for EDI efforts, and discussed the importance of collaborating with other departments. In particular, panelists highlighted specific grants and fellowships from governmental bodies, private foundations, and professional organizations that focus on promoting diversity and inclusion in education. Some participants shared personal success stories of activities they had organized, such as mentorship programs, diversity workshops, and outreach events, which helped foster a more inclusive environment within their institutions. These shared experiences provided valuable insights into the practical steps that can be taken to overcome barriers and make meaningful progress in advancing EDI in mathematical education.

Panelists also shared their experiences with students who, despite being admitted to college, struggle to catch up with the demands of college-level courses due to weak foundational backgrounds. Educators are increasingly faced with the challenge of teaching students with widely varying levels of preparedness within the same class, creating a difficult balancing act between providing sufficient challenge and ensuring the material remains understandable. This disparity often makes it challenging to implement effective teaching strategies, as instructors must find ways to support students who may not be ready for the rigorous content while also keeping more advanced learners engaged and appropriately challenged. The panel discussed several potential solutions to address these challenges, such as by offering preparatory or remedial courses, implementing differentiated instruction within the classroom, and incorporating peer mentoring programs. They also emphasized the use of technology, such as online resources and learning management systems, to provide personalized learning pathways and additional practice for students who need it. Instructors were encouraged to adopt formative assessment techniques to regularly gauge student progress, allowing for early identification of gaps in understanding and timely interventions. Finally, the panel recommended fostering a growth mindset among students, encouraging them to view challenges as opportunities for growth, which could help build resilience and motivation among those who initially struggle with the material.

8 Comments on the Hybrid Format

We believe that in-person participation adds more value for all participants due to the possibility of further interaction during meal times and breaks. For this reason, we stuck to our commitment to include a large number of early career participants. That said, we recognize the importance of online participation to greatly broaden the reach of a BIRS workshop by allowing for a significantly larger number of participants, and also to allow people to participate who cannot travel to Banff for various personal or professional reasons. There were two issues that arose during the talks that we would like to raise as potential areas for future improvement.

- From the perspective of the in-person organizers the speakers, it was unsettling to not know who was attending a talk virtually. For example, it frequently becomes necessary to reduce the scope and content of the talks, and without knowing who is listening remotely, it is hard to properly adjust comments to best suite the audience. We suggest including on the screen in the back of the room a list of participants in the Zoom call. Alternatively, maybe having an extra laptop given to the session moderator that is connected to the Zoom call so that they can relay this information to the speaker and also see allow for online participants to add questions via chat.

- The online participants were able to follow slide talks talks without difficulty. However, some online participants found that, while the automatic framing worked very well, the lighting on the board seemed insufficient, to the point that it was not uncommon that online participants could not read what had been written there.

Finally, we remark that we noted a high level of online participation amongst those who signed up to participate that way. There were also many instances where discussions were held between speakers and online participants during the question period and the ensuing coffee break. We also had remote participation during the working group period, though this was facilitated via participant laptops rather than BIRS equipment. We did not know whether there was equipment available outside of the lecture rooms for virtual participation in working groups, but do not think that this is truly necessary.