# Contracting with a Present-Biased Agent: Sannikov meets Laibson

**Alejandro Rivera**[1]

[1]UT-Dallas

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# Outline

# Motivation

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  2. Martingale techniques in continuous-time formulation to characterize incentive compatibility as constraint on volatility of cont. value (Sannikov, 2008).
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  3. $\implies$ Standard stochastic control problem (very tractable).
- By and large, modeling done under neoclassical exponential discounting.

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  - IC-constraint links volatility of perceived continuation value with actual discount factor.
  - PCC-constraint links volatility of perceived continuation value with perceived discount factor.

# Contribution to the Literature

| Setting | Two-period model | Continuous-time model |
|---|---|---|
| Exp. discounting | (IC)-constraint: Reward agent with higher consumption if "high" output is realized. Holmström (1979). | (IC)-constraint: Use sensitivity of agent's continuation value to output to incentivize effort. Sannikov (2008). |
| Present-biased | (PCC)-constraint: Rewards incentivize agent's perceived choice under his (wrongly) anticipated future present-bias $\hat{\beta}$. Heidhues and Kőszegi (2010). | (PCC)-constraint: Use sensitivity of agent's *perceived* continuation value to incentivize agent's perceived choice using $\hat{\beta}$ as discount factor. This paper. |

Table: Contract theory with present-bias and in continuous-time.

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# Model

- Continuous-time, infinite horizon setting.
- Risk-neutral, deep pocketed principal.
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- Risk-neutral, deep pocketed principal.
- Risk-neutral, limited liability, and present-biased agent.
- Present-bias following IG Model of Harris and Laibson (2013).
- Principal needs to contract with agent to manage a project with cash flows $Y_t$:

$$dY_t = a_t \mu dt + \sigma dZ_t^a, \tag{1}$$

where agent's effort $a_t$ is his private information.

# Agent's Problem

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- Agent's (perceived) continuation utility $\hat{V}$ (under exponential discounting):

$$\hat{V}_t = E_t^{\hat{a}} \left[ \int_t^{\tau} e^{-\gamma(s-t)} (dC_s - g(\hat{a}_s)) ds \right]. \qquad (2)$$

- Expected value is computed under the $\mathbb{P}^{\hat{a}}$.
- Agent (incorrectly) anticipates his future selves to exert effort policy $\hat{a}$.

# Agent's Problem

- Following Sannikov (2008) apply the MRT such that evolution of $\hat{V}$:

$$d\hat{V}_t = \gamma \hat{V}_t dt - \left(dC_t - g(\hat{a}_t)dt\right) + \phi_t \left(dY_t - \hat{a}_t \mu dt\right). \quad (3)$$

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- First term captures appreciation due to long-term exponential discounting.
- Second term captures utility anticipated from consumption net of effort costs.
- Last term captures measure sensitivity to output realizations: $\phi_t = d\hat{V}_t / dY_t$ is a measure of the contract's incentives.

# Agent's Problem (IC)

- Definition: Contract $\Gamma = (C, \tau, a, \hat{a})$ is (IC) if optimal for agent's current self $t$ to exert effort $a_t$ when it anticipates his future selves to exert effort $\hat{a}_s$, for all $s > t$.
- Lemma 1: $\Gamma = (C, \tau, a, \hat{a})$ is (IC) iff:

$$g'(a_t) = \beta \phi_t \mu \iff a_t = \frac{\beta \mu \phi_t}{\theta} \tag{IC}$$

for all $t$, where $\phi$ comes from the dynamics of $\hat{V}$ given in equation (3).

# Agent's Problem (PCC)

- Definition: $\Gamma = (C, \tau, a, \hat{a})$ satisfies (PCC) if the 0-self agent thinks *it will be optimal* for all his future selves to choose $\hat{a}_t$ for all $t > 0$.
- Lemma 2: $\Gamma = (C, \tau, a, \hat{a})$ satisfies (PPC) iff:

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- Equation (PCC) is new in the literature and captures (PCC) constraint in recursive settings!

# Principal's Problem

- Principal solves:

$$\max_{\Gamma} \mathbb{E}^{a} \left[ \int_0^{\tau} e^{-rt} \left( dY_t - dC_t \right) + e^{-r\tau} L \right] \tag{4}$$

subject to (IC), (PCC), and (PC).

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- Constraints only require keeping track of $\hat{V}$, which follows:

$$d\hat{V}_t = \gamma \hat{V}_t dt - (dC_t - g(\hat{a}_t)dt) + \underline{\phi_t \mu(a_t - \hat{a}_t)dt} + \phi_t \sigma dZ_t^a, \quad (5)$$

under $\mathbb{P}^a$ used by the principal.

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- Solve standard control problem formulating HJB for $F(\hat{V})$.

# Outline

# Signing Bonus and Payout Boundary

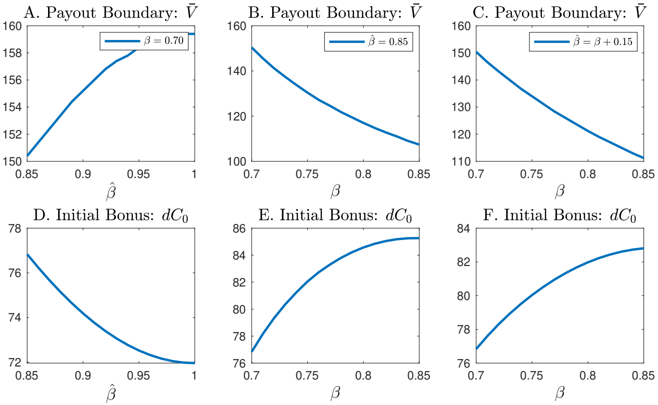


Figure: **Comparative statics for the payout boundary and initial bonus.**

# Value Function, Incentives, and Effort

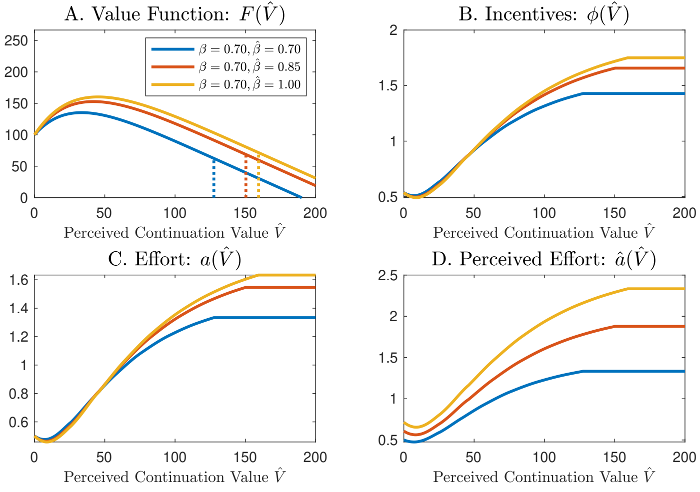


Figure: **Comparative statics with respect to $\hat{\beta}$.**

# Exploitation Effect

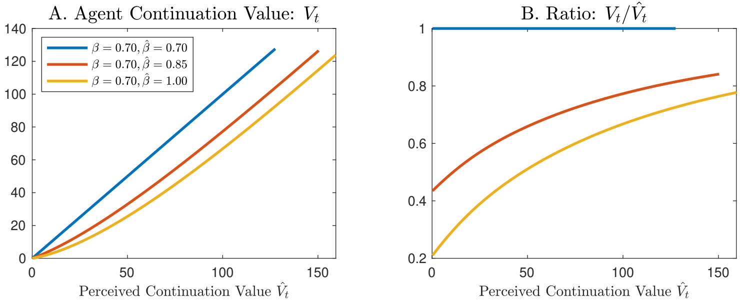


Figure: **Continuation value versus perceived continuation value.**

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1. Motivation
2. Model
3. Economic Insights
4. Conclusion

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  2. Link volatility of cont. value and actual discount factor to capture IC (as in Sannikov (2008)).
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# Conclusion

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  2. Link volatility of cont. value and actual discount factor to capture IC (as in Sannikov (2008)).
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- Present-bias gives rise to:
  1. Signing bonus.
  2. Naivete leads to more back-loaded contracts.
  3. Naivete leads to higher powered incentives.
  4. Agent is "exploited" with rewards for unrealistically high performance that are unlikely to materialize.

THANK YOU!!!

# Agent's Problem (PC)

- Agent's participation constraint (PC) states that the perceived payoff from the contract at $t = 0$ must be larger than an exogenous initial outside option denoted $\underline{\hat{V}}$:

$$\beta E^{\hat{a}} \left[ \int_{0_+}^{\tau} e^{-\gamma s} (dC_s - g(\hat{a}_s)) ds \right] + dC_0 = \beta \hat{V}_{0_+} + dC_0 \geqslant \underline{\hat{V}}. \tag{PC}$$

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- Characterizing IC via equation (IC), PCC via equation (PCC), and PC via equation (PC) allow us to write the principal's problem recursively with the agent's perceived continuation value $\hat{V}$ as a state variable.

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- Denote principal value as $F(\hat{V})$.
- Conjecture $dC_t = 0$ whenever $\hat{V}_t \in [0, \bar{V}))$ and reflect $\hat{V}_t$ down by $dC_t > 0$ whenever $\hat{V}_t = \bar{V}$.
- $F(\hat{V})$ satisfies for $\hat{V} \in [0, \bar{V}]$:

$$rF(\hat{V}) = \max_{\phi} \left\{ a\mu + F'(\hat{V})(\gamma\hat{V} + g(\hat{a}) + \phi\mu(a - \hat{a})) \right. \tag{6}$$

$$\left. + \frac{1}{2}F''(\hat{V})\phi^2\sigma^2 \right\} \tag{7}$$

$$F(0) = L, \qquad F'(\bar{V}) = -1, \qquad F''(\bar{V}) = 0, \tag{8}$$

where $a = \frac{\beta\mu\phi}{\theta}$ (IC) and $\hat{a} = \frac{\hat{\beta}\mu\phi}{\theta}$ (PCC).

# Value function at $t = 0$:

- Recall disproportional valuation of current self utility.
- Need to solve optimal initial payment $dC_0$.
- Formally given by

$$\max_{dC_0} F(\hat{V}_{0^+}) - dC_0, \tag{9}$$

subject to the participation constraint (PC).

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subject to the participation constraint (PC).

- Substituting (PC) yields

$$dC_0 = \begin{cases} 0, & \text{if } 0 \leqslant \underline{\hat{V}} < \tilde{V}, \\ \underline{\hat{V}} - \tilde{V}, & \text{if } \underline{\hat{V}} \geqslant \tilde{V}, \end{cases} \tag{10}$$

where $\tilde{V}$ solves $F'(\tilde{V}) = -\beta$.

# Value Function, Incentives, and Effort

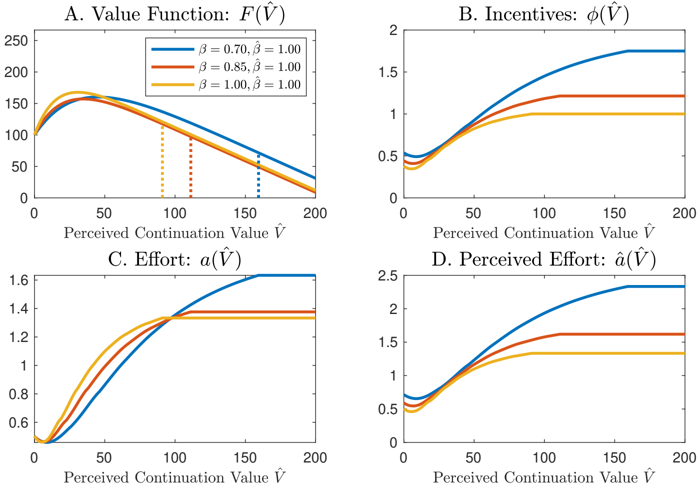


Figure: **Comparative statics with respect to β.**

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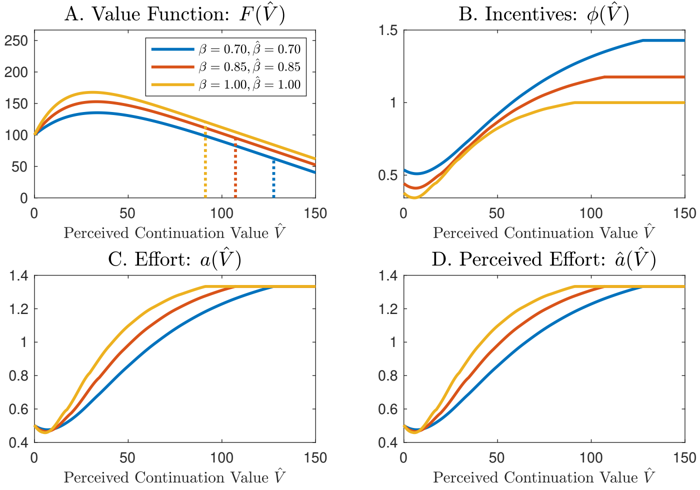


Figure: **Comparative statics with respect to $\beta$ and $\hat{\beta}$ simultaneously.**

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