Geometry in the service of equity: moving frames in learning analytics

Debra Lewis
Mathematics Department, UCSC

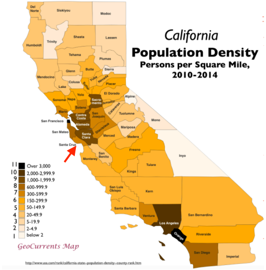
November 22, 2021

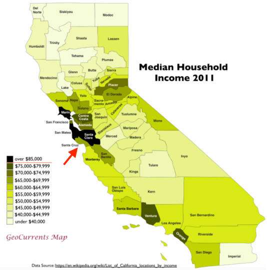
Serving diverse communities

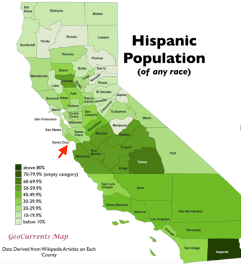
Silicon Valley is roughly 20 miles away from UCSC.
So is Watsonville.

Many UCSC students are from affluent, highly educated, STEM-centric communities.

Many are from low-income, agricultural communities with inadequate educational and communications infrastructure.







Key demographics at UCSC

Ethnicity

- White: 30%
- Asian: 28%
- Latinx: 27%
- International: 8%
- Other underserved groups: 5%

EOP—educationally disadvantaged

- First Generation $\approx \frac{1}{3}$
- Low income $\approx \frac{1}{3}$
- Undocumented, veteran, ...

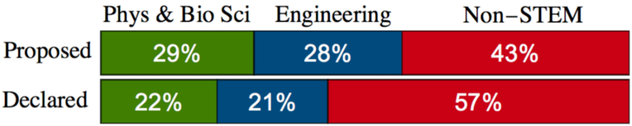
Gender

- 60% female in life sciences majors
- 32% female in physical sciences majors

Leaks, eddies, and bottlenecks in STEM pipelines

Half the proposed STEM students graduate within four years.
One fourth don't graduate within six years.

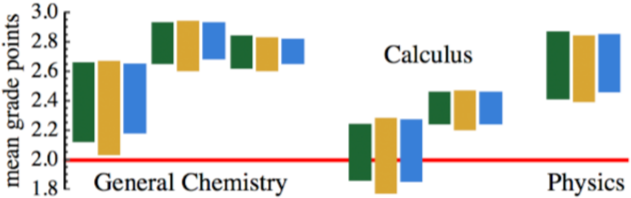
Many students switch to non-STEM majors.



Key concerns:

- time to declaration of major
- time to graduation
- STEM persistence
- # of courses repeated
- # of courses taken that satisfy requirements

Grade gaps in core STEM sequences, 2015–18



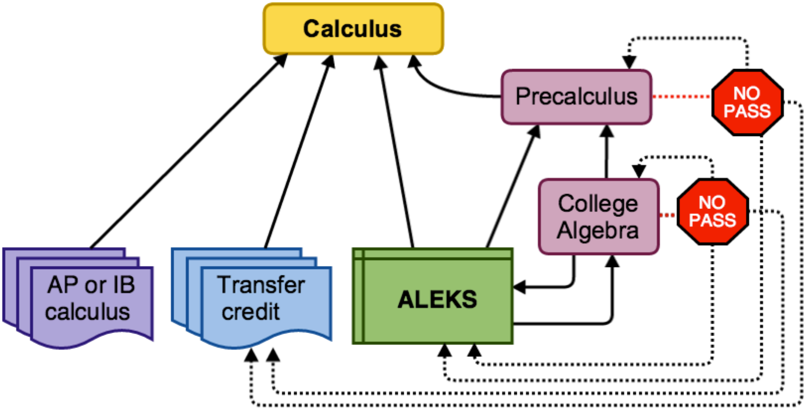
Bottom of bar: mean grade for underserved group
Top of bar: mean grade for students outside group.

Green: EOP (educationally disadvantaged)
Gold: Underserved minorities
Blue: First Generation.

Are gaps largely due to differences in math skills on arrival?

Sequences shown are for life sciences & chemistry majors.

Pathways to calculus (and chemistry) at UCSC



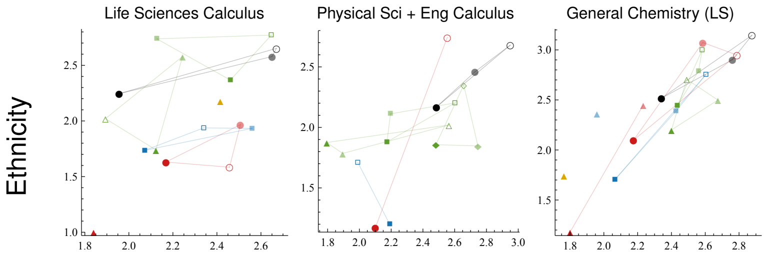
Students who aren't calculus-eligible after their initial placement can review and reassess in ALEKS PPL and/or do preparatory math coursework.

Grade gaps by math placement & prep pathway

First grade in sequence, aggregated by ethnicity and placement + prep pathway.

'Constellations' link outcomes for the same pathway.

Opaque: Latinx, semi-opaque: White, outlined: Asian.



First STEM class: Fall 2015 or 16 (horiz), Fall 2017 or 18 (vert).

Graph decoder

Shapes indicate the eligibility after the ***initial* math placement**:

- △ College Algebra
- □ Precalculus
- ◇ life science calculus, but not phys sci + eng calculus
- ○ the relevant sequence

Colors indicate the **preparation pathway** taken:

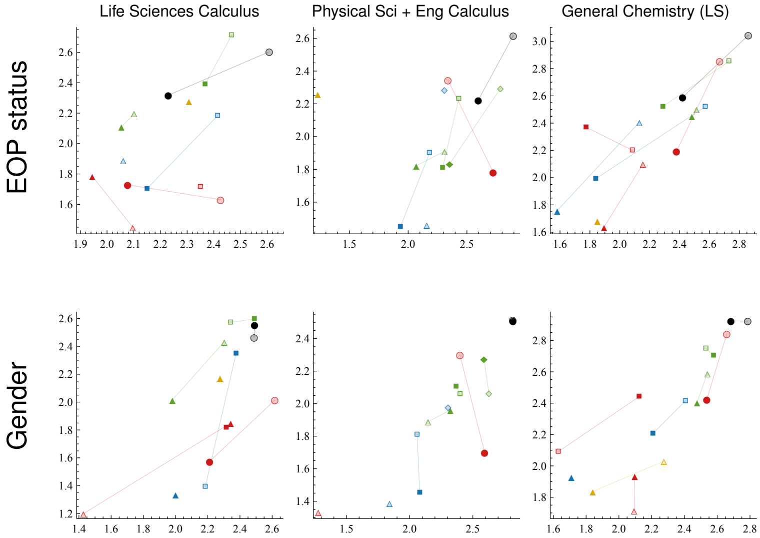
Black: Eligible for the sequence on the initial assessment, no prep courses.

Green: Reassessed in ALEKS PPL and became sequence eligible, no prep courses.

Blue: Took Precalculus (but not College Algebra) as placed.

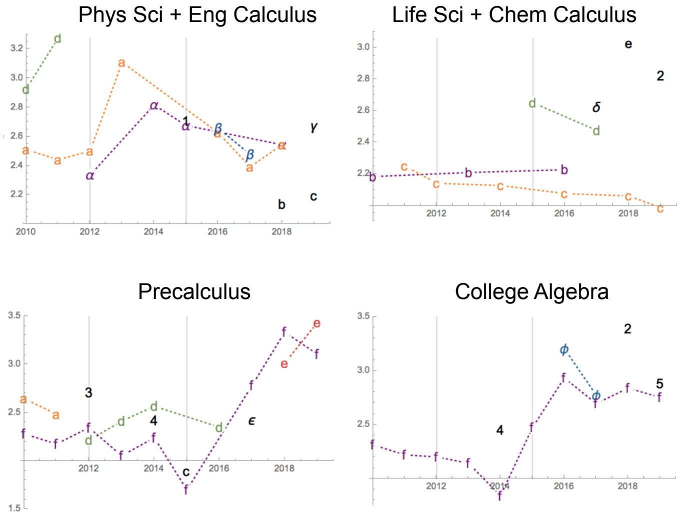
Yellow: Took Precalculus and College Algebra, as placed.

Red: Took Precalculus even though eligible for the relevant sequence.



Life Sciences Calculus
Physical Sci + Eng Calculus
General Chemistry (LS)
EOP status
Gender

Do mean GPA’s mean anything?



Course means for fall offerings: 2010-19.
Labels indicate instructor.

Standardized scores and moving frames

The group $G = \mathbb{R}^+ \ltimes \mathbb{R}$, with

$$(s, t)(s', t') = (s\, s', s\, t' + t),$$

acts on $M = \mathbb{R}^n \backslash (\mathbb{R}\, \boldsymbol{f})$, where $\boldsymbol{f} = \frac{1}{\sqrt{n}}(1, \ldots, 1)^T$, by

$$(s, t) \cdot \boldsymbol{x} = s\, \boldsymbol{x} + t\, \boldsymbol{f}.$$

The left-equivariant moving frame

$$\rho(\boldsymbol{x}) = (\sigma(\boldsymbol{x}), \boldsymbol{x}^T \boldsymbol{f}),$$

where σ denotes the standard deviation, determines a section

$$[\boldsymbol{x}] \mapsto \rho(\boldsymbol{x})^{-1} \cdot \boldsymbol{x} = \frac{1}{\sigma(\boldsymbol{x})}(\boldsymbol{x} - (\boldsymbol{x}^T \boldsymbol{f})\boldsymbol{f}),$$

the standardization of $\boldsymbol{x}$.

What do we mean by 'equity'?

If we represent k outcomes for n sub-cohorts using a matrix $A \in \mathbb{R}^{k \times n}$, what properties of A indicate an equitable process?

A 'one size fits all' process yielding the same outcomes for all sub-cohorts determines a matrix with equal columns:

$$A = \boldsymbol{u}\boldsymbol{f}^T \qquad \text{for some} \quad \boldsymbol{u} \in \mathbb{R}^k.$$

Student priorities (e.g. GPA, courseload, time to graduation) are influenced by

- financial and/or temporal constraints
- societal and/or family expectations
- career goals...

so different outcomes of equal value may be more equitable, if everyone gets what they want/need.

Investigating 'one size fits all' equity

The rank one modification

$$\zeta(A) := A - (A\boldsymbol{f})\boldsymbol{f}^T$$

yields a matrix whose columns sum to zero.

$$\zeta(A) = 0 \qquad \Longleftrightarrow \qquad A = \boldsymbol{u}\boldsymbol{f}^T,$$

so $\|\zeta(A)\|$ captures the 'inequity' of the process.

The group $G = \mathbb{R}^+ \ltimes \mathbb{R}^k$, with

$$(s, \boldsymbol{t})(s', \boldsymbol{t}') = (s\,s', s\,\boldsymbol{t}' + \boldsymbol{t}),$$

acts on $M = \mathbb{R}^{k\times n} \backslash \zeta^{-1}(0)$ by

$$(s, \boldsymbol{t}) \cdot A = s\,A + \boldsymbol{t}\,\boldsymbol{f}^T.$$

An unfair(ness) and mean moving frame

The left-equivariant moving frame

$$\rho(A) = (\|\zeta(A)\|, A\boldsymbol{f})$$

determines a section

$$[A] \mapsto \rho(A)^{-1} \cdot A = \frac{\zeta(A)}{\|\zeta(A)\|}.$$

The moving frame ρ can be used to compare the equity and efficacy of various processes.

The section captures crucial aspects of the structure of unequal outcomes, but encodes a *lot* of information.

Even ρ may output more numbers than stakeholders want.

In the following graphs, data for a 'portfolio' of six outcomes is further compressed by mapping $\rho(A)$ to

$$\left(\frac{\|A\boldsymbol{f}\|}{\|A_{\mathrm{all}}\boldsymbol{f}\|}, \|\zeta(A)\| \right),$$

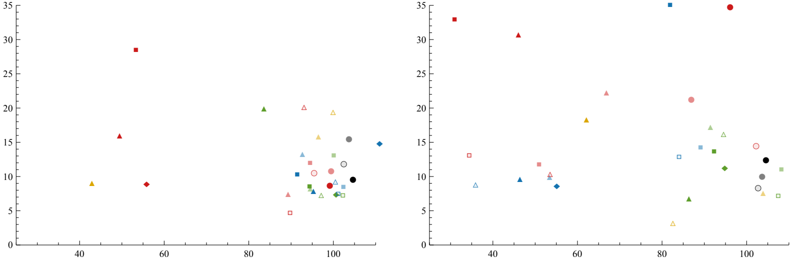
where A_{all} is the outcome matrix for all demographic groups and all pathways for the course in question.

The first four outcomes capture key information about 'flow' through the sequence—the percentage of students

- who *pass at least one course* in the sequence,
- who *successfully complete* the sequence,
- with *no non-passing grades* in the sequence,
- with *at most one non-passing grade* in the sequence.

The remaining two outcomes are the *mean grades* for the *first course* and *over all courses* taken in the sequence.

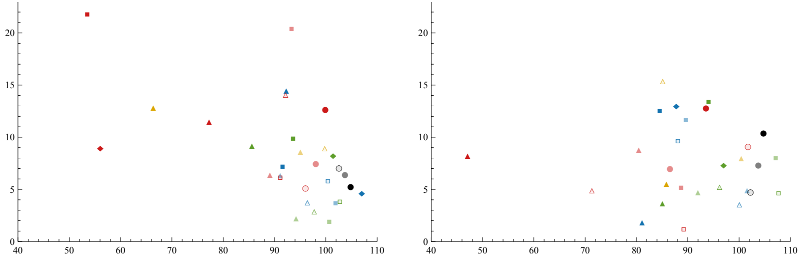
Ethnicity



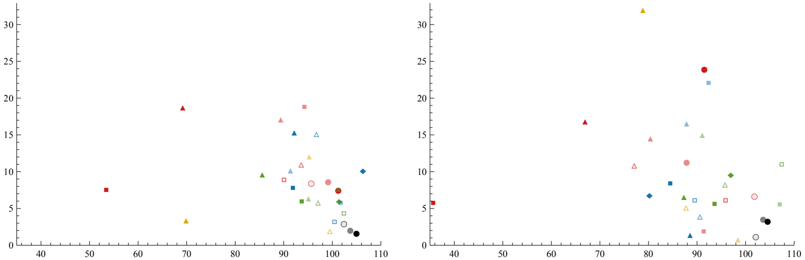
Left: Started STEM in Fall 2015 or 16; right: 2017 or 18.

Opaque: Calculus for physical sciences + engineering
Semi-opaque: Calculus for life sciences + chemistry
Outlined: General chemistry

EOP status



Gender



How to measure ‘To each their own’ equity?

If A is nearly rank one, with $\sigma_1 \gg \sigma_2$, but $\boldsymbol{v}_1$ isn’t close to $\boldsymbol{f}$, the associated process is inequitable!

If $\tilde{A}$ is a (somehow) known optimal, equitable outcome matrix and $A = s\tilde{A}$ for some $s \in \mathbb{R}^+$, the associated process is equitable.

Could matrix decompositions, e.g. SVD or QR, be used to usefully compare outcome matrices?

Since SVD and the QR decomposition aren’t unique, why talk about them in a moving frames talk?

Partial moving frames

A partial moving frame $\phi : M \to G$ acts like a moving frame, with wriggle room for isotropy. (L., Nigam, Olver, 2003)

$\phi : M \to G$ is a left-equivariant partial moving frame if

$$\phi(g \cdot m)^{-1}(g \cdot m) = \phi(m)^{-1} \cdot m$$

for all $g \in G$ and $m \in M$.

A left-equivariant partial moving frame determines a section

$$[m] \mapsto \phi(m)^{-1} \cdot m,$$

$$(UQ)\Sigma(VQ)^T = U\Sigma V^T \quad \Longleftrightarrow \quad Q\Sigma = \Sigma Q$$

The rank one decomposition and a section

$$\beta : \left(S^n \times S^n\right)/\mathbb{Z}_2 \to S^n \times S^n$$

determine a partial moving frame ϕ for the action of $O(n)^2$ on matrices in $\mathbb{R}^{n\times n}$ with non-degenerate singular values:

$$(U, V) = \phi\left(\sum_{j=1}^{n} \sigma_j \boldsymbol{u}_j \boldsymbol{v}_j^T\right) = ([\boldsymbol{u}_1 \cdots \boldsymbol{u}_n], [\boldsymbol{v}_1 \cdots \boldsymbol{v}_n])$$

for

$$(\boldsymbol{u}_j, \boldsymbol{v}_j) = \beta([(\boldsymbol{u}_j, \boldsymbol{v}_j)]).$$

The associated section maps matrices to their singular values.

Questions?

Advice?