Values, Temperatures, and Enumeration of Placement Games

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Joint work with Neil McKay, Lexi Nash, and Craig Tennenhouse

Combinatorial Game Theory

Combinatorial Game: 2-player, perfect information, no chance

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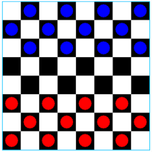

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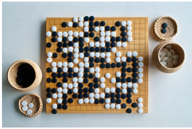

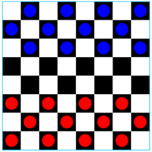

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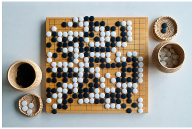

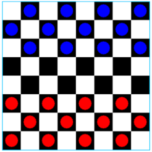

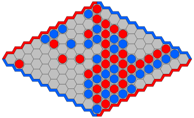

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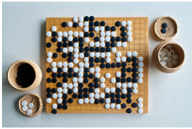
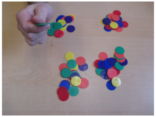
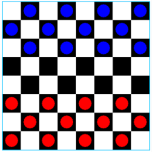
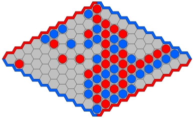

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- Two players are called Left (female, positive, bLue) and Right (male, negative, Red)

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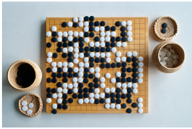
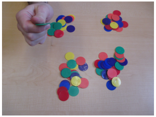
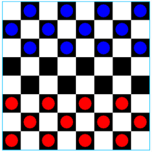
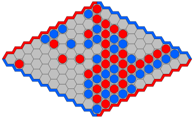

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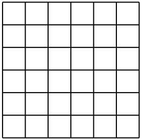

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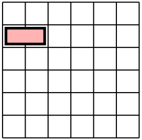

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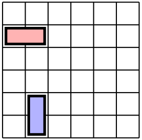

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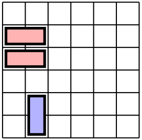

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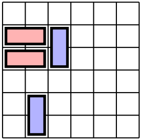

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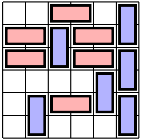

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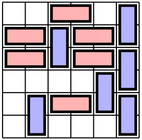

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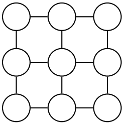

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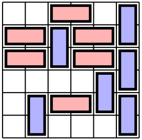

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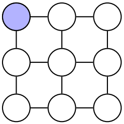

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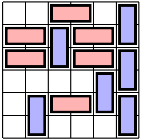

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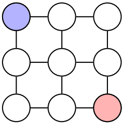

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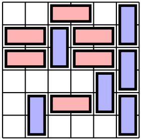

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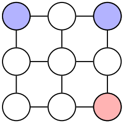

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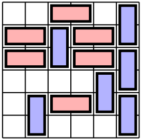

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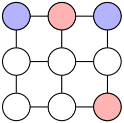

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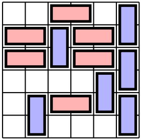

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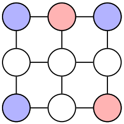

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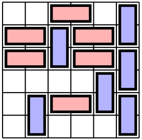

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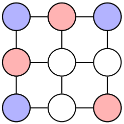

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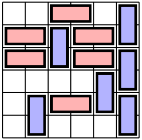

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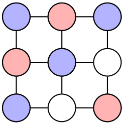

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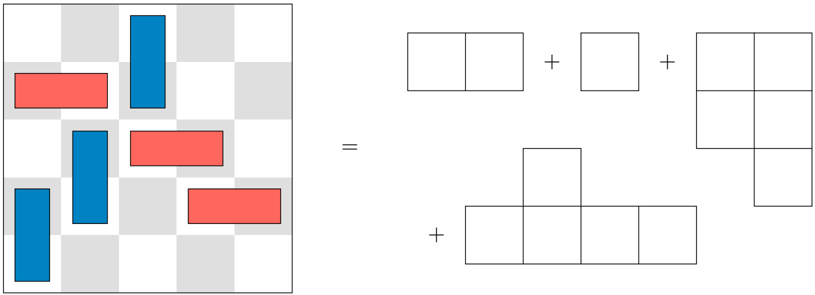
  - Misère Play: Win if unable to move

Combinatorial Game Theory - Disjunctive Sum



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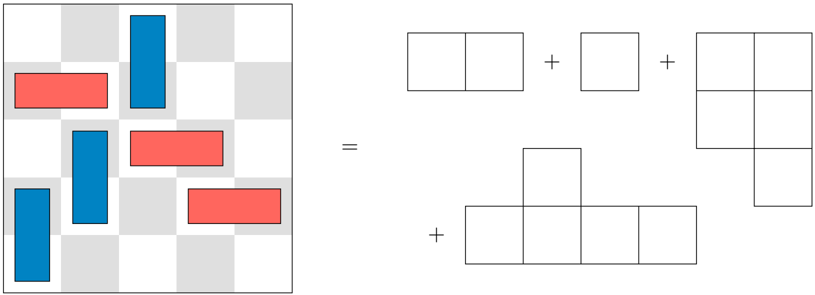

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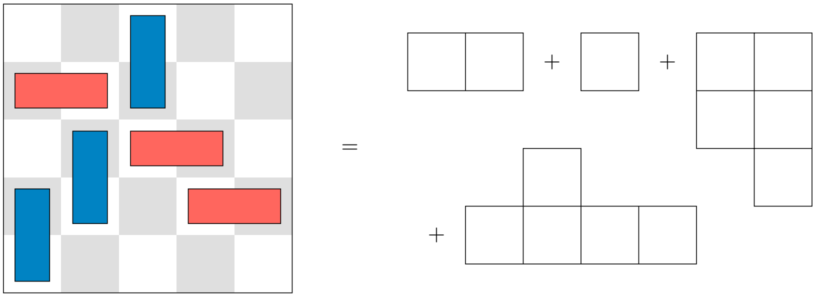

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- Can get non-alternating play in one component

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- COL only has numbers or numbers plus $*$
- Lexi Nash generalized and showed that many COL-like games also only have those values
- DOMINEERING has received a lot of attention, but still unknown

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Are SP-games universal?

- Yes: Every combinatorial game is equal to an SP-game
- No: Might be able to simplify game value calculations for SP-games

Game Values

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- SP-games likely to be good restricted universe
- Recent advances for DOMINEERING by Dwyer, Milley, and Willette

Temperature

Main Question 3: How urgent is it to move in a certain component?

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Theorem (–, Nowakowski, Santos, 2021)

Let S be a class of short games and J, K be two non-negative numbers. If for all $G \in S$, we have $\ell(G) \leq K$ and for all G^L and G^R that $\ell(G^L), \ell(G^R) \leq J$, then

$$BP(S) \leq \frac{K}{2} + J.$$

Research Problem 2.1

What is the boiling point of distance games?

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What is the boiling point of distance games?

- For Col it is 0
- For Snort it is infinite in general
  - Appears that for specific board it is bounded by polynomial in degree and 2-degree

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Research Problem 2.2

Is the boiling point of DOMINEERING 2?

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- Grid structure of the board at core of this?
  - Working with McKay and Tennenhouse on PARTIZAN ARCKAYLES
  - Using a genetic algorithm, we found a position with temperature $5/2$

Enumeration of Positions

- Go: Farr (2003), Tromp and Farnebäck (2007), Farr and Schmidt (2008)

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- On paths: Brown et al. (2019)

Enumeration

Main Question 4: How complex is a complete analysis of a game?

- To estimate this, we enumerate all possible positions

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- To estimate this, we enumerate all possible positions
- Polynomial profile
  - Bivariate: $P_G(x, y) = \sum_{i,j} f_{i,j} x^i y^j$
  - Can be used to find the number of positions both in purely alternating play and in non-alternating play

Enumeration of Positions

Research Problem 3.1

Determine the bipartite independence polynomial of graph products.

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- For games such as COL or SNORT the auxiliary board is a graph product
- Brown et al. (2019) determined generating function for polynomial profile of COL and SNORT on paths
  - Generalized with Lexi Nash to other distance games and other boards

Enumeration of Positions

Problem 3.2

Enumerate bipartite matchings.

Enumeration of Positions

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Enumerate bipartite matchings.

Theorem (–, McKay, 2021)

The polynomial profile of DOMINEERING *on an* $m \times n$ *board is the (1,1) entry of* $G_{0,n}^m$ *where*

$$G_{0,q+1} = \begin{bmatrix} G_{0,q} + yG_{1,q} & xG_{0,q} \\ G_{0,q} & \mathbf{0} \end{bmatrix} \quad G_{1,q+1} = \begin{bmatrix} G_{0,q} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$$

Play Positions

n	Number of play positions	Ratio of play positions
1	1	1
2	5	0.71428
3	75	0.57251
4	4,632	0.46264
5	1,076,492	0.38299
6	963,182,263	0.32222
7	3,317,770,165,381	0.27774
8	43,809,083,383,524,391	0.24367
9	2,209,112,327,971,366,587,064	0.21689
10	424,273,291,301,040,427,702,718,109	0.19532

Snort and Col on Complete Bipartite

m/n	0	1	2	3	4	5	6	7	8	9	10	11	12	13
0	1	3	9	27	81	243	729	2187	6561	19683	59049	177147	531441	1594323
1	3	7	17	43	113	307	857	2443	7073	20707	61097	181243	539633	
2	9	17	35	77	179	437	1115	2957	8099	22757	65195	189437		
3	27	43	77	151	317	703	1637	3991	10157	26863	73397			
4	81	113	179	317	611	1253	2699	6077	14291					
5	243	307	437	703	1253	2407	4877	10303						
6	729	857	1115	1637	2699	4877	9395							
7	2187	2443	2957	3991	6077	10303								
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11	177147	181243	189437											
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Conjecture (–, Nash, 2022)

The number of positions when playing $\textsc{Col}$ *or* $\textsc{Snort}$ *on the complete bipartite graph* $K_{m,n}$ *are recursively given by*

$$P_{\textsc{Col},K_{m,n}}(1) = 5P_{\textsc{Col},K_{m,n-1}}(1) - 6P_{\textsc{Col},K_{m,n-2}}(1) + c_m$$

with c_m *given by the OEIS sequence A260217 (first few terms are* $c_2 = 4$, $c_3 = 24$, $c_4 = 100$, $c_5 = 360$, *and* $c_6 = 1204$*).*

Other Research Projects

- Games played on designs (with Melissa Huggan and Brett Stevens)

Other Research Projects

- Games played on designs (with Melissa Huggan and Brett Stevens)
- Computational complexity of sums and thermographs (with Kyle Burke, Matt Ferland, and Shanghua Teng)

Thank you!

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