Algebraic Dynamics and its Connections to Difference and Differential Equations (Hybrid)

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First of all, we wish to thank BIRS for supporting this 5-day workshop which had to be done in a hybrid format due to travel restrictions caused by the pandemic.

1 Overview

This workshop brings together (in a hybrid setting) researchers in Algebraic Dynamics and Algebraic Differential and Difference Equations. Each is a highly interesting area in its own and an important connection between them lies in the model theory of difference fields. There have been several significant results in both sides recently. In November 2020, our 5-day workshop with the same title was switched to the online format and the current workshop was an opportunity for a "sequel" with an in-person component. Due to the fourth wave of COVID-19, a state of public health emergency was declared in Alberta in late September 2021 and several participants decided not to join the in-person activities. At the end, there are 11 in-person participants and more than 30 online participants. There are 11 talks in total featuring a wide range of topics. 8 talks are given by in-person participants at the Banff Centre and available via Zoom while the remaining 3 talks are given by online participants. 2 of the talks are given by graduate students. There are also 2 problems sessions. While there are many further discussions among the in-person participants, there are rather limited interactions between them and the group of online participants outside the talks.

2 Presentation Highlights

Instead of choosing one or two talks as the highlight of the workshop, we decide to include every talk in the highlight:

- "Walks, difference equations and elliptic curves" by Michael Singer. The speaker considers the generating functions of certain random walks and explains how to relate several algebraic differential properties of those functions to orbits of points on algebraic curves using Galois theory of linear difference equations.

- "Uniformity in the Dynamical Bogomolov Conjecture" by Myrto Mavraki. The speaker presents uniform versions of the Dynamical Manin-Mumford and Dynamical Bogomolov Conjectures across 1-parameter families of certain split maps and curves.

- "The Zariski dense conjecture over fields of positive characteristic" by Sina Saleh. The speaker studies the existence of a point with Zariski dense orbit under the algebraic self-map of a semiabelian varieties over a field of positive characteristic.

- "A new invariant for difference fields" by Zoé Chatzidakis. The speaker introduces the limit degree and inverse limit degree of a tuple over a difference field.

- "Commutative bidifferential algebra" by Rahim Moosa. The speaker introduces the theory of commutative unital rings equipped with a binary operation which is a derivation in each argument.

- "The degree of nonminimality and differential equations" by James Freitag. The speaker introduces a new technique for showing that a nonlinear algebraic differential equation is strongly minimal based on the recently developed notion of the degree of nonminimality.

- "A Dynamical Shafarevich theorem for endomorphisms of $\mathbb{P}^N$" by Jamie Juul. The speaker proves a dynamical analogue of the Shafarevich conjecture for endomorphisms of $\mathbb{P}^N$ of degree at least 2 over a number field.

- "Rational maps with transcendental dynamical degrees" by Jeffrey Diller. The speaker presents first examples of rational maps and birational maps with transcendental dynamical degrees.

- "Toward a general Ax-Schanuel theorem for geometric structures" by Joel Nagloo. The speaker highlights the role of the model theory of differentially closed fields towards proving the Ax-Schanuel theorem for uniformizers of geometric structures.

- "Stochastic dynamics and equidistribution" by Paul Fili. The speaker introduces the canonical heights of certain stochastic algebraic dynamical systems and study equidistribution properties.

- "The Zeta Functions for endomorphisms of positive characteristic tori " by Keira Gunn. The speaker gives a complete classification of when the the Artin-Mazur zeta function associated to a linear endomorphism of a positive characteristic torus is algebraic.

3 Further comments

The workshop brings together researchers from two different areas and our goal is to stimulate discussions and exchange of ideas between the two groups. Professor Michael Singer sent us the following comments.

"I have not been actively engaged in questions concerning arithmetic dynamics but nonetheless found the talks on this subject accessible and stimulating. In particular, Jeffrey Diller's talk included questions concerning rational approximation of solutions of algebraic differential equations. We were able to further discuss this aspect after his talk and I was able to point to several papers on this subject of which he was unaware. I do plan on contacting him soon to see if these were useful and if there are other related differential algebra questions we could discuss. I also had discussions with James Freitag and Joel Nagloo concerning questions of Liouvillian first integrals of differential equations. These continued via email after the conference and we were able to resolve some of the questions they had asked."

References

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