Stability of BPS states at weak coupling

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One expectation that comes out of the Swampland program is that if a theory has a U(1) gauge symmetry with gauge coupling g, then in the $g \to 0$ limit an infinite tower of charged states (with increasing charges) should become light relative to the Planck mass.

$$m_q \sim gqM_p$$

[Arkani-Hamed, Motl, Nicolis, Vafa ’06][Heidenreich, Reece, Rudelius ‘15][Grimm, Palti, Valenzuela ‘18][Andriolo, Junghans, Noumi, Shiu ‘18][Weigand, Lee, Lerche ‘18] …

We can test this in a controlled setting in type IIB string theory compactifications on Calabi-Yau 3-folds, yielding a four-dimensional $N = 2$ supergravity

U(1) gauge symmetries coming from the RR sector, whose gauge coupling is controlled by the complex-structure moduli

The charged states are D3 branes wrapping 3-cycles in the CY

Will focus on those which are supersymmetric, so BPS

It was shown that approaching any weak-coupling limit, these BPS states indeed have a mass which behaves like the gauge coupling (and is exponentially small in the proper distance in moduli space)

[Grimm, Palti, Valenzuela ’18]

- Open question: are the charges actually populated by BPS states in the theory?

Even though the BPS states never become the lightest in a controlled supergravity regime, it was proposed that they do offer a dual description of some of the string physics – specifically the infinite distance in the moduli space as well as the moduli and gauge fields themselves: “Emergence Proposal”

[Grimm, Palti, Valenzuela ’18]

See also [Harlow ’15][Heidenreich, Reece, Rudelius ‘17]

If correct, would expect some correspondence in properties of:

Complex-structure moduli space ⟷ BPS states

Classic example: conifold singularity in moduli space

The work is taking some steps towards making such connections more quantitative and concrete

The results stand for themselves for those interested in properties of BPS states and Black Holes in general

The mass of the BPS states is

$$M_{D3} \sim \frac{M_s}{g_s} \mathcal{V}_{\mathcal{C}} \sim \frac{M_s}{g_s} \sqrt{\mathcal{V}_s} Z(\mathbf{q}) \sim Z(\mathbf{q}) M_p$$

Where the central charge, which is a function of the complex-structure moduli, is

$$\eta = \begin{pmatrix} 0 & \mathbb{1}_{n_V+1} \\ -\mathbb{1}_{n_V+1} & 0 \end{pmatrix}$$

$$Z(\mathbf{q}) = e^{\frac{K}{2}} \mathbf{q}^T \cdot \eta \cdot \mathbf{\Pi}$$

$$\langle \mathbf{q}, \mathbf{p} \rangle \equiv \mathbf{q}^T \cdot \eta \cdot \mathbf{p}$$

$$K = -\log i \left\langle \mathbf{\Pi}, \overline{\mathbf{\Pi}} \right\rangle$$

Note: The states are never lighter than string states in the supergravity regime

See, Emergent String Conjecture [Weigand, Lee, Lerche '18]

Upon variation of the gauge couplings, or moduli fields, the BPS states can decay or combine.

This occurs along co-dimension 1 loci in moduli space called Walls of Marginal Stability where the triangle inequality is saturated

$$A \to B + C \qquad \mathbf{q}_A = \mathbf{q}_B + \mathbf{q}_C \qquad Z\left(\mathbf{q}_A\right) = Z\left(\mathbf{q}_B\right) + Z\left(\mathbf{q}_C\right)$$

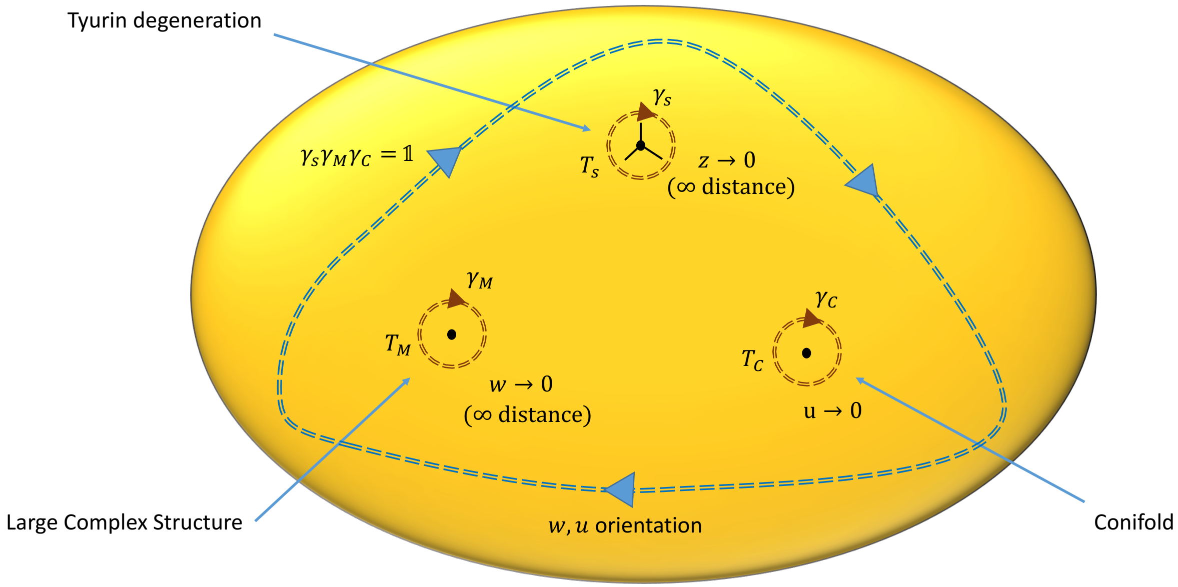
$$Z\left(\mathbf{q}\right) = M\left(\mathbf{q}\right) e^{i\alpha(\mathbf{q})} \qquad \mathrm{WMS}_{A\to B+C} \; : \; \alpha\left(\mathbf{q}_B\right) = \alpha\left(\mathbf{q}_C\right)$$

The <u>change</u> in the BPS index across a wall is given by the KS-formula. A simple (primitive) version of it is:

$$\Delta\Omega\left(\mathbf{q}_A, z\right) = (-1)^{\langle \mathbf{q}_B, \mathbf{q}_C\rangle - 1} \left|\langle \mathbf{q}_B, \mathbf{q}_C\rangle\right| \Omega\left(\mathbf{q}_B, z_{ms}\right) \Omega\left(\mathbf{q}_C, z_{ms}\right)$$

[Denef, Moore ’07]

The moduli space is controlled by Monodromy matrices



$$\gamma \cdot \mathbf{\Pi}(z) = \mathbf{\Pi}\left(z e^{2\pi i}\right) = T \cdot \mathbf{\Pi}(z) \qquad T = e^N \qquad \mathbf{\Pi}(z) = \mathrm{Exp}\left[N \frac{\log z}{2\pi i}\right] \sum_{p=0}^{\infty} \mathbf{a}_p z^p$$

An infinite distance $g \to 0$ limit is controlled by the Nilpotency of N:

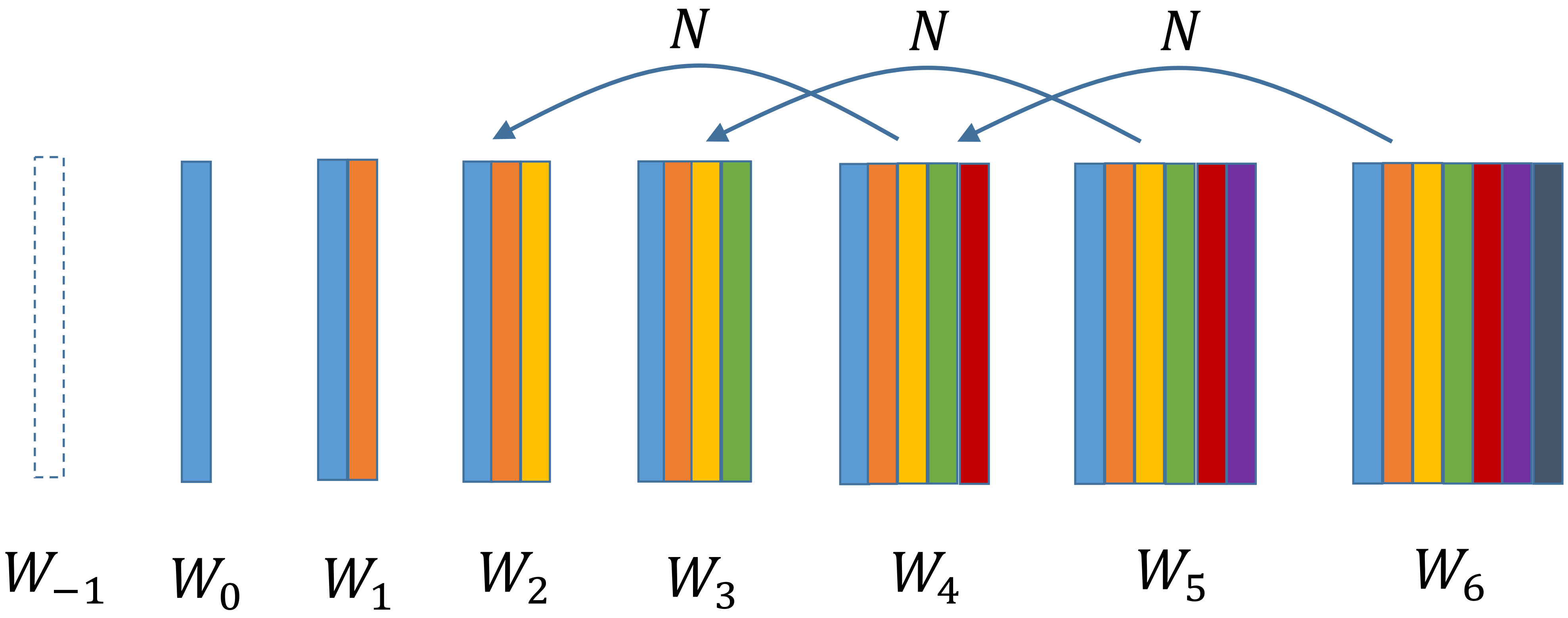
$$d \ : \ N^{d+1}\mathbf{a}_0 = 0 \ , \quad N^d \mathbf{a}_0 \neq 0$$

[Grimm, Palti, Valenzuela ’18]

N is an integer matrix which acts on the BPS charges in a natural way determining a filtration (Monodromy Weight Filtration):

$$\mathbf{q} \in V(\mathbb{Q}) \qquad W_{-1} \equiv 0 \subset W_0 \subset W_1 \subset \ldots W_6 = V$$

$$W_0 = \text{im}\, N^3 \ , \quad W_1 = \text{im}\, N^2 \cap \ker N \ , \ \ldots \ , \quad W_5 = \ker N^3$$



The results of the work are then:

Within any Nilpotent orbit approximation of the moduli space

$$\mathbf{\Pi}\left(z\right) = \mathrm{Exp}\left[N \frac{\log z}{2\pi i}\right] \sum_{p=0}^{\infty} \mathbf{a}_p z^p$$

a BPS state can only decay to constituents which do not have a higher weight.

* Proven for $d = 1$ one-parameter, and evidence that holds generally.

* Weight of charge is not quite as refined: $n \; : \; N^{n+1} = 0 \; , \quad N^n \neq 0 \; .$

$$\mathrm{s} - \mathrm{weight}\left[\mathbf{q}\right] = \mathrm{Largest\ integer}\ w\ \mathrm{such\ that}\ \mathbf{q} \notin W_{2(w+1)-n} \; .$$

Complex-structure moduli space ⟷ BPS states

Infinite distance $g \to 0$ limit | An ordering / filtration of BPS stability

Example: $d = 1$, have electric and dyonic states with masses going as

$$\mathrm{m_e} \sim g\, \boldsymbol{q_e}\, M_p \qquad\qquad \mathrm{m_d} \sim \frac{1}{g}\, \boldsymbol{q_d}\, M_p$$

An electric state cannot decay to two dyonic states:

$$(q_e, 0) \nrightarrow (q_e^1, q_m^1) + (q_e^2, -q_m^1)$$

True irrespective of the mass of the electric state (could be super-massive BH)

A decay is allowed in the sense of:

- Mass and Charge conservation
- Existence of appropriate Wall of Marginal Stability

The obstruction is due to the population of BPS states in the spectrum

Complex-structure moduli space ⟷ BPS states

Infinite distance $g \to 0$ limit | An ordering / filtration of BPS stability

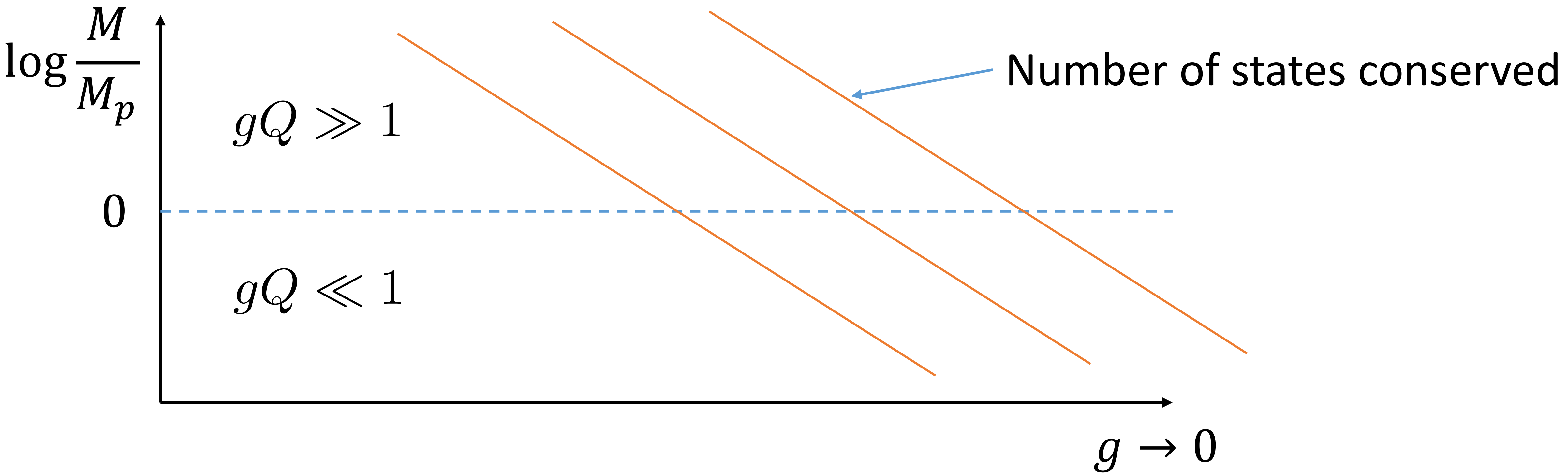
Note: “Fundamentality” of states is determined by their mass at $g = 0$, not their mass at any finite g

Similar to ideas of “moduli space holography”

[Grimm ’20][Grimm, Monnee, van Heisteeg ‘21]

If BPS electrically charged black holes exist in the theory - non-trivial due to diverging attractor flow

Then the light tower of BPS states is populated, and the light states are exactly the black hole microstates (their multiplicity remains exactly the same as the associated black hole)



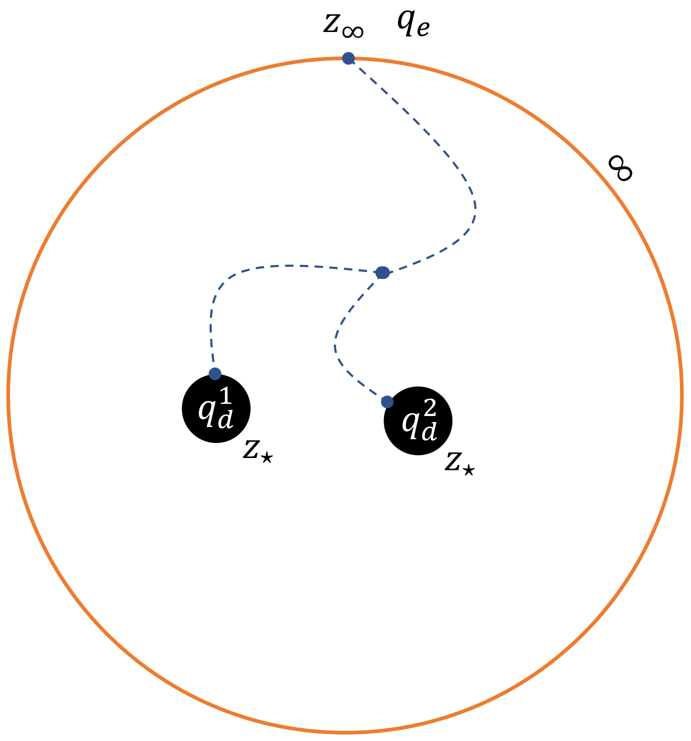
Evidence / Proof

In $N = 2$ supergravity expect a correspondence between BPS states and BPS black holes: If an attractor flow exists for the Black Hole of charge qM, then the BPS state with charge q exists.

[Moore '03; Denef '04]

In this correspondence, a decay across a WMS corresponds to a two-centre black hole, with split attractor flow

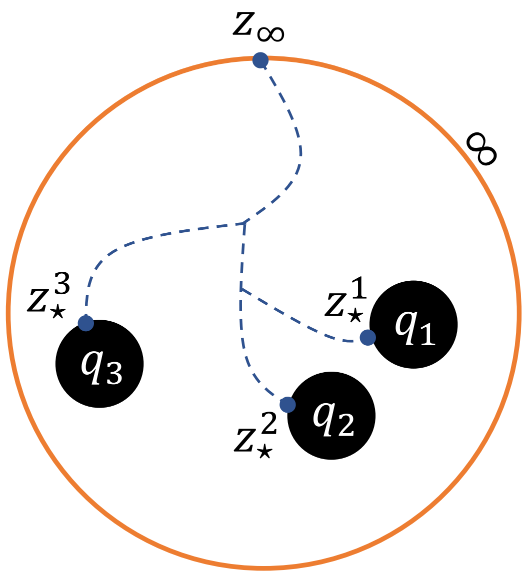
$$t = \frac{\log z}{2\pi i} \qquad \mathbf{q}^A = \mathbf{q}^B + \mathbf{q}^C$$



Find obstruction to such flows:

$$\frac{\text{Im } t_\star^B - \text{Im } t_\star^C}{\text{Im } t_\star^B + \text{Im } t_{WMS}} \geq 1$$

Further, find that further splits in the attractor flow…



…cannot evade the obstruction, so

$$\mathrm{Im}\ t_\star^A < 0\ ,\quad \mathrm{Im}\ t_\star^B > 0\ ,\quad \mathrm{Im}\ t_\star^C > 0$$

is not possible.

This seems to be showing that in a decay which violates the filtration, one of the decay products is not in the spectrum

We can see this another way: ask if the mass of one of the decay products vanishes somewhere on the wall of marginal stability. Find:

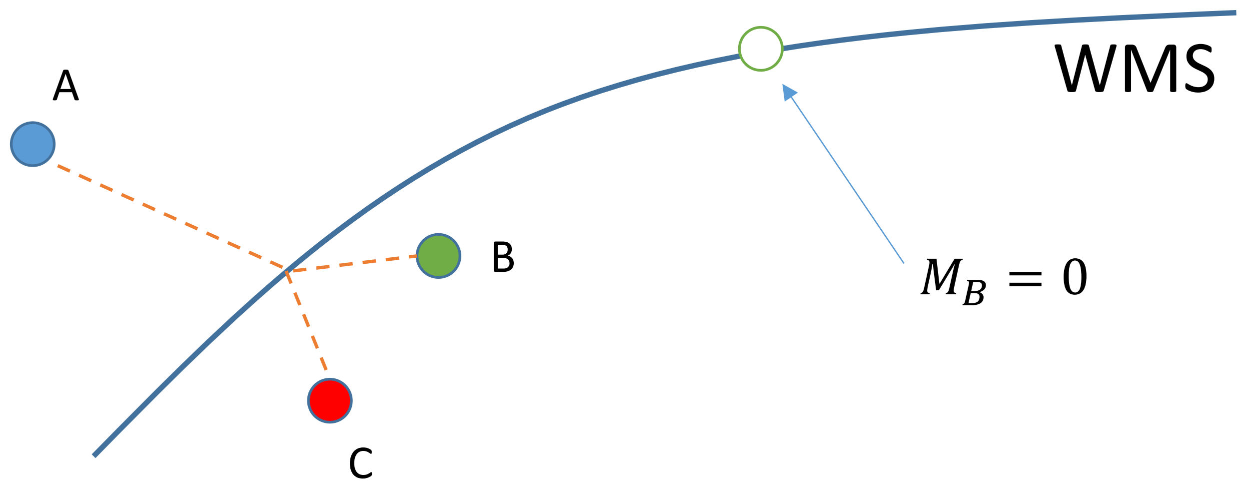
$$Z(\mathbf{q}, \mathrm{Im}\ t_0)|_{WMS} = 0$$

$$\mathrm{Im}\ t_0^B = -\left[\frac{-q_N^{B,-} q^{B,+} + q^{B,-} q_N^{B,+}}{\left|q_N^B\right|^2}\right] = -\mathrm{Im}\ t_\star^B ,$$

$$\mathrm{Im}\ t_0^C = -\left[\frac{-q_N^{B,-} q^{B,+} + q^{B,-} q_N^{B,+} - q^{A,-} q_N^{B,+} + q^{A,+} q_N^{B,-}}{\left|q_N^B\right|^2}\right] = -\mathrm{Im}\ t_\star^C$$

The points of vanishing mass, are exactly the negative of the attractor loci

So, if we had a negative attractor locus for one of the decay products, its mass will vanish somewhere on the WMS.

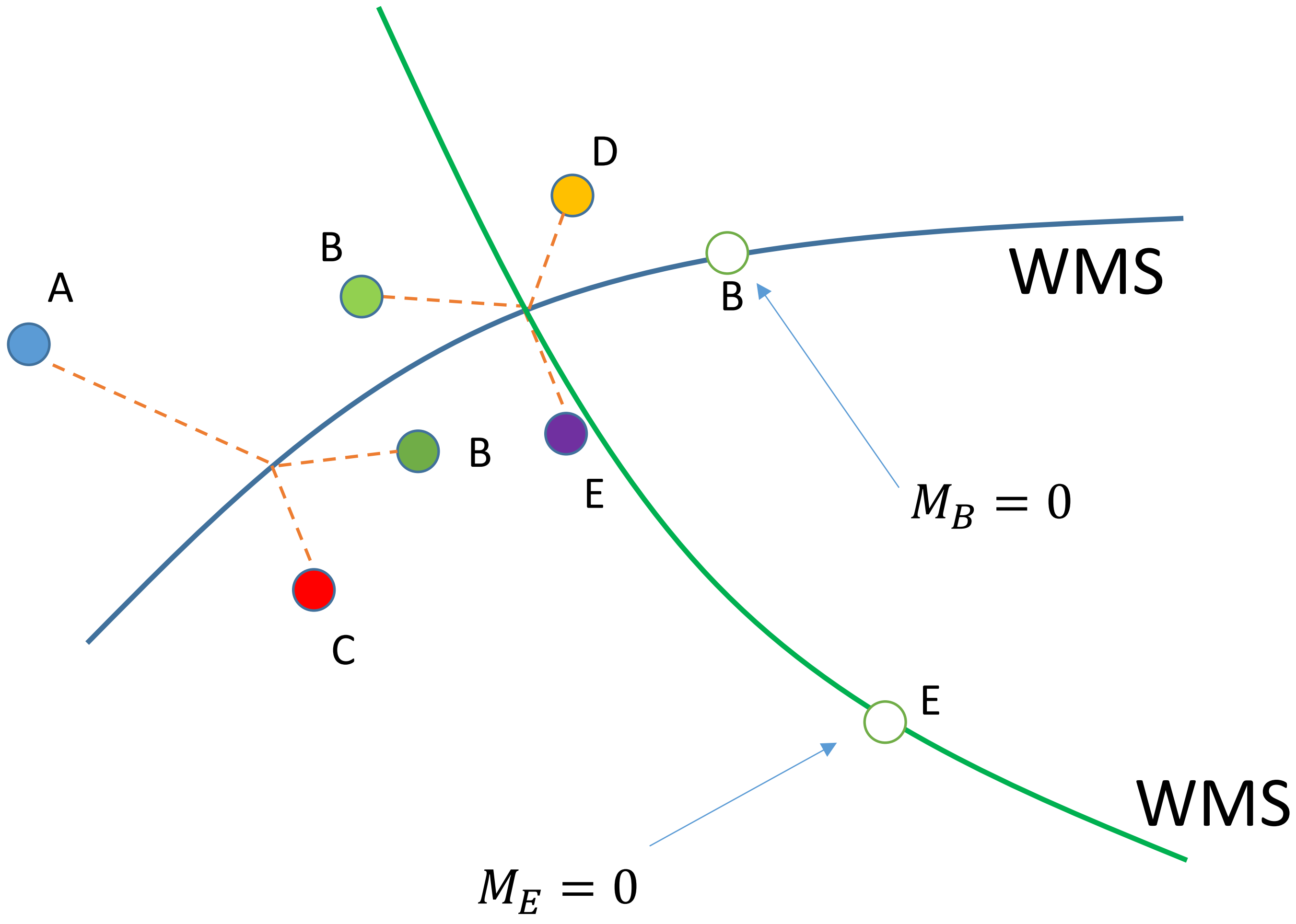


However, we know that no BPS states become massless within the Nilpotent orbit: the only singularity is the one at infinite distance

Therefore, any state which has a vanishing mass within the Nilpotent orbit must be absent from the spectrum

The decay is therefore not physically possible – matching the black hole results

If the state decays before it reaches the point of vanishing mass, one of its decay products will have a vanishing mass along the WMS of that decay



It is possible to show one of the decay products becomes massless for any number of moduli and any d:

First, one can show that the WMS of any decay which violates the filtration stretches all the way to $g = 0$

Along the wall we have mass equality between the initial state and its decay products

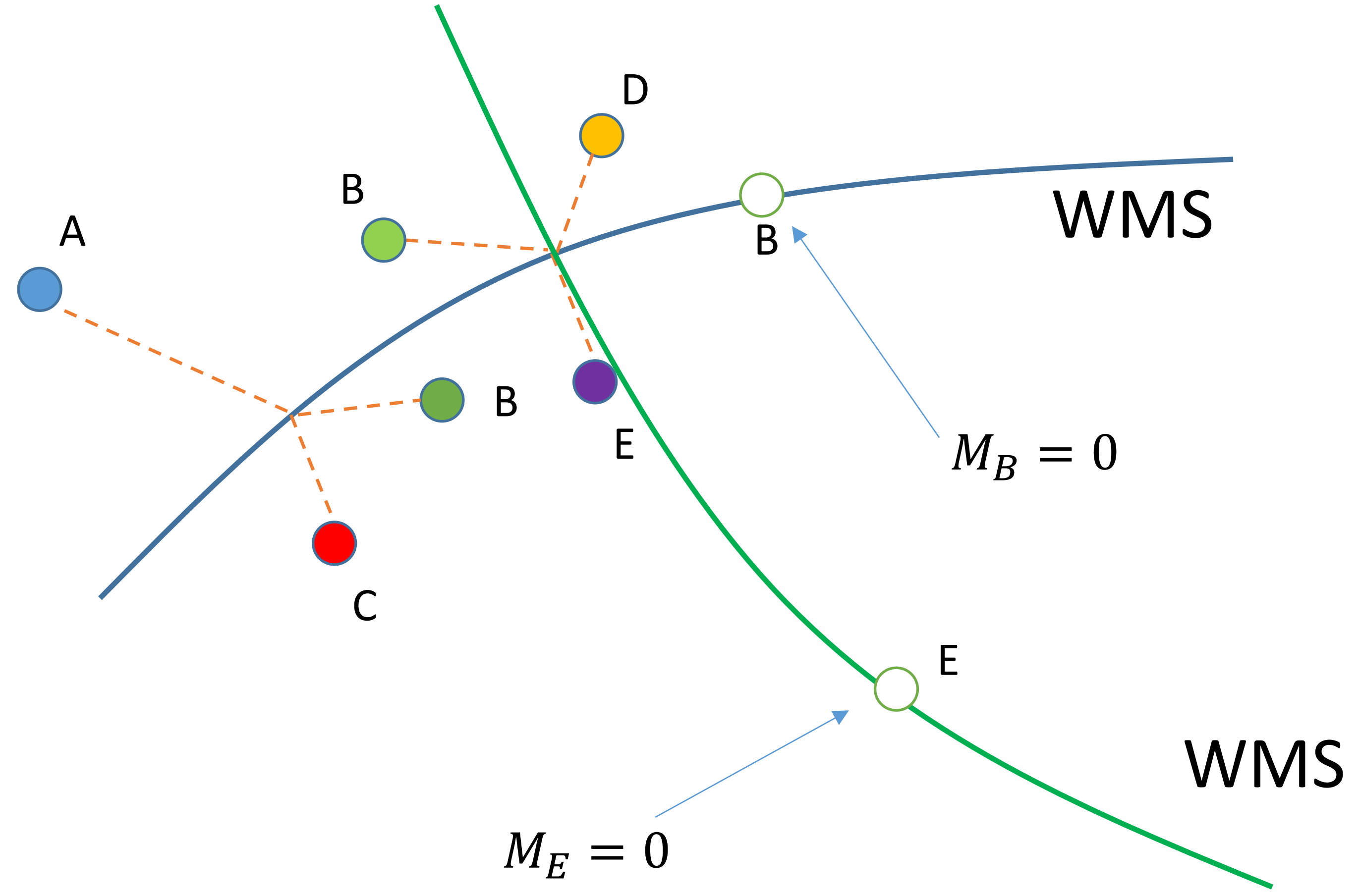
$$M_A\left(g_c\right) = M_B\left(g_c\right) + M_C\left(g_c\right)$$

If somewhere along the wall one of the decay products becomes more massive than the initial state, the other decay product must be massless

The absence of moduli space singularities implies therefore that there cannot be a decay to a state which becomes heavier in the $g \to 0$ limit

The remaining step to a full proof is to show, for $d > 1$, that it is not possible for the state to decay from the spectrum before it reaches the zero

[Work in progress…]



Summary

Evidence/proof that BPS states satisfy a stability filtration which is determined by the monodromy about infinite distance / zero coupling.

Consistent with ideas of Emergence: a qualitative connection between moduli space geometry and the BPS spectrum

Infinite distance $g \to 0$ limit $\Longleftrightarrow$ An ordering / filtration of BPS stability

Stability of BPS states has a microscopic description – interesting to explore how filtration / weak-coupling is described in that picture

Proof of distance conjecture, if electric black holes exist: Tower is exactly black hole microstates.

Thank You