Cosmology from entanglement

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BIRS

Jun 2023

w/ S. Antonini, B. Swingle *[2306.xxxxx]*

Context

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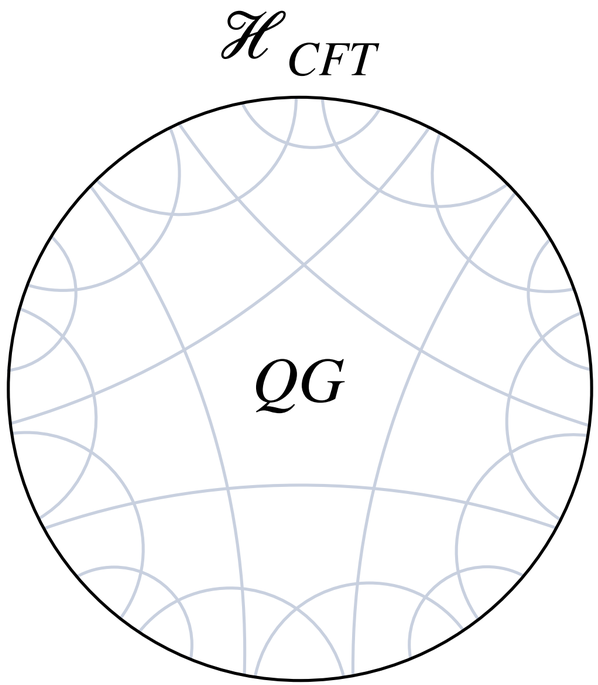

$$\mathscr{H}_{QG} \equiv \mathscr{H}_{CFT}$$

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- Some proposals exist for eternally inflating de Sitter space. No clear non-perturbative understanding.

[Strominger] [Susskind]

- Other cosmologies still less understood. Some proposals also exist, for big-bang/big-crunch AdS cosmologies.

[van Raamsdonk et al.]

Outline

- I will propose a way to describe states of big-bang/big-crunch cosmologies as microstates of the CFT.

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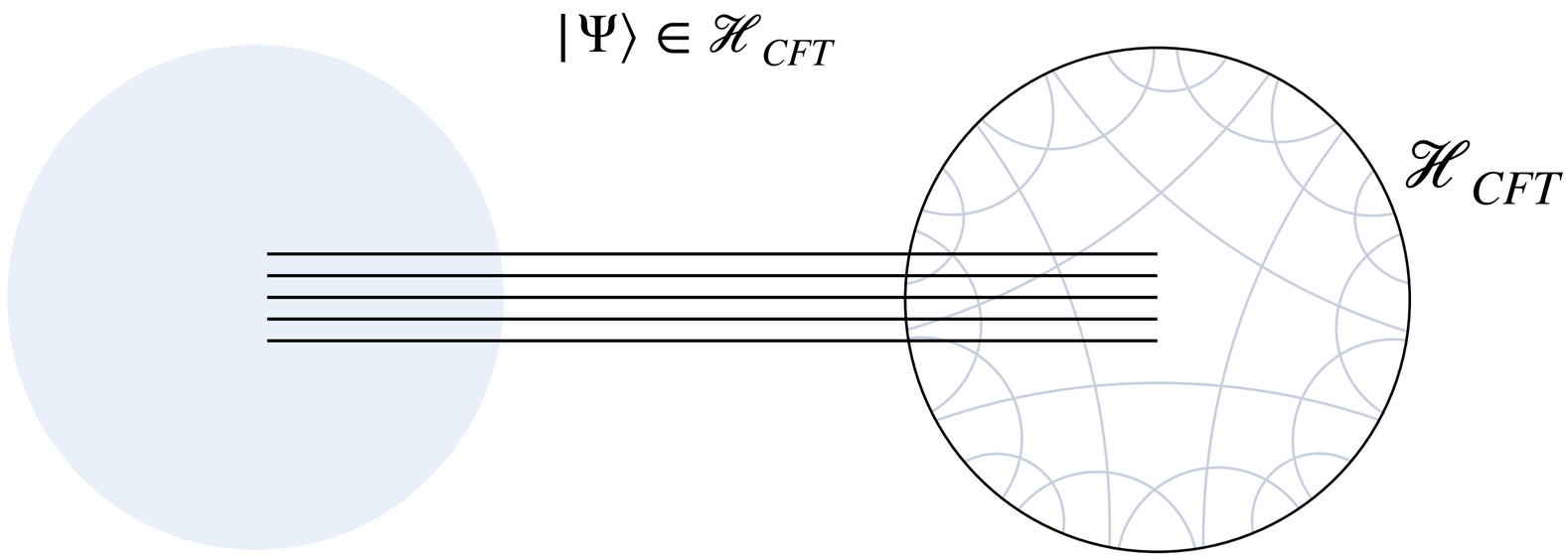

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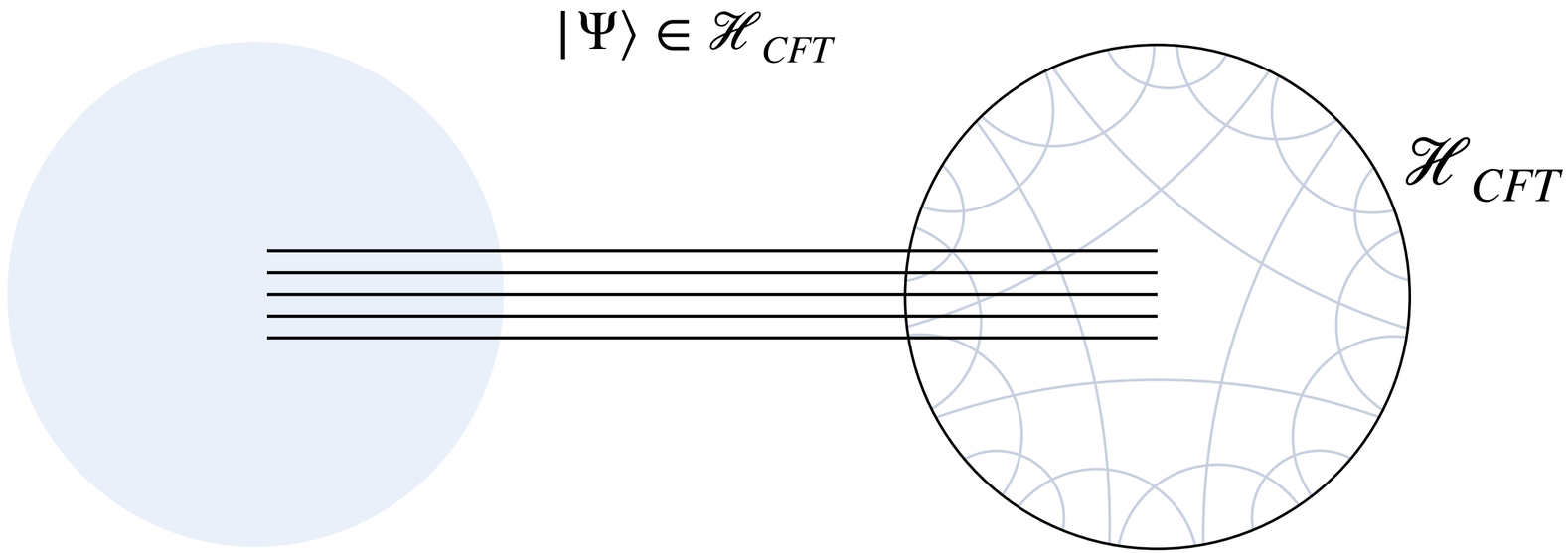


- The idea is to entangle the closed cosmology to some holographic system, which defines an `observer'. The holographic description of the cosmology becomes that of what the observer can see.

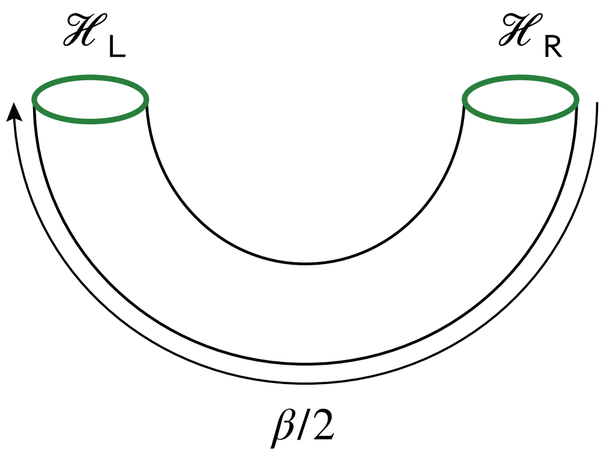
- The observers in the cosmology are encoded in the form of an entanglement island.

[Penington] [Almheiri et al.]

Cosmology as an entangled microstate

- A standard way to prepare entangled microstates is to use the Euclidean CFT path integral.

$$|TFD\rangle = \frac{1}{\sqrt{Z(\beta)}} \sum_n e^{-\beta E_n/2} |E_n\rangle_{\mathsf{L}}^* \otimes |E_n\rangle_{\mathsf{R}}$$



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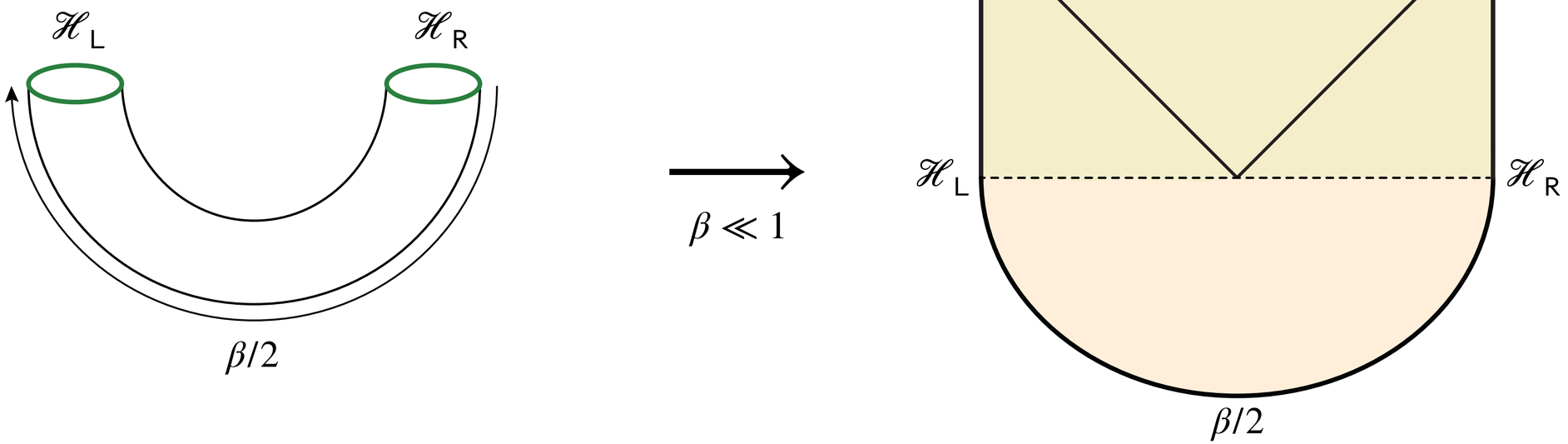
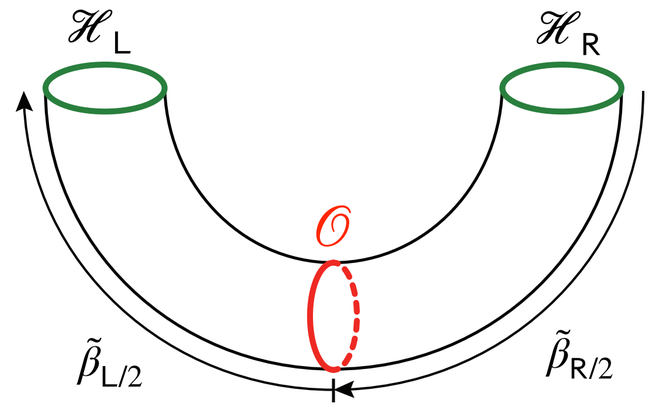


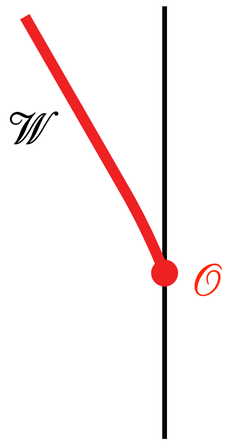
Cosmology as an entangled microstate

- A different microstate can be prepared by inserting an operator $\mathcal{O}$ in between the left/right Euclidean evolutions. The matrix elements of the operator enter as the coefficients of the state.



$$|\Psi_{\mathcal{O}}\rangle = \frac{1}{\sqrt{Z_1}} \sum_{n,m} e^{-\tilde{\beta}_L E_n/2 - \tilde{\beta}_R E_m/2} \mathcal{O}_{nm} |E_n\rangle_L^* \otimes |E_n\rangle_R$$

- Among semiclassical operators $\mathcal{O}$, the simplest choice is an operator which creates a spherical thin domain wall of matter particles.



$$I[X] = -\frac{1}{16\pi G}\int_X (R - 2\Lambda) + \frac{1}{8\pi G}\int_{\partial X} K + I_{matter} + \int_{\mathscr{W}} \sigma$$

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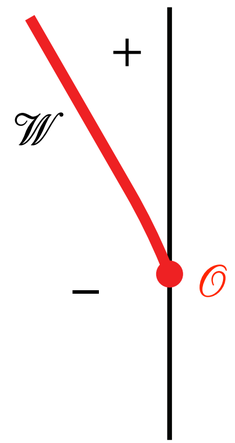

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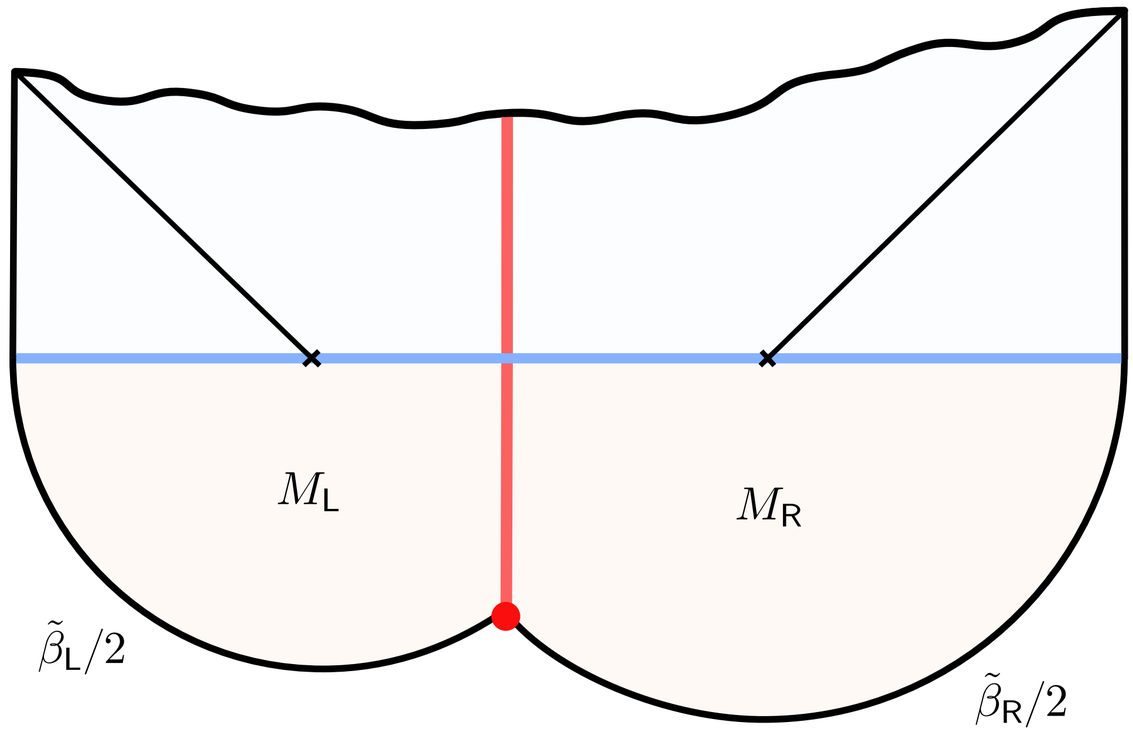
- The thin shell propagates as a spherical 'particle' of mass m and backreacts on the spacetime. The solution to Einstein's equations can be found exactly by a cut-and-glue procedure:

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} \qquad \leftrightarrow \qquad \begin{cases} h^+_{ab} - h^-_{ab} = 0 \\ \\ K^+_{ab} - K^-_{ab} = 8\pi G\sigma \end{cases}$$

Cosmology as an entangled microstate

- For $\tilde{\beta}_{\mathrm{L},R} \lesssim 1$, the bulk semiclassical state dual to $|\Psi_{\mathcal{O}}\rangle$ is the initial data on a long ER bridge supported by the thin shell.



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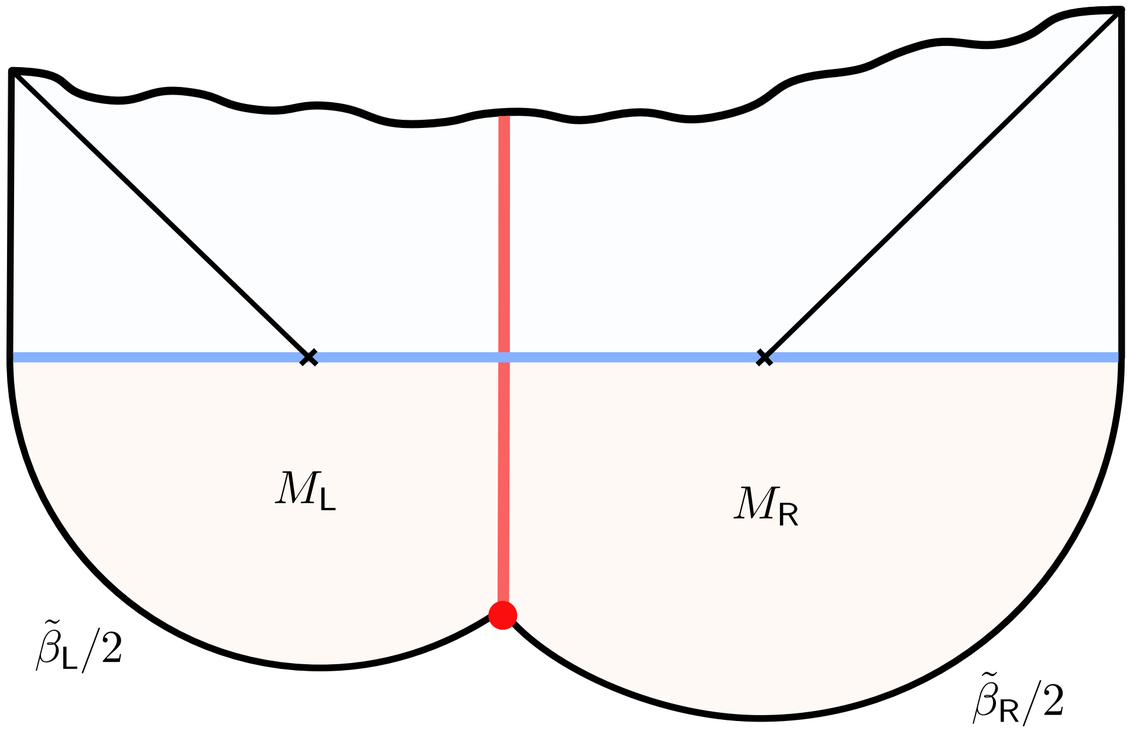

Physical temperatures:

$$\beta_R = \tilde{\beta}_R + \Delta\tau_R$$

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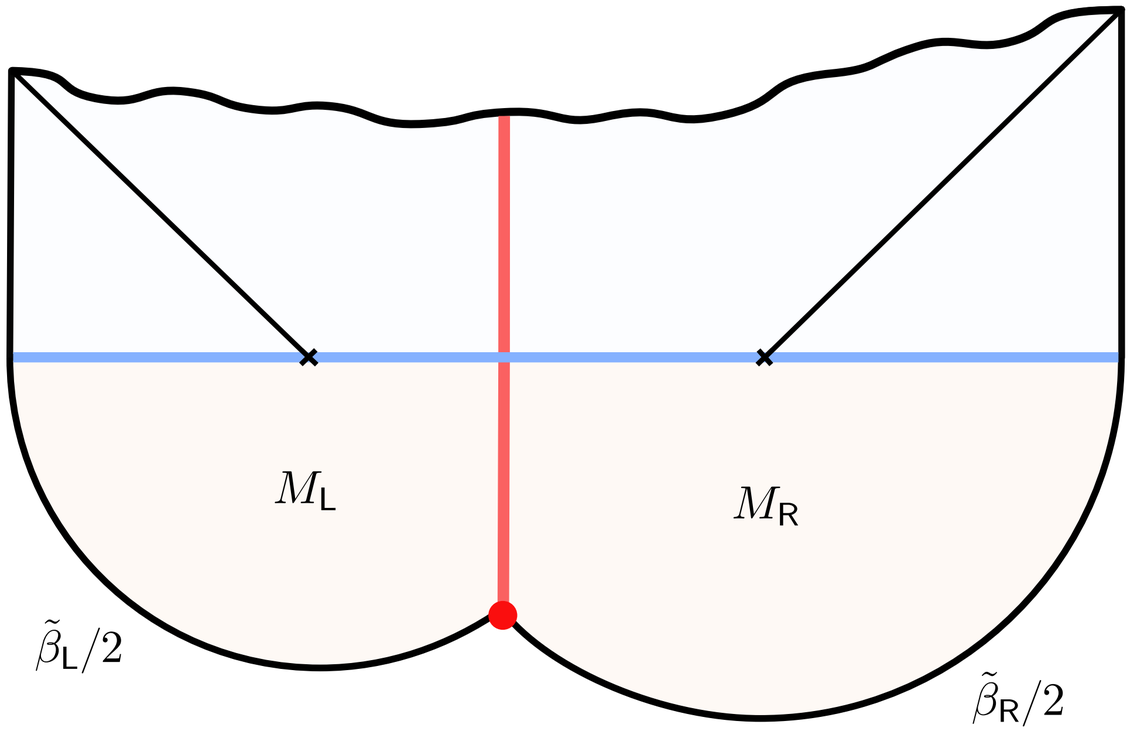

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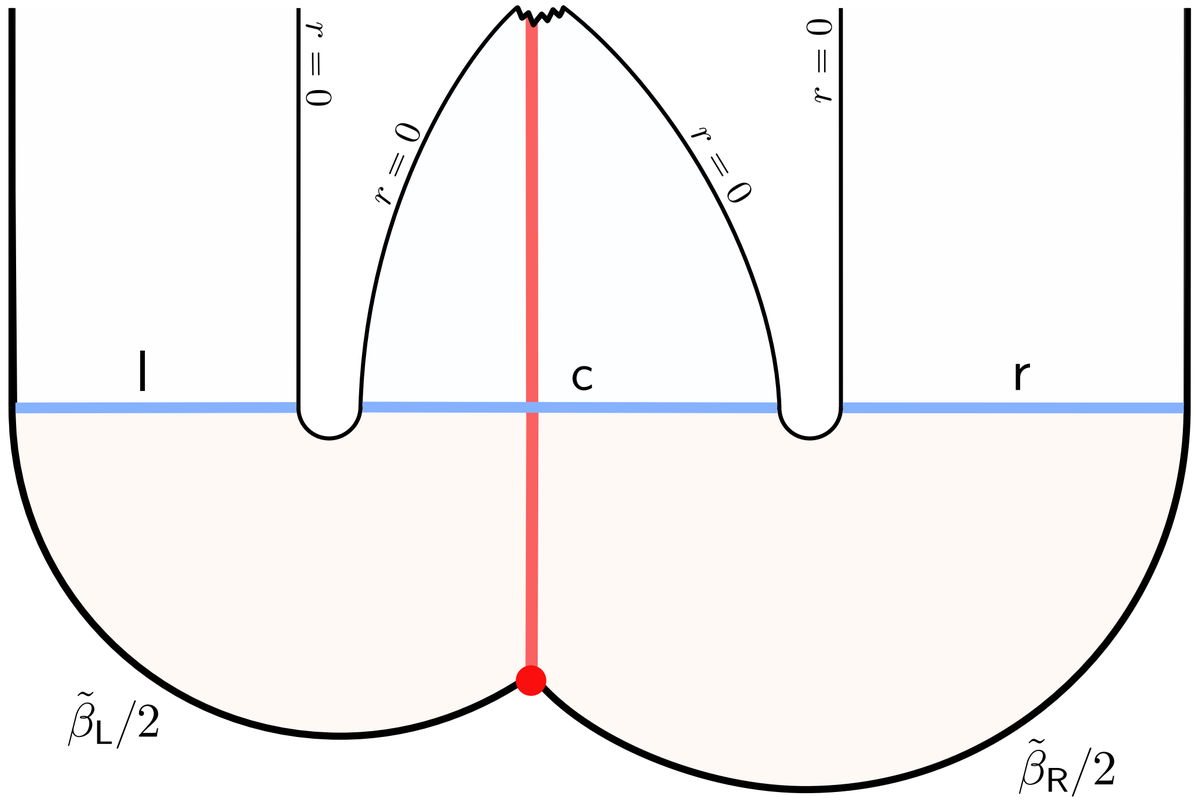
- The mass of the operator m can take any value given the physical masses of the two black holes $M_{\mathrm{L,R}}$. The infinite family $|\Psi_m\rangle$ causes sharp `bag-of-gold' paradox for neutral black holes.

[Balasubramanian, Lawrence, Magan, M.S.]

Cosmology as an entangled microstate

- For $\tilde{\beta}_{L,R} \gtrsim 1$, the CFT confines and the dual state consists of the initial data on two global AdS slices, and a third closed AdS cosmology.



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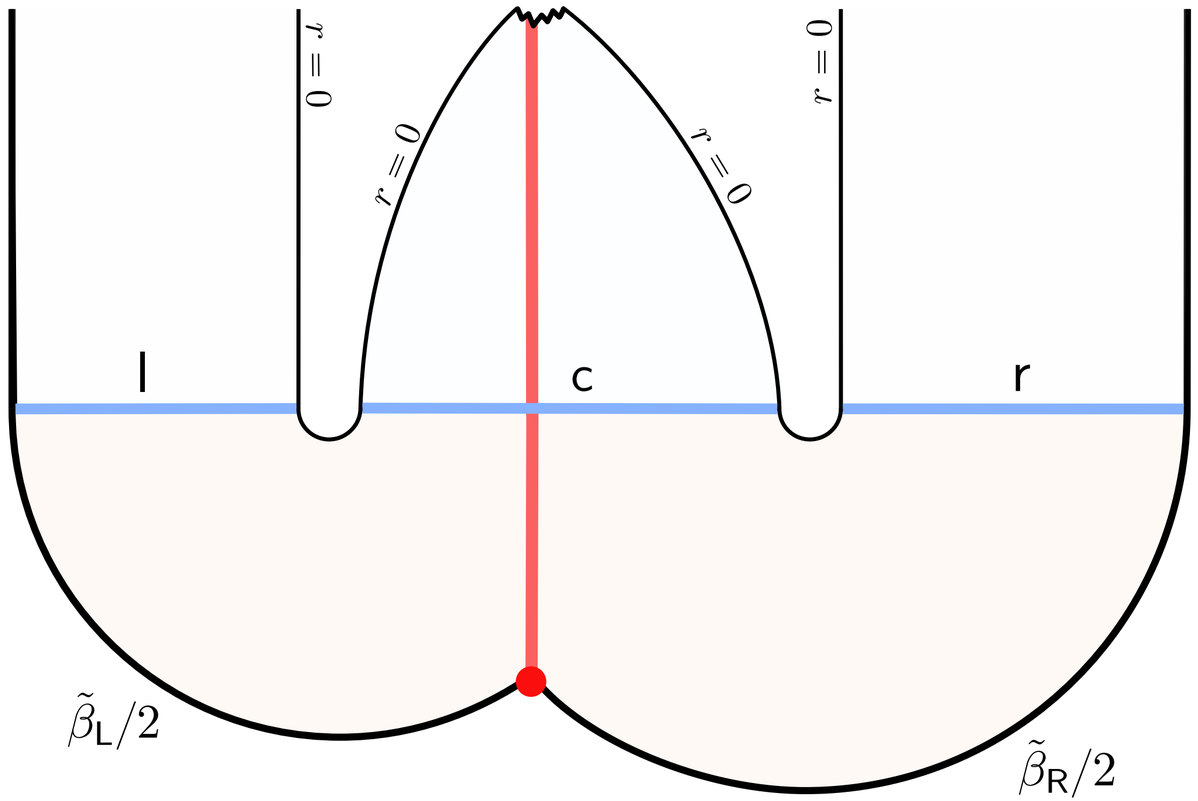

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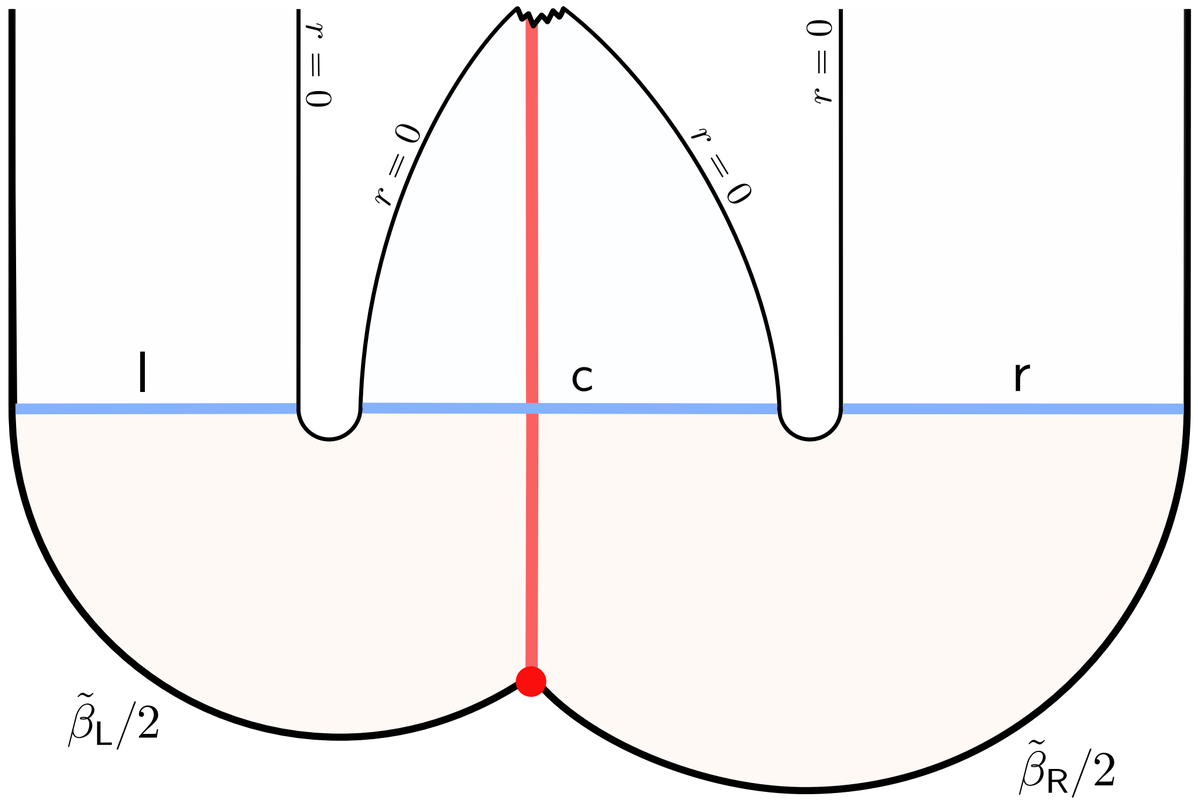

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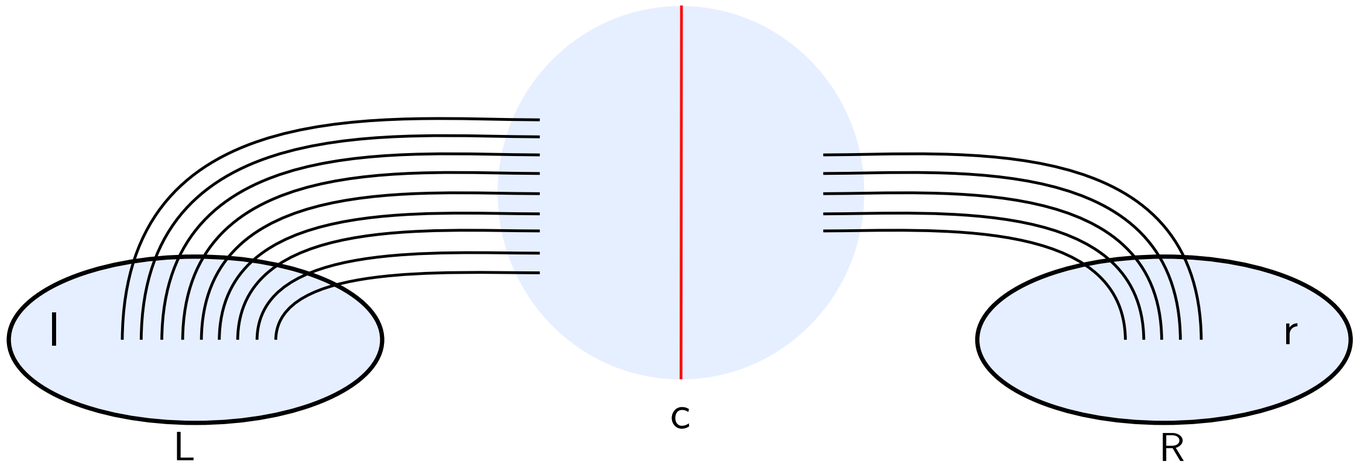
- The cosmology develops a big-crunch singularity towards the future. It corresponds roughly to two FRW AdS cosmologies. Topologically, the spatial slices of the cosmology are closed (spheres).

Cosmology as an entangled microstate

- The cosmology is ‘attached’ to the AdS regions along the entanglement lines of the quantum fields. It partially purifies the thermal gases of particles of the l, r AdS regions.



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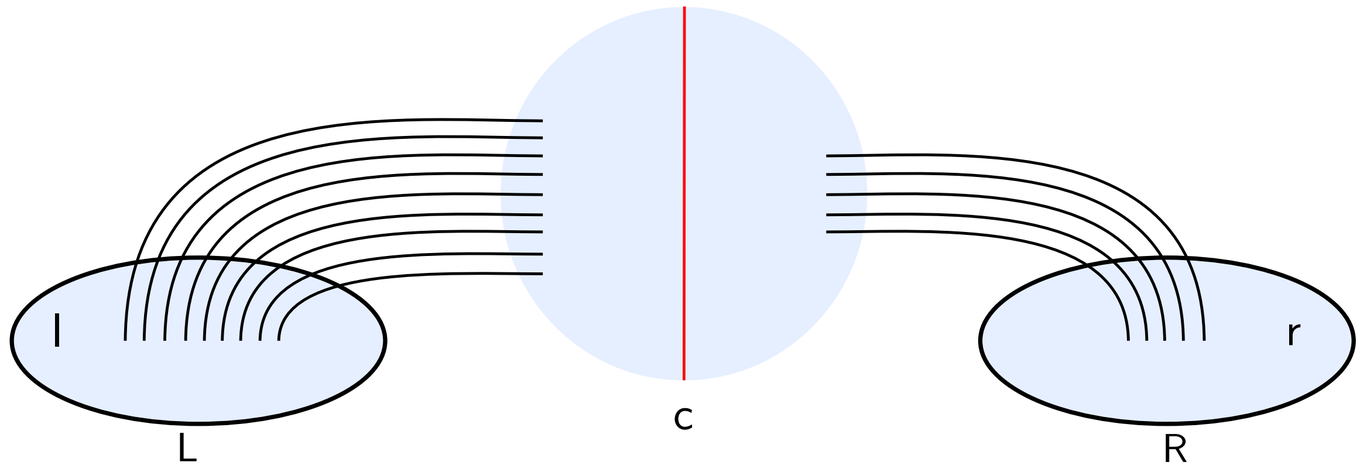

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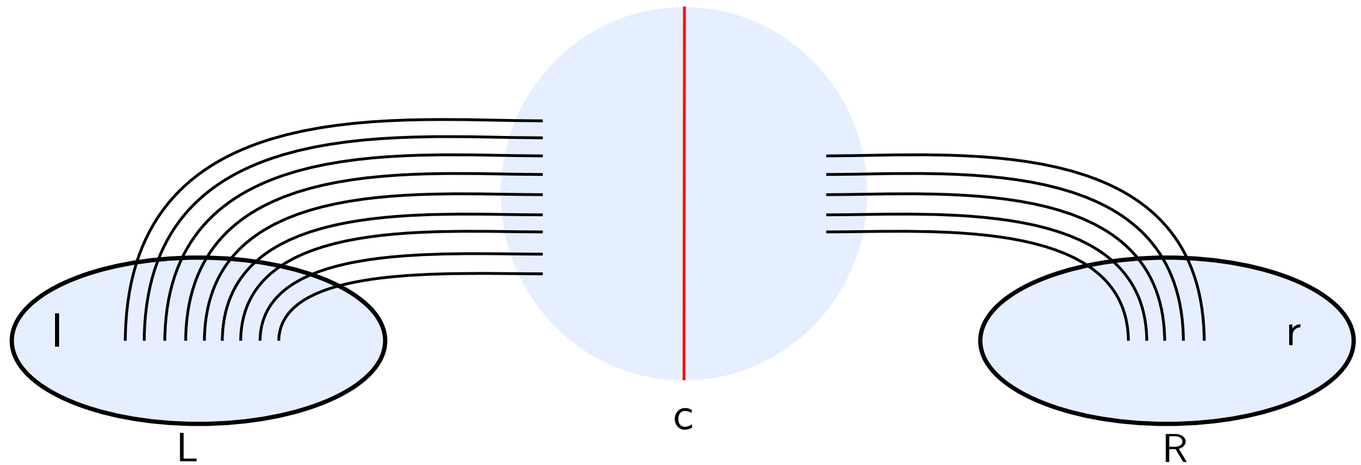

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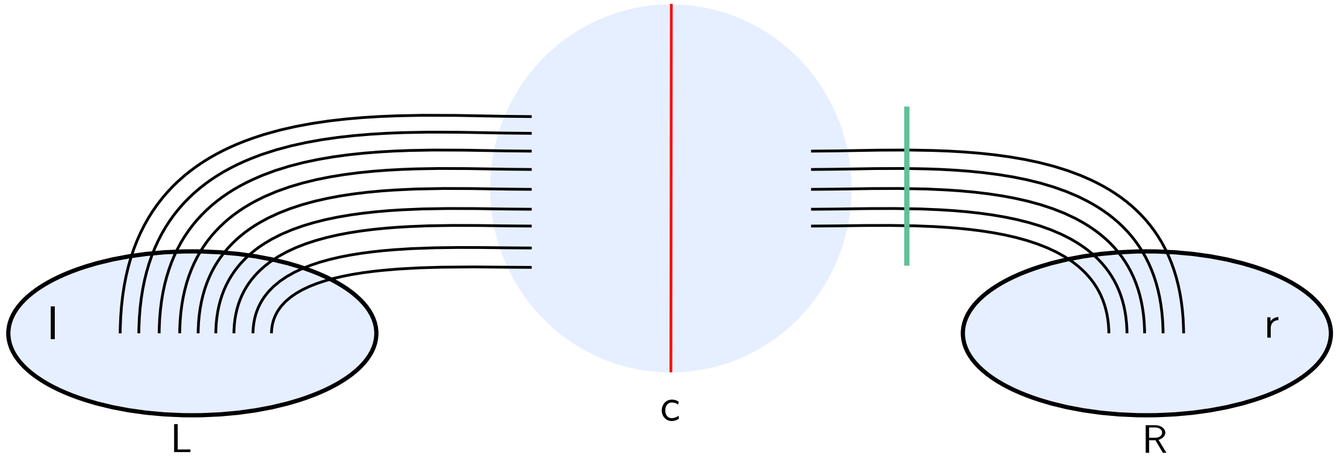
- *Disclaimer*: this model is not realistic, but it opens up the possibility of creating more realistic states, such as those which incorporate initial accelerated expansion. [van Raamsdonk, Swingle, …]

Cosmology in the island

- The entanglement entropy of CFT L is given by the number of EPR pairs cut by the green line.



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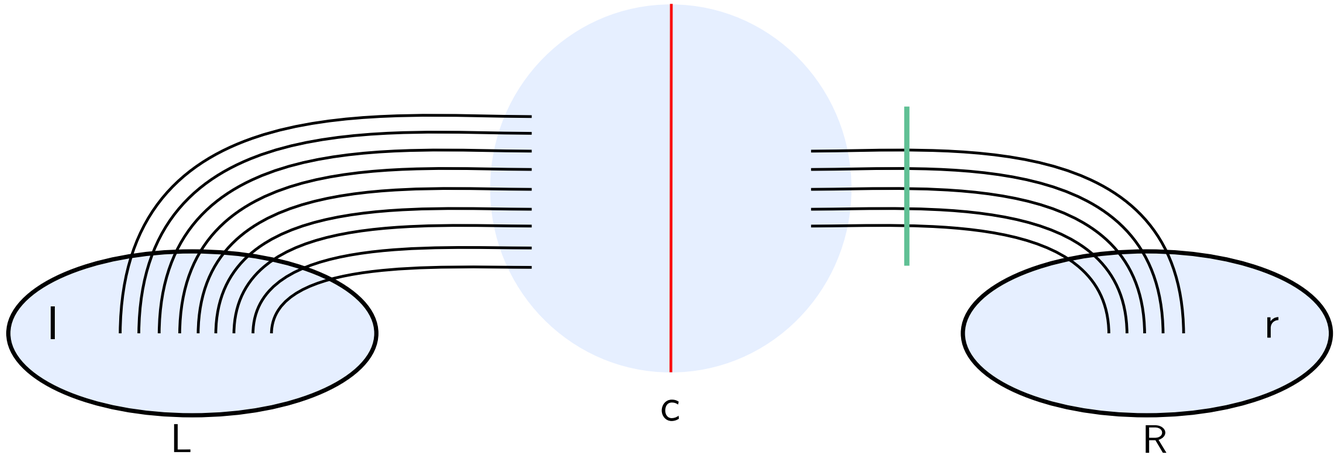
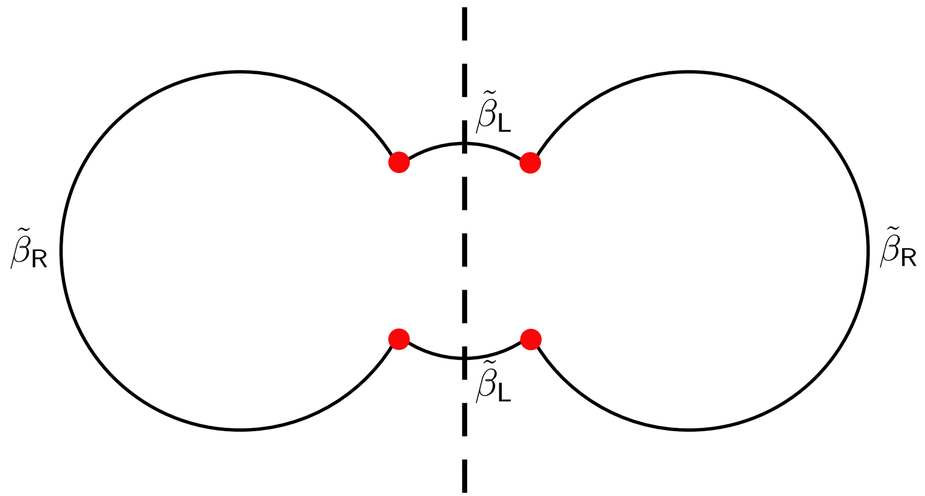


$$S(\rho_L) \approx S(\rho_r) = S(\rho_{l \cup c})$$

- This is the so-called `island formula' for the cosmological microstate. In this context it can be derived without coupling the CFT to an external system, avoiding potential issues with the mass of the graviton.

- To compute the entanglement spectrum, we use CFT path integrals over replicated manifolds.

$\mathrm{Tr}(\hat{\rho}_{\mathrm{L}}^2) =$



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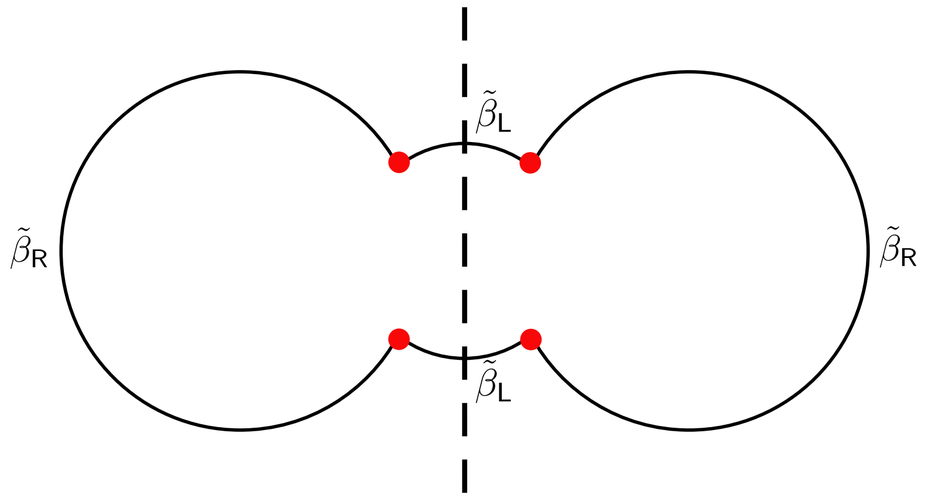

- The rule is to fill in the bulk for the Renyis, an analytically continue the replica-symmetric saddlepoints $X/\mathbf{Z}_n$ off-shell, to get

$$S(\rho_{\mathrm{L}}) = -\lim_{n\to 1^+} \partial_n \log \mathrm{Tr}\rho_{\mathrm{L}}^n$$

[Gibbons, Hawking]

[Lewkowycz, Maldacena]

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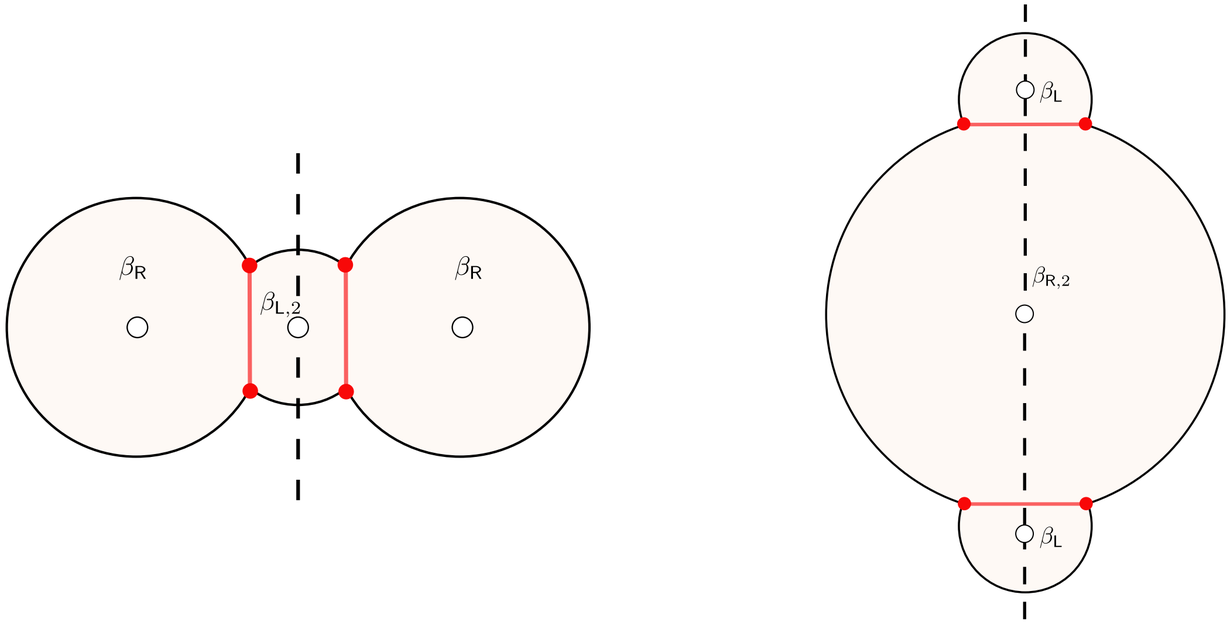
- There are two saddlepoints contributing to the purity. One of them has the interpretation of a `replica wormhole'.

[Penington, Shenker, Stanford, Yang]



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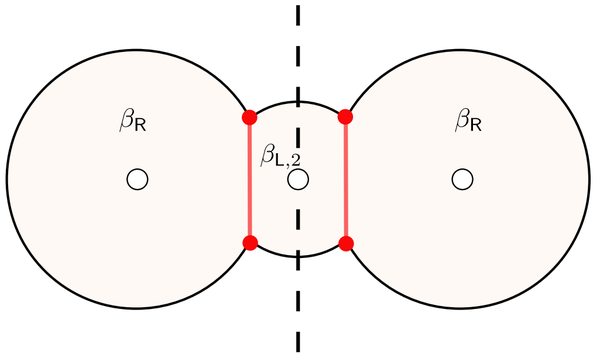

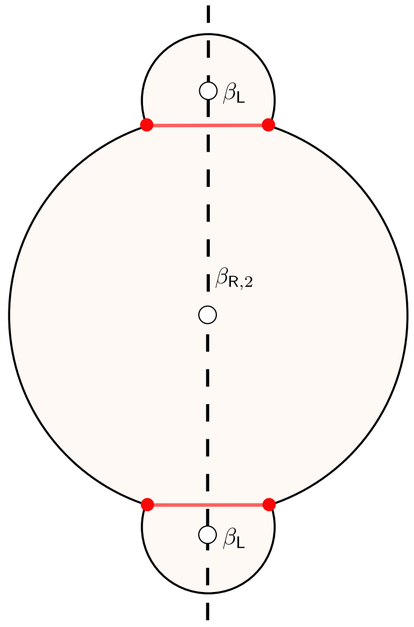

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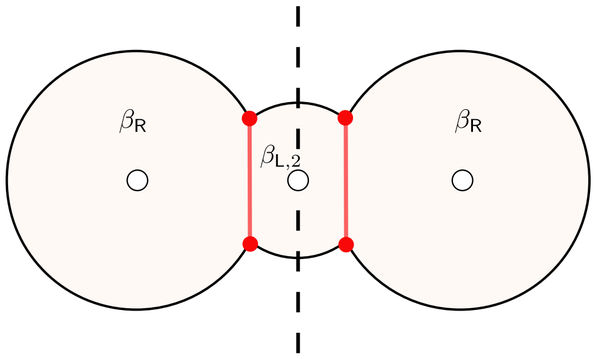

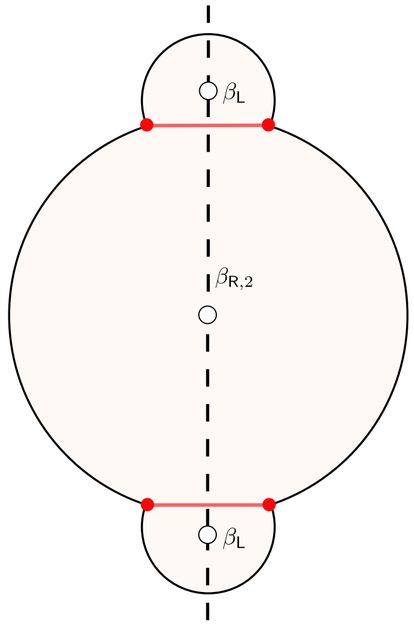

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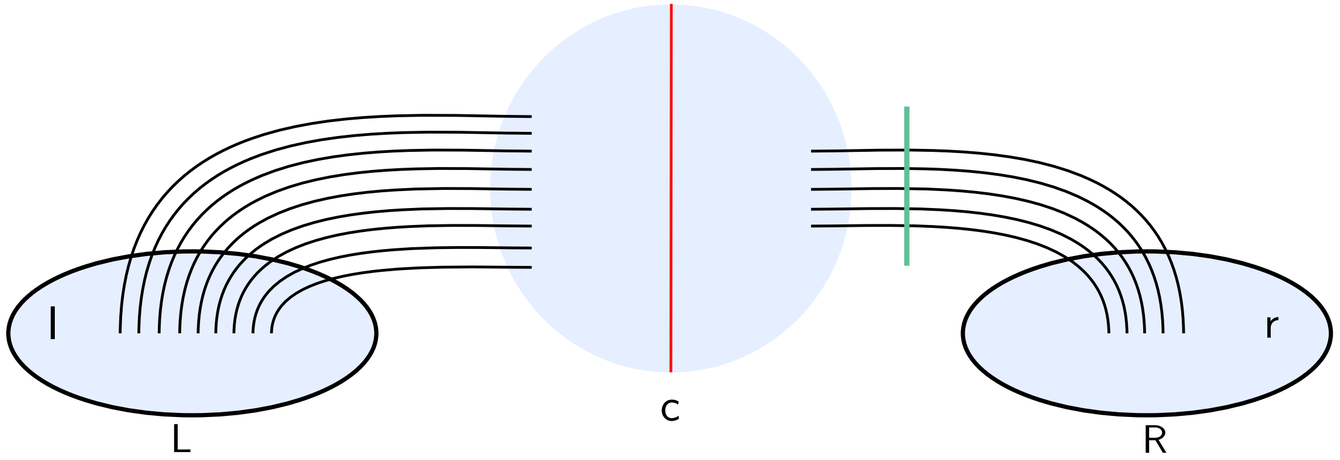
- This gives the island formula for the purity of the CFT L. If $\beta_L \ll \beta_R$

$$\boxed{S_2(\rho_L) \approx S_2(\rho_r) = S_2(\rho_{l\cup c})}$$

Cosmology in the island

- For the entanglement entropy, the calculation is a little more involved, but it is a standard FLM replica trick:



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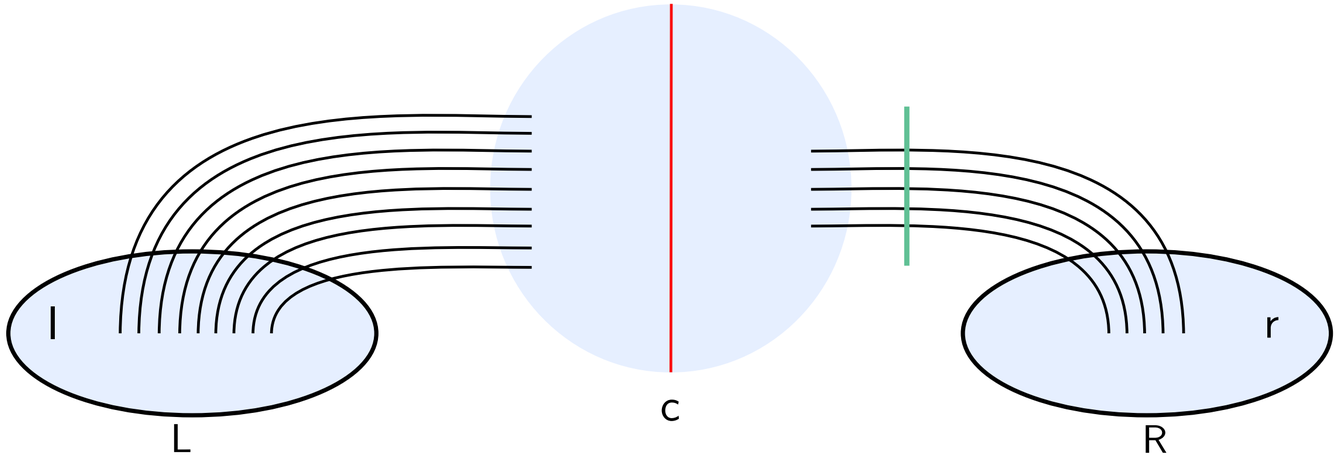

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- Replica wormholes arise in the CFT from a 'generalized ETH ansatz' for the operator $\mathcal{O}$. Entanglement entropy is more subtle.

Cosmological Hilbert space dimension

- The microscopic resources to describe the cosmology are limited to the `observers' with finite density of states. Is it possible to compute the microcanonical Hilbert space?

$$\mathcal{F}_n = \{ |\Psi_{m_1}\rangle, \ldots, |\Psi_{m_n}\rangle \}$$

[Balasubramanian, Lawrence, Magan, M.S.]

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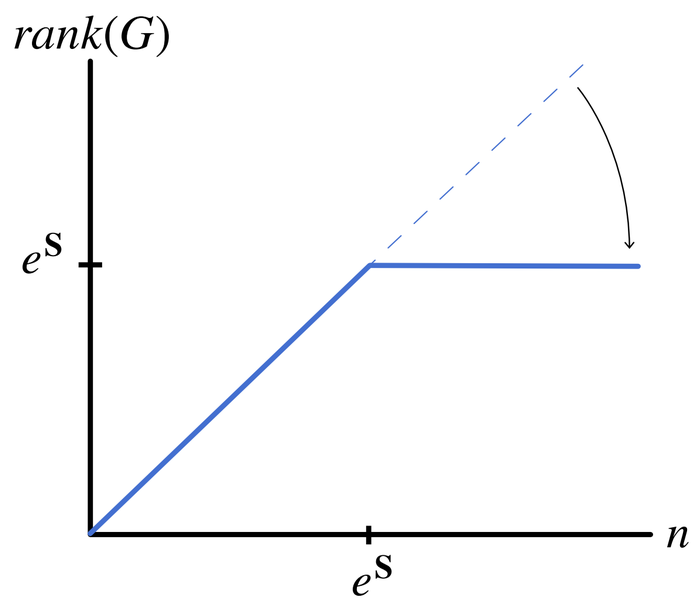
- Euclidean wormholes predict a maximum entropy:

$$\mathbf{S} = S(E_{\mathsf{L}}) + S(E_{\mathsf{R}})$$



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- Does the cosmology (or the black hole interior) have its own inherent description, without any reference to the exterior?

Thanks!