Euclidean Wormholes and String Theory

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Why Euclidean Wormholes?

- Theoretical laboratory for sharpening concepts such as **locality** in gravitational systems.
- Play crucial roles in Holography, especially from the **quantum information** perspective.
- Explicit role in producing the **Page-curve** of entanglement entropy, as a fine-grained entropy, of the black hole radiation degrees of freedom. [Penington];[Almheiri, Mahajan, Maldacena, Zhao];..
- Results demonstrating the utility of wormholes in black hole information were obtained in 2D gravity in which explicit calculations are under control.
- Do Euclidean wormholes play similar roles in $D \geq 4$ where gravity is dynamical?
- Do they arise as Euclidean saddles of UV complete theories such as string theory?
    - Are they genuine saddle points? Perturbatively stable?
    - Can we construct Euclidean wormholes from compactification of string theory?

Plan of the Talk

- *Complex saddles and Euclidean wormholes in the Lorentzian path integral* [Loges, GS, Sudhir, '22]:
  - Evaluate Lorentzian path integral using Picard-Lefschetz theory: Lorentzian, Euclidean & complex saddles treated democratically. Allowability criterion [Kontsevich, Segal, '21]; [Witten, '21].
  - **Wormhole stability:** Consider **gauge invariant** perturbations and correct **boundary conditions**. No homogenous perturbation (pure gauge) while inhomogeneous modes **increase** the Euclidean action. Axionic wormhole is perturbatively stable.
- *A 10d construction of Euclidean axion wormholes in flat and AdS space* [Loges, GS, Van Riet '23]:
  - Explicit 10d embedding of Euclidean axion wormhole from universal hypermultiplet of IIA compactification on T^6.
  - Explicit 10d embedding of Euclidean axion wormholes in $AdS_5 \times T^{1,1}$: test of positivity bound $\mathrm{Tr}(F \pm \star F)^2 \geq 0$ in the dual CFT.

Giddings-Strominger Wormhole

Giddings-Strominger Solutions

- Consider the following Euclidean action in d ≥ 3 dimensions:

$$S = \frac{1}{2\kappa_d^2}\int\left(\star(\mathcal{R}-2\Lambda)-\frac{1}{2}G_{ij}(\varphi)\mathrm{d}\varphi^i\wedge\star\mathrm{d}\varphi^j\right)$$

- A simple set of solutions with O(d) symmetry take the form [Giddings, Strominger, '88]:

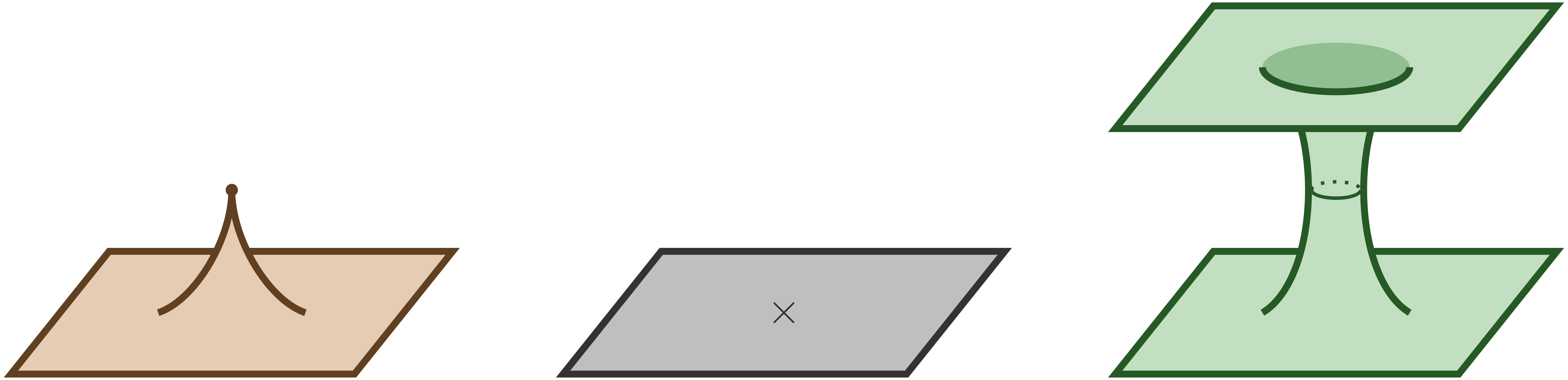
$$\mathrm{d}s^2 = f(r)^2\,\mathrm{d}r^2 + a(r)^2\,\mathrm{d}\Omega_{d-1}^2\,,$$

$$\left(\frac{a'}{f}\right)^2 = 1+\frac{a^2}{\ell^2}+\frac{\mathsf{c}}{2(d-1)(d-2)a^{2d-4}}\,,$$

$$\mathsf{c} = G_{ij}(\varphi)\frac{\mathrm{d}\varphi^i}{\mathrm{d}h}\frac{\mathrm{d}\varphi^j}{\mathrm{d}h} \quad = \text{constant}$$

- $h(r)$ is a harmonic function, normalized to $h' = f/a^{d-1}$ so that $\star\, h = \mathrm{vol}_{d-1}$; plays the role of affine parameter along the geodesic.

Three Classes of Euclidean Geometries

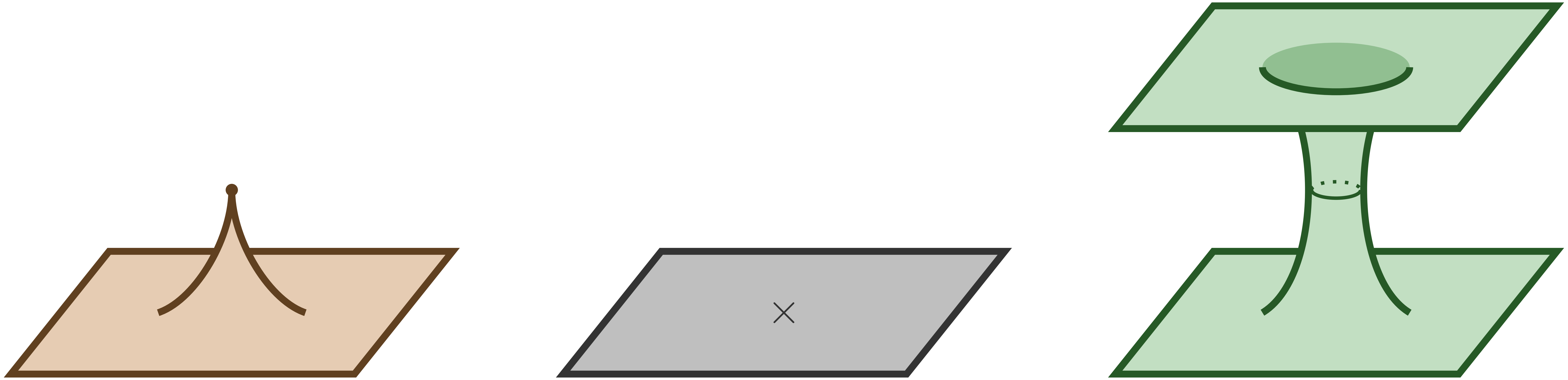


$c > 0$	$c = 0$	$c < 0$
space-like geodesic	null geodesic	time-like geodesic
Core instanton	Extremal instanton, e.g. D-instanton	Wormhole

$$G_{ij}(\varphi)\, \mathrm{d}\varphi^i \mathrm{d}\varphi^j = -\mathrm{d}\chi^2$$

only time-like geodesics

Three Classes of Euclidean Geometries



c > 0
space-like geodesic

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Core instanton

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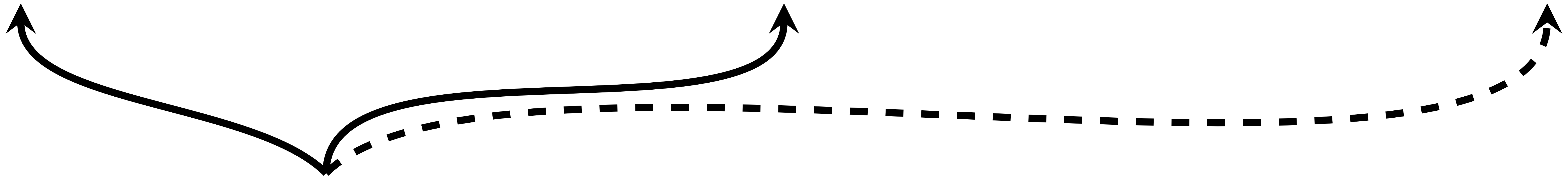
Wormhole



$$G_{ij}(\varphi)\,\mathrm{d}\varphi^i \mathrm{d}\varphi^j = \mathrm{d}\phi^2 - e^{\beta\phi}\,\mathrm{d}\chi^2$$

c $\gtreqless 0$ all possible, but longest time-like geodesic has length $\frac{2\pi}{|\beta|}$

Wormhole Regularity

- Required geodesic length for wormholes only depends on the wormhole size in AdS units:

$$D_d\left(\frac{a_0}{\ell}\right) = \text{“length of geodesic required by geometry”}$$

- $D_d(q_0)$ is monotonic in $q_0 \equiv a_0/\ell$:

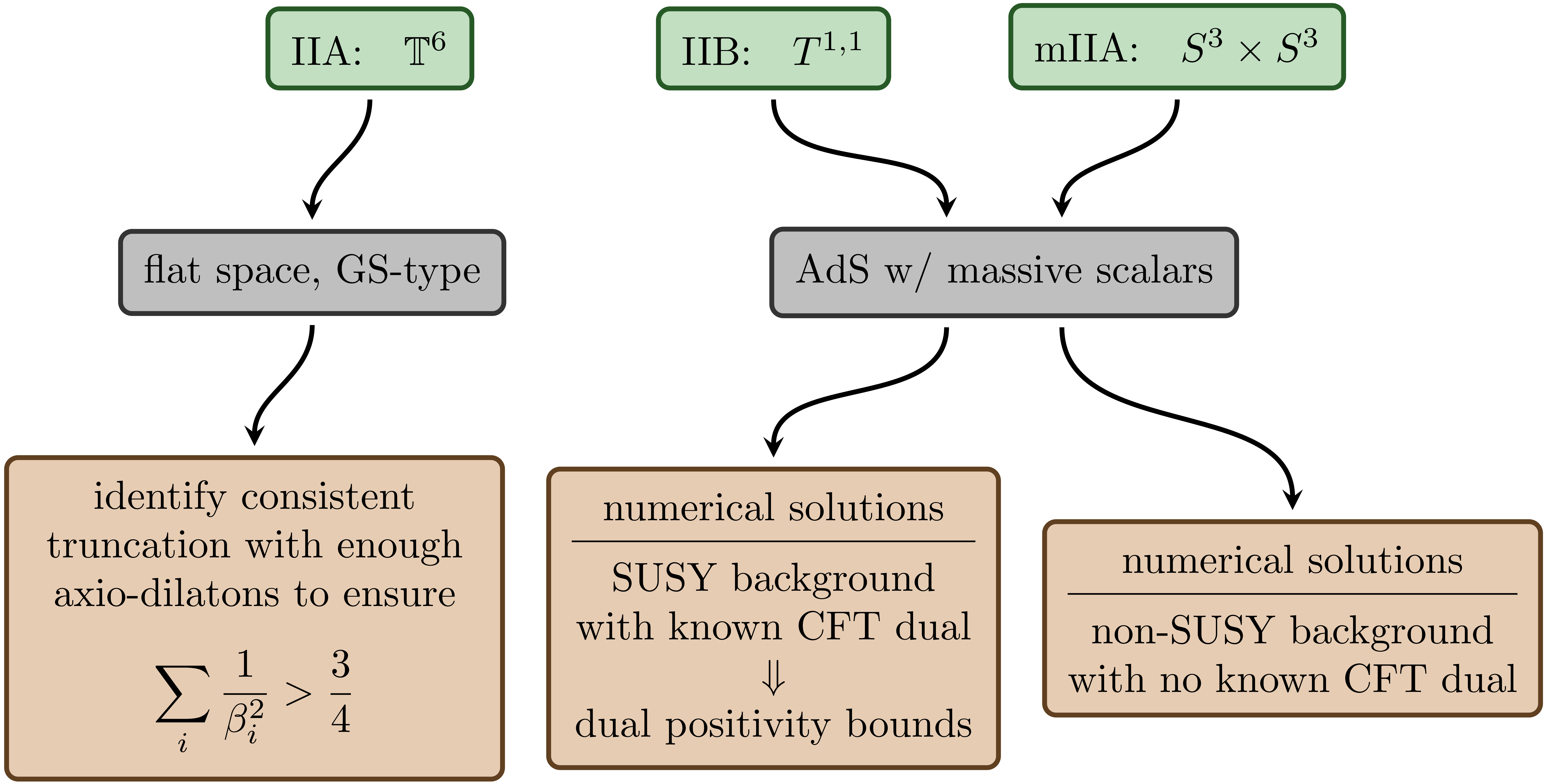
$$2\pi\sqrt{\frac{d-1}{2(d-2)}} = D_d(0) \quad \geq \quad D_d(q_0) \quad \geq \quad D_d(\infty) = 2\pi\sqrt{\frac{d-2}{2(d-1)}}$$

- There must exist a time-like geodesic longer than $D_d(q_0)$ [Arkani-Hamed, Orgera, Polchinski, ‘07]

$$\sum_i \frac{1}{\beta_i^2} > \frac{d-1}{2(d-2)}$$

Euclidean Axion Wormholes in String Theory

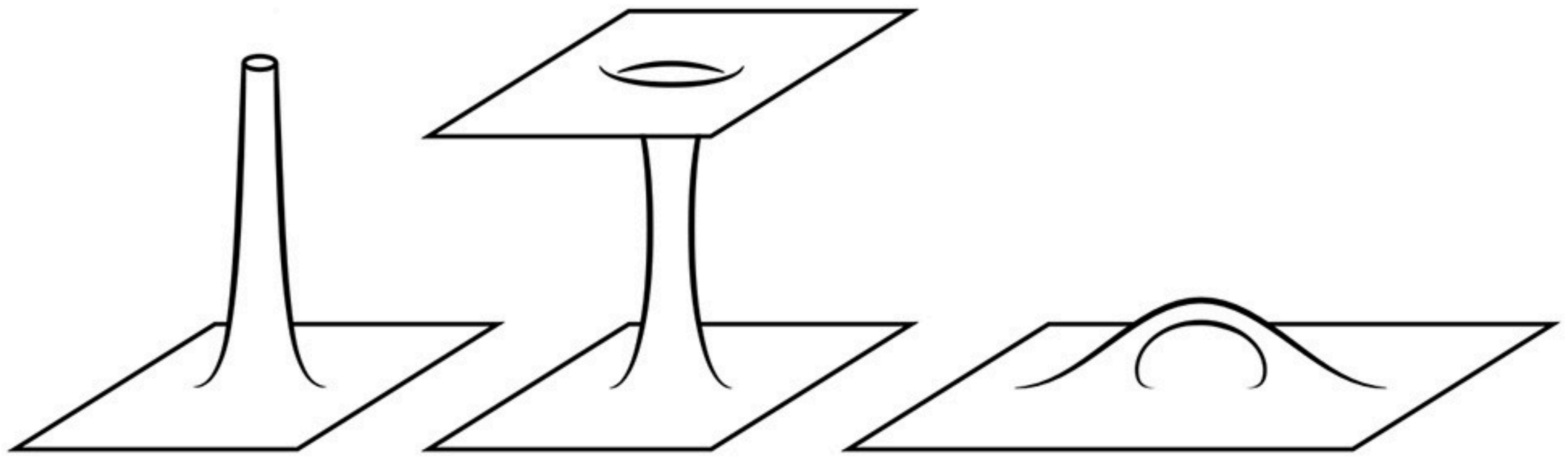
[Loges, GS, Van Riet, '23]



Wormholes and Quantum Gravity

Euclidean Wormholes

- These wormholes lead to a **breakdown of locality**:



$$S_{WH} = -\frac{1}{2}\sum_{I,J}\int d^D x \int d^D y \mathcal{O}_I(x) C^{IJ} \mathcal{O}_J(y)$$

- **Coleman's α-parameters** [Coleman, '89]:

$$e^{-S_{WH}} = \int d\alpha_I \, e^{-\frac{1}{2}\alpha_I (C^{-1})^{IJ}\alpha_J} e^{-\int d^D x \sum_I \alpha_I \mathcal{O}_I(x)}$$

ensembles

- Euclidean wormhole, if embeddable into AdS compactification of string theory, poses a puzzle for AdS/CFT as they jeopardize factorization of the two bdy CFTs [Maldacena, Maoz, '04].

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Wormholes and Quantum Gravity

- 👍 Wormholes breaks global symmetries by Planck suppressed operators.
- 👍 They play a key role in the axionic Weak Gravity Conjecture [Brown, Cottrell, GS, Soler, '15];[Montero, Valenzuela, Uranga, '15]; [Heidenreich, Reece, Rudelius, '15]; [Hebecker,Mangat-Theissen-Witkowski, '16]; [Hebecker, Mikhail, Soler, '18]; …

$$f \cdot S_{instanton} \lesssim M_P$$

  which constrains some large field inflation models.

- 👍 Derivative corrections lower the wormhole action, giving support to the axionic WGC [Andriolo, Huang, Noumi, Ooguri, GS, '20]; [Andriolo, GS, Soler, Van Riet, '22].
- 👎 α-parameter interpretation leads to -1 form global symmetries [McNamara, Vafa, '20]
- 👎 Factorization and AdS/CFT (ensemble average).
- 👎 $\exists$ AdS wormholes which violate positivity bound $\mathrm{Tr}(F \pm \star F)^2 \geq 0$ in the dual CFT: [Katmadas, Ruggeri, Trigiante, VR, '18];[Loges, GS, Van Riet, '23]

Wormhole Stability

Wormhole Stability

[Loges, GS, Sudhir, '22]

- Previous works (25+ years) on perturbative stability of axion wormholes have led to contradictory claims, casting doubts on their contributions to the Euclidean path integral.

	Frame	**Stable**	**Gauge-inv**	**j=0,1**	**B.C.**
Rubakov, Shvedov, '96	axion	No	No	physical	✗
Alonso, Urbano, '17	axion	Yes	Yes	physical	✗
Hertog, Truijen, Van Riet, '18	axion	No	Yes	pure gauge	✗
Loges, GS, Sudhir, '22	3-form	Yes	Yes	pure gauge	✓
Hertog, Meanaut, Tielemans, Van Riet, to appear	axion	Yes	Yes	pure gauge	✓

Boundary Conditions and Gauge Invariance

- Under diffeomorphism, metric and axion/3-form perturbations are mixed. Physically meaningful conclusions can only be drawn on gauge-invariant perturbations.

- In analyzing scalar perturbations around the GS wormhole, the boundary conditions in the 3-form picture can be imposed more straightforwardly. Finite energy perturbations:

$$\int \delta H \wedge \star \delta H < \infty \,,$$

  which corresponds to:

$$\int \mathrm{d}\delta\theta \wedge \star \mathrm{d}\delta\theta < \infty \,,$$

- Metric perturbations vanish at the boundaries. Gauge invariant perturbations are Dirichlet in the H_3 picture [Loges, GS, Sudhir, '22], while in the θ picture, gauge invariant perturbations involve mixed b.c. [Hertog, Meanaut, Tielemans, Van Riet, to appear].

Wormhole Stability

- We determine the stability of GS wormhole by carrying out the following steps:
  - Parametrization of scalar perturbations and their boundary conditions.
  - Diffeomorphisms and physical degrees of freedom.
  - Quadratic action.
  - Integrate out non-dynamical and unphysical, gauge-dependent modes.
  - Analyze spectrum of remaining physical modes.

Steps akin to analyzing gauge invariant perturbations in inflationary cosmology.
But as we shall show, not only is the spectrum of perturbations
but **on-shell value of the quadratic action** is important for determining stability.

Scalar Perturbations

$$\mathrm{d}s^2 = a(\eta)^2\left[-(1+2\phi)\,\mathrm{d}\eta^2 + 2\partial_i B\,\mathrm{d}\eta\mathrm{d}x^i + \big((1-2\psi)\gamma_{ij} + 2\nabla_i\partial_j E\big)\,\mathrm{d}x^i\mathrm{d}x^j\right]$$

$$H = \frac{n}{2\pi^2}\left[(1+s)\mathrm{vol}_3 + \mathrm{d}\eta\wedge\big(\tfrac{1}{2}\sqrt{\gamma}\epsilon_{ijk}\partial^i w\,\mathrm{d}x^j\wedge\mathrm{d}x^k\big)\right]$$

- 6 scalar perturbations: ϕ, ψ, E, B, s, w.
- **Dirichlet boundary conditions:** perturbations must go to zero.

Diffeomorphisms

- Some of these perturbations are unphysical and only represent the freedom to perform diffeomorphism.
- Under a diffeomorphism generated by $\xi = \zeta^0 \partial_0 + \gamma^{ij}(\partial_i \zeta)\partial_j$ parametrized by two scalar functions ζ^0, ζ, the perturbations transform:

$$\delta_\xi \phi = \dot{\zeta}^0 + \mathcal{H}\zeta^0 \qquad \delta_\xi B = -\zeta^0 + \dot{\zeta} \qquad \delta_\xi s = \Delta\zeta$$

$$\delta_\xi \psi = -\mathcal{H}\zeta^0 \qquad \delta_\xi E = \zeta \qquad \delta_\xi w = \dot{\zeta}$$

- **Only one physical scalar mode**. Convenient to pick:

$$\mathcal{S} = s - \Delta E \qquad \delta_\xi \mathcal{S} = 0$$

Quadratic Action

- Expanding the action to quadratic order in perturbations:

$$S = \int \left(\frac{1}{2} \star R - \frac{1}{2} H \wedge \star H \right) + (\text{boundary terms}) \longrightarrow S\big|_{\text{bkgd}} + S_2 + \cdots$$

$$\begin{aligned} S_2 = \int \mathrm{d}\eta \mathrm{d}^3x \, \sqrt{\gamma}\, a^2 \Big\{ & -3\big(\dot{\psi} + \mathcal{H}\phi\big)^2 + \big(B - \dot{E}\big)\Delta\big(B - \dot{E}\big) - 2\big(\dot{\psi} + \mathcal{H}\phi\big)\Delta\big(B - \dot{E}\big) \\ & - 3\big(1 + \mathcal{H}^2\big)\big[(\phi + 3\psi - \Delta E + s)^2 - \phi^2 + (B - w)\Delta(B - w)\big] \\ & + (2\phi - \psi)(\Delta + 3)\psi \Big\} + \sqrt{6}\,\tilde{n} \int \mathrm{d}\eta \mathrm{d}^3x \, \sqrt{\gamma}\, (\dot{s} - \Delta w)\theta \,, \end{aligned}$$

- Note: not all perturbations are dynamical (non-dynamical perturbations impose constraints)
- Expand perturbations in angular momentum eigenstates:

$$\Delta \to \lambda_j = j(j+2) \in \{0,3,8,15,\ldots\}$$

Homogeneous Modes ($j = 0$)

- In terms of conjugate momentum of the only dynamical field ψ:

$$\mathcal{L}\big|_{j=0} = a^2\Big[-3\big(\dot{\psi}_0 + \mathcal{H}\phi_0\big)^2 - 9\big(1+\mathcal{H}^2\big)(2\phi_0 + 3\psi_0)\psi_0 + 3(2\phi_0 - \psi_0)\psi_0\Big]$$

$$= \Pi_0^\psi \dot{\psi}_0 + \frac{(\Pi_0^\psi)^2}{12a^2} - 3a^2\big(10 + 9\mathcal{H}^2\big)\psi_0^2 + \big[\underbrace{\mathcal{H}\Pi_0^\psi - 6a^2\big(2+3\mathcal{H}^2\big)\psi_0}_{\text{gauge-invariant}}\big]\phi_0$$

- Integrate out non-dynamical field ϕ_0:

$$\mathcal{L}\big|_{j=0} = 6a^2\mathcal{H}^{-1}\big[\big(2+3\mathcal{H}^2\big)\psi_0\dot{\psi}_0 + \big(2+\mathcal{H}^2\big)\psi_0^2\big] = \frac{\mathrm{d}}{\mathrm{d}\eta}\big[3a^2\mathcal{H}^{-1}\big(2+3\mathcal{H}^2\big)\psi_0^2\big]$$

- **No physical degrees of freedom** (no conformal factor problem!). Similarly, $j = 1$ mode is pure gauge.

Quadratic Action for Physical Perturbations

- For each $j \geq 2$ there is one physical degree of freedom $(\mathcal{S}_j = s_j + \lambda_j E_j)$

$$S_2 = \int \mathrm{d}\eta \sum_{j\geq 2} \frac{3a^2}{\lambda_j\left(\frac{9}{\lambda_j-3}+\frac{1}{1+\mathcal{H}^2}\right)} \left[\dot{S}_j^2 + \frac{6\lambda_j\left(1+\mathcal{H}^2\right)}{(\lambda_j-3)\mathcal{H}} S_j\dot{S}_j - \frac{\lambda_j}{\mathcal{H}^2}\left(\frac{\lambda_j-9}{\lambda_j-3}(1+\mathcal{H}^2)-1\right)S_j^2\right],$$

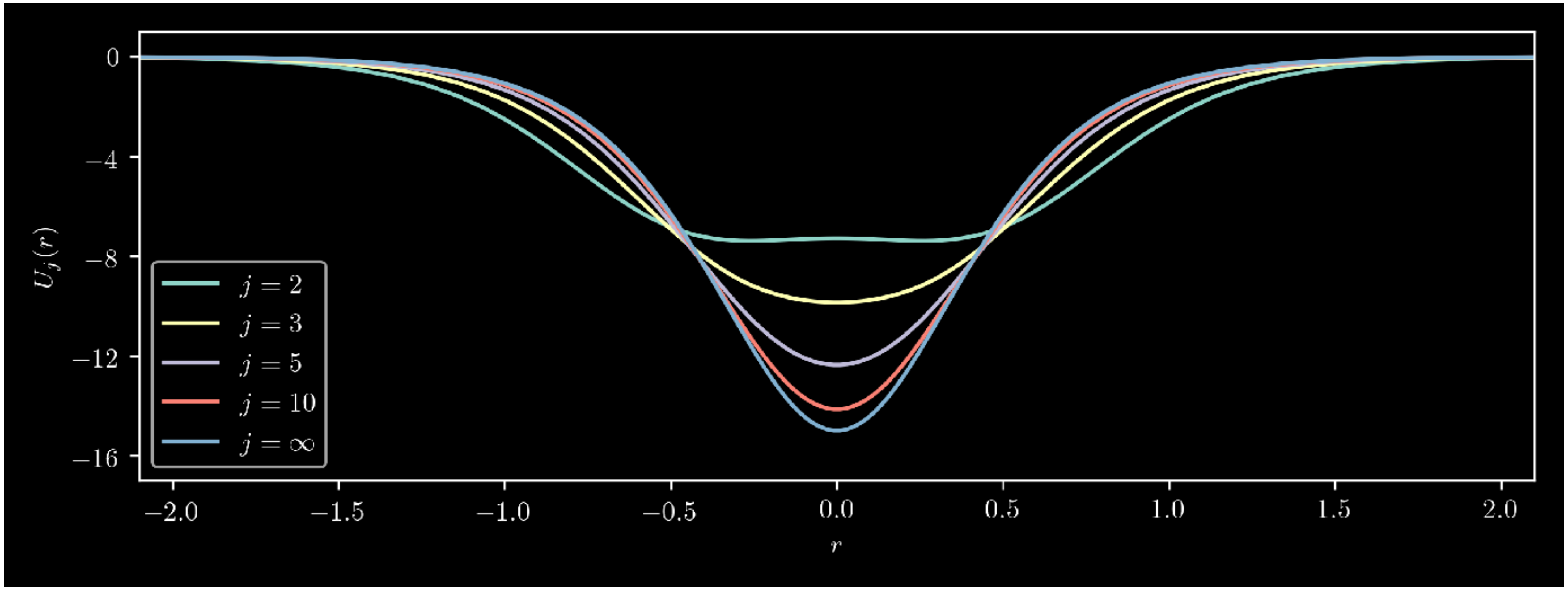
- Wick rotate $\eta \to -ir$ to **GS wormhole**: $a(r) \propto \sqrt{\cosh(2r)}$ and $\mathcal{H}_E(r) = -i\mathcal{H}(ir) = \tanh(2r)$
- Canonical normalize $\mathcal{Q}_j = (\cdots)\mathcal{S}_j$:

$$S_2^{\mathrm{E}} = \int \mathrm{d}r \sum_{j\geq 2}\left(\frac{1}{2}(\mathcal{Q}_j')^2 + \frac{1}{2}\underbrace{\left(U_j^{\mathrm{E}}+\lambda_j+1\right)}_{>0}\mathcal{Q}_j^2 + G_j^{\mathrm{E}}\right)$$

- S_2^E looks positive definite, but we will have to check the boundary term G_j^E.

Eigenvalue Problem

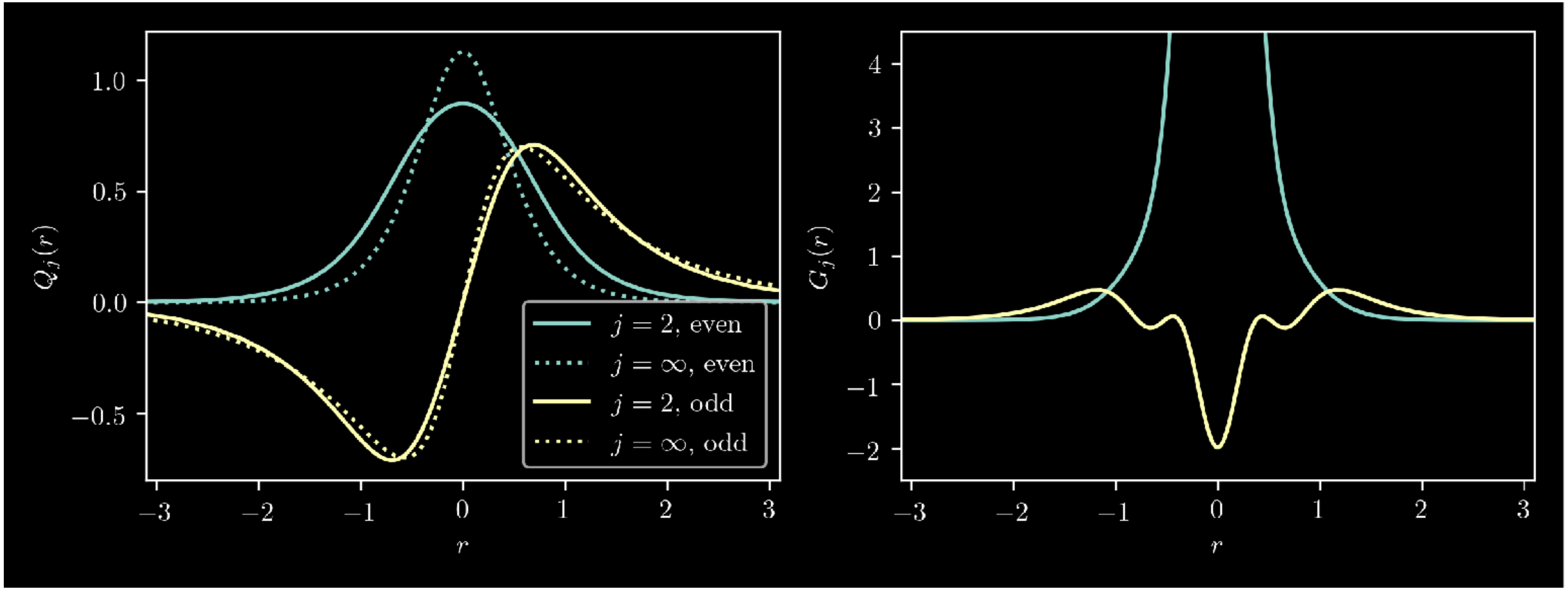
- Schrodinger-like problem: $$\mathcal{Q}_j^{(k)\prime\prime} + U_j^{\mathrm{E}}(r)\,\mathcal{Q}_j^{(k)} = \omega_j^{(k)}\,\mathcal{Q}_j^{(k)}$$



- $\mathcal{S} \to 0 \implies \mathcal{Q}_j^{(k)}$ must go to zero ***faster*** than $e^{-|r|}$. Look for bound states with $\omega_j^{(k)} < -1$.

Eigenfunctions

- There is exactly one even and one odd bound state for each $j \geq 2$:



- Total derivative term G_j^E is not integrable for the even eigenfunctions: $S_2[\mathcal{Q}^{(\text{even})}] \to +\infty$

Spectrum and Stability

j	$\omega_j^{(\mathrm{odd})}$	$\omega_j^{(\mathrm{odd})} + \lambda_j + 1$	j	$\omega_j^{(\mathrm{odd})}$	$\omega_j^{(\mathrm{odd})} + \lambda_j + 1$
2	-1.5335	7.4665	6	-1.0921	47.9079
3	-1.2873	14.7127	7	-1.0705	62.9295
4	-1.1817	23.8183	8	-1.0556	79.9444
5	-1.1256	34.8744	9	-1.0450	98.9550

$$\mathcal{Q} = \sum_{j\geq 2} c_j \, \mathcal{Q}_j^{(\mathrm{odd})}(r) Y_j(\Omega) \quad \Longrightarrow \quad S_2 = \sum_{j\geq 2} \frac{1}{2} \left(\omega_j^{(\mathrm{odd})} + \lambda_j + 1\right) c_j^2 > 0$$

The Euclidean action only ever **increases** under scalar perturbations:

the GS wormhole is perturbatively stable.

String Theory Embeddings

[Loges, GS, Van Riet, ’23]

Euclidean Axion Wormholes in Flat Space

[Loges, GS, Van Riet, '23]

- Reduction ansatz motivated by the extremal solution:

$$\mathrm{d}s_{10}^2 = e^{-6b\varphi}\,\mathrm{d}s_4^2 + e^{2b\varphi}R^2\mathcal{M}_{ij}\,\mathrm{d}\theta^i\mathrm{d}\theta^j$$

$$\mathcal{M}_{ij} = \mathrm{diag}\big(e^{\vec{\beta}_1\cdot\vec{\Phi}}, e^{\vec{\beta}_2\cdot\vec{\Phi}}, \dots, e^{\vec{\beta}_6\cdot\vec{\Phi}}\big)$$

$$C_3 = \chi_1\,\mathrm{d}\theta^{123} + \chi_2\,\mathrm{d}\theta^{145} + \chi_3\,\mathrm{d}\theta^{256} + \chi_4\,\mathrm{d}\theta^{346}$$

	1	2	3	4	5	6
$D2_1$	×	×	×			
$D2_2$	×			×	×	
$D2_3$		×			×	×
$D2_4$			×	×		×

- 4d theory contains 11 scalars:

No Wick rotation that turns them into Lorentzian "overextremal" branes.

$$S_4 = \frac{1}{2\kappa_4^2}\int\left[-\mathcal{R} + \frac{1}{2}\sum_{i=1}^{4}\left[(\partial s_i)^2 + e^{2s_i}(\partial\chi_i)^2\right] + \frac{1}{2}\sum_{i=5}^{7}(\partial s_i)^2\right]$$

There are four decoupled axio-dilaton pairs with $\beta = 2$:

$$\sum_{i=1}^{4}\frac{1}{\beta_i^2} = 1 \quad > \quad \frac{3}{4} \qquad \checkmark$$

Euclidean Axion Wormholes in AdS Space

[Loges, GS, Van Riet, '23]

$$T^{1,1} = [\mathrm{SU}(2)\times\mathrm{SU}(2)]/\,\mathrm{U}(1) \quad \sim \quad \overbrace{S^2}^{\int_{S^2}(B_2,C_2)\ \text{axions}} \times S^3$$

$\underbrace{S^2 \times S^3}$ → F_5 flux

S^2 → $\int_{S^2}(B_2, C_2)$ axions

Background solution:

$$\mathrm{d}s^2_{10} = \ell^2\,\mathrm{d}s^2_5 + \ell^2\big(\mathrm{d}s^2_{\mathrm{KE}} + \eta^2\big)$$
$$e^{\Phi} = g_{\mathrm{s}}$$
$$B_2 = 0$$
$$C_0 = 0$$
$$C_2 = 0$$
$$F_5 = 4\ell^2(1 - i\star)\mathrm{vol}_{T^{1,1}}$$

Reduction ansatz;

$$\mathrm{d}s^2_{10} = \ell^2 e^{-\frac{2}{3}(4u+v)}\,\mathrm{d}s^2_5 + \ell^2\big(e^{2u}\,\mathrm{d}s^2_{\mathrm{KE}} + e^{2v}\eta^2\big)$$
$$e^{\Phi} = g_{\mathrm{s}}e^{\phi}$$
$$B_2 = \ell^2 g_{\mathrm{s}}^{1/2} b\,\Phi_2$$
$$C_0 = i g_{\mathrm{s}}^{-1}\chi$$
$$C_2 = i\ell^2 g_{\mathrm{s}}^{-1/2} c\,\Phi_2$$
$$F_5 = 4\ell^2(1 - i\star)\mathrm{vol}_{T^{1,1}}$$

Consistent Reduction to 5D

[Loges, GS, Van Riet, '23]

string dilaton

$H_3 \sim \mathrm{d}b \wedge \Phi_2$

$F_1 \sim \mathrm{d}\chi$

$F_3 \sim (\mathrm{d}c - \chi\, \mathrm{d}b) \wedge \Phi_2$
$(\mathrm{d}F_3 = H_3 \wedge F_1)$

$$S_5 = \frac{1}{2\kappa_5^2} \int \mathrm{d}^5x \, \sqrt{|g|} \Big[-\mathcal{R} + \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2} e^{-4u-\phi} (\partial b)^2 + \tfrac{1}{2} e^{2\phi} (\partial\chi)^2 + \tfrac{1}{2} e^{-4u+\phi} (\partial c - \chi\, \partial b)^2$$

$$+ \tfrac{28}{3}(\partial u)^2 + \tfrac{8}{3}\partial u \partial v + \tfrac{4}{3}(\partial v)^2 + \underbrace{2e^{-\frac{8}{3}(4u+v)}\left(2e^{4u+4v} - 12e^{6u+2v} + 4\right)}_{\mathcal{V}} \Big]$$

$e^{2u}\, \mathrm{d}s^2_{\mathrm{KE}} + e^{2v}\, \eta^2$

[Cassani, Dall'Agata, Faedo – '10]

[Cassani, Faedo – '11]

Not Giddings-Strominger wormhole!

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$$S_5 = \frac{1}{2\kappa_5^2} \int \mathrm{d}^5x \sqrt{|g|} \Big[-\mathcal{R} + \tfrac{1}{2}(\partial\phi)^2 + \tfrac{1}{2}\boxed{e^{-4u-\phi}}(\partial b)^2 + \tfrac{1}{2}\boxed{e^{2\phi}}(\partial\chi)^2 + \tfrac{1}{2}\boxed{e^{-4u+\phi}}(\partial c - \chi\,\partial b)^2$$

$$+ \tfrac{28}{3}(\partial u)^2 + \tfrac{8}{3}\partial u \partial v + \tfrac{4}{3}(\partial v)^2 + \underbrace{2e^{-\frac{8}{3}(4u+v)}\left(2e^{4u+4v} - 12e^{6u+2v} + 4\right)}_{\mathcal{V}} \Big]$$

$e^{2u}\,\mathrm{d}s^2_{\mathrm{KE}} + e^{2v}\,\eta^2$

[Cassani, Dall'Agata, Faedo – '10]

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Not Giddings-Strominger wormhole!

Dual CFT and Operators Positivity

[Loges, GS, Van Riet, '23]

Type IIB on $T^{1,1}$ is dual to an $\mathcal{N}=1$ quiver CFT with two nodes [Klebanov, Witten – '98]

$$e^{-\Phi} \longleftrightarrow \frac{1}{g_1^2}+\frac{1}{g_2^2} \qquad\qquad C_0 \longleftrightarrow \theta_1+\theta_2$$

$$\int_{S^2} B_2 \longleftrightarrow \frac{1}{g_1^2}-\frac{1}{g_2^2} \qquad\qquad \int_{S^2}\tilde{C}_2 \longleftrightarrow \theta_1-\theta_2$$

$$(\mathrm{d}\tilde{C}_2=\mathrm{d}C_2-C_0\,\mathrm{d}B_2)$$

Dual operators:

$$\mathcal{O}_\Phi=\mathrm{Tr}(F_1\wedge\star F_1+F_2\wedge\star F_2) \qquad \mathcal{O}_{C_0}=\mathrm{Tr}(F_1\wedge F_1+F_2\wedge F_2)$$

$$\mathcal{O}_{B_2}=\mathrm{Tr}(F_1\wedge\star F_1-F_2\wedge\star F_2) \qquad \mathcal{O}_{\tilde{C}_2}=\mathrm{Tr}(F_1\wedge F_1-F_2\wedge F_2)$$

Operator positivity:

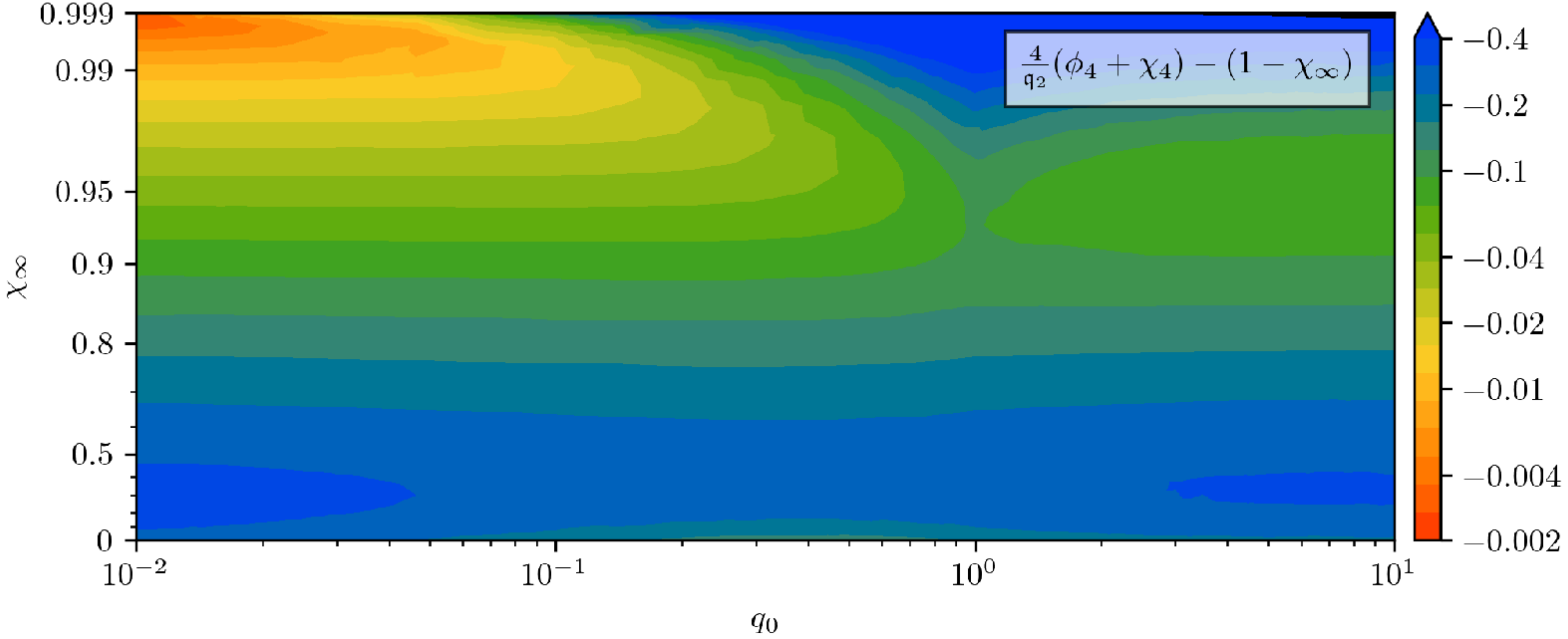
$$\langle\mathrm{Tr}[(F_i\pm\star F_i)^2]\rangle\geq 0 \qquad\Longrightarrow\qquad \langle\mathcal{O}_\Phi\rangle\pm\langle\mathcal{O}_{B_2}\rangle\geq\langle\mathcal{O}_{C_0}\rangle\pm\langle\mathcal{O}_{\tilde{C}_2}\rangle$$

Violation of Positivity Bounds

[Loges, GS, Van Riet, '23]

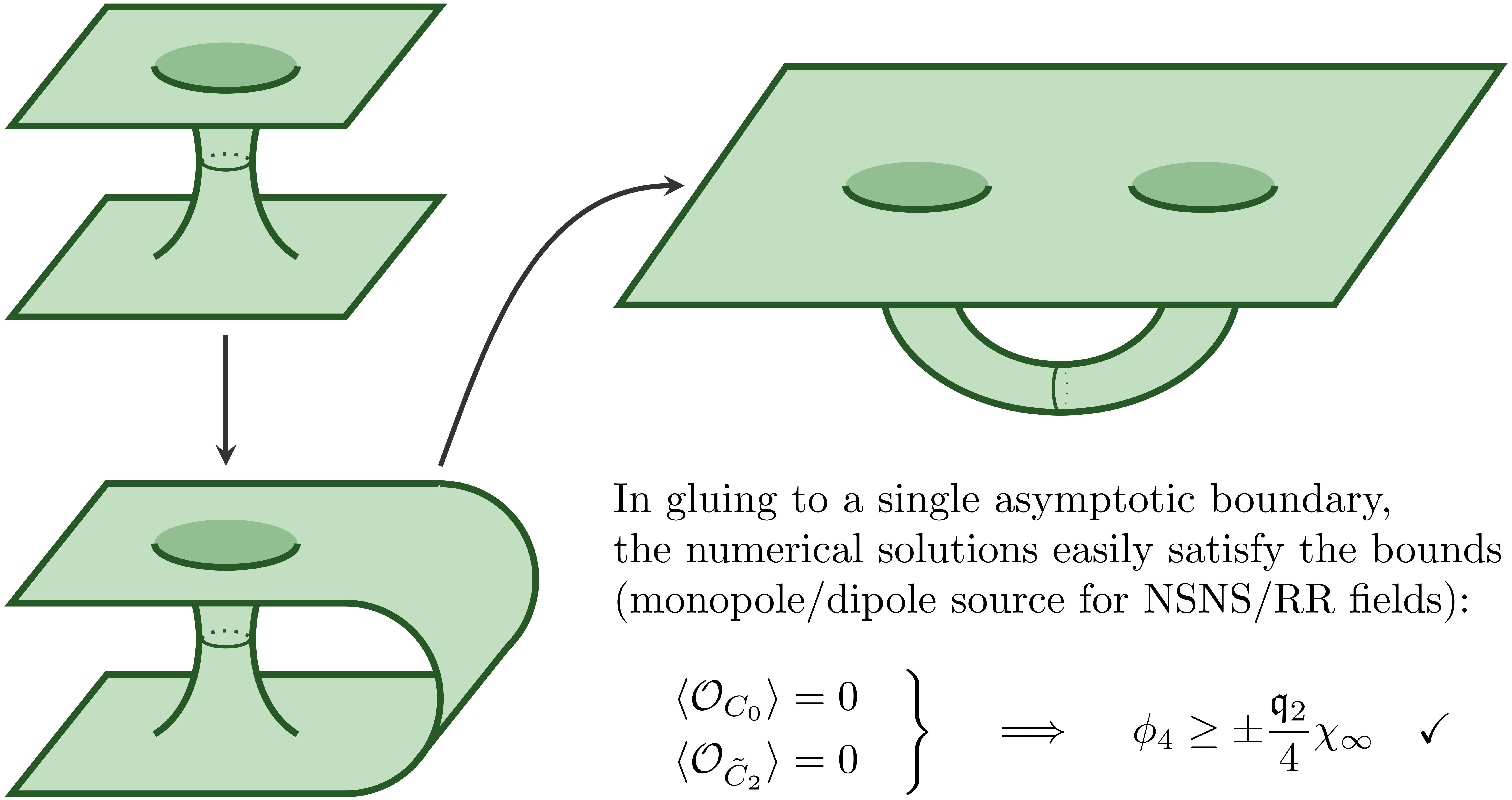
With the fully explicit 10d gravity solution, we can check whether $\langle \mathcal{O}_\Phi \rangle \pm \langle \mathcal{O}_{B_2} \rangle \geq \langle \mathcal{O}_{C_0} \rangle \pm \langle \mathcal{O}_{\tilde{C}_2} \rangle$

This is **always violated** (for all q_0 and χ_∞)!



One boundary vs two?

[Loges, GS, Van Riet, ’23]



In gluing to a single asymptotic boundary, the numerical solutions easily satisfy the bounds (monopole/dipole source for NSNS/RR fields):

$$\left.\begin{aligned}\langle\mathcal{O}_{C_0}\rangle &= 0\\ \langle\mathcal{O}_{\tilde{C}_2}\rangle &= 0\end{aligned}\right\} \implies \phi_4 \geq \pm\frac{\mathfrak{q}_2}{4}\chi_\infty \quad \checkmark$$

Summary

Summary

- Establish that GS wormhole is perturbatively stable. The 3-form picture makes gauge invariance and proper boundary boundary conditions transparent.
- Conclusion of stability may carry over to AdS space since the (physical) perturbations are localized to the wormhole throat whose size is much less than the AdS curvature.
- Construct explicit Euclidean axion wormholes in flat and AdS space from string theory:
  - Flat space wormholes from type IIA on T^6: cannot Wick rotate to only Lorentzian branes.
  - AdS space wormholes from type IIB on $T^{1,1}$
    - Not Giddings-Strominger type: saxions have a potential and are sourced by the axions.
    - Known CFT dual: violation of positivity bounds in the CFT state for two-boundary solutions.
    - Massive scalars u, v dual to irrelevant operators may play a crucial role in identifying such CFT state.
- Other conceptual issues remain, e.g., α-parameters? Baby universes? For small wormholes (in AdS units) where one might integrate out wormhole effects a la Coleman, the solutions break down.