Quantum circuit and matter collisions in the bulk

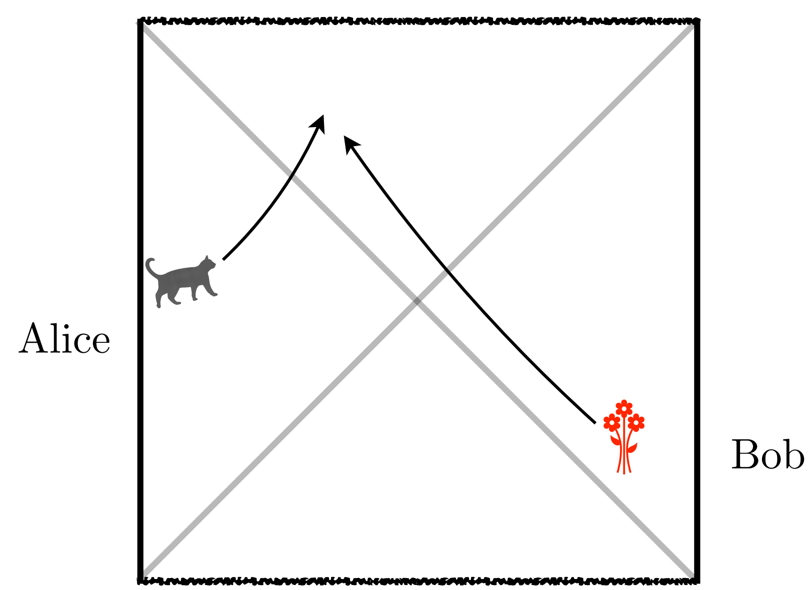
Y.Z. arXiv:2011.06016
Felix Haehl and Y.Z. arXiv:2104.02736
Felix Haehl, Alex Streicher, Y.Z. arXiv:2105.12755
Felix Haehl and Y.Z. Work in Progress

J. Maldacena, hep-th/0106112
M. Raamsdonk, arXiv:1015.3035
J. Maldacena, L. Susskind: arXiv:1306.0533

Motivation

- Two distant black holes can be connected in the interior through a wormhole.

- Such an Einstein-Rosen bridge can be interpreted as an entangled state: ER = EPR



D. Marolf, A. Wall, arXiv:1210.3590

- A mystery: Two signals sent in from the two different boundaries can meet in the interior. On the other hand, there are no boundary Interactions, so what governs the law according to which the two signals interact?

- In particular, signals should interact in the same way no matter they come from the same boundary or come from two different boundaries.

This is required in gravity: equivalence principle, but not clear from boundary point of view.

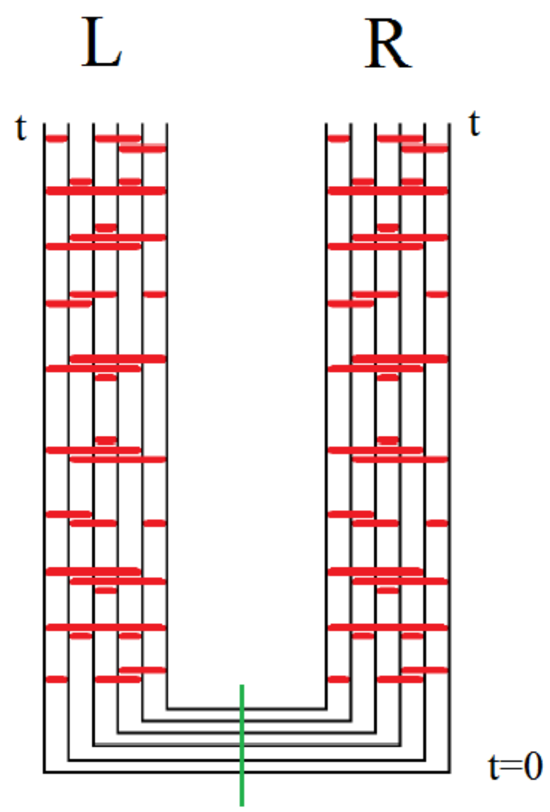
Outline

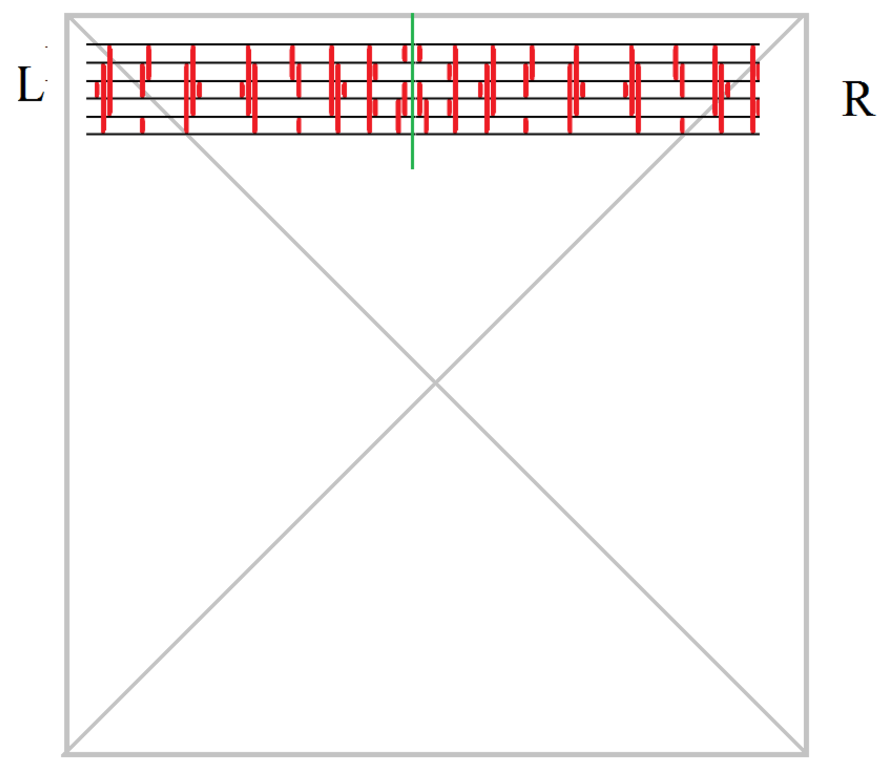
- Thermal field double and quantum circuit in the interior of eternal black hole
- A meeting in the interior of a wormhole and overlap of perturbations in the quantum circuit
- The overlap region in the quantum circuit and post-collision region
- Comparison with collisions between signals coming from one boundary
- Collisions between localized shocks
- Future directions and unanswered questions

Bulk tensor network and quantum circuit

B. Swingle arXiv:1209.3304
T. Hartman, J. Maldacena arXiv:1303.1080
L. Susskind arXiv:1411.0690

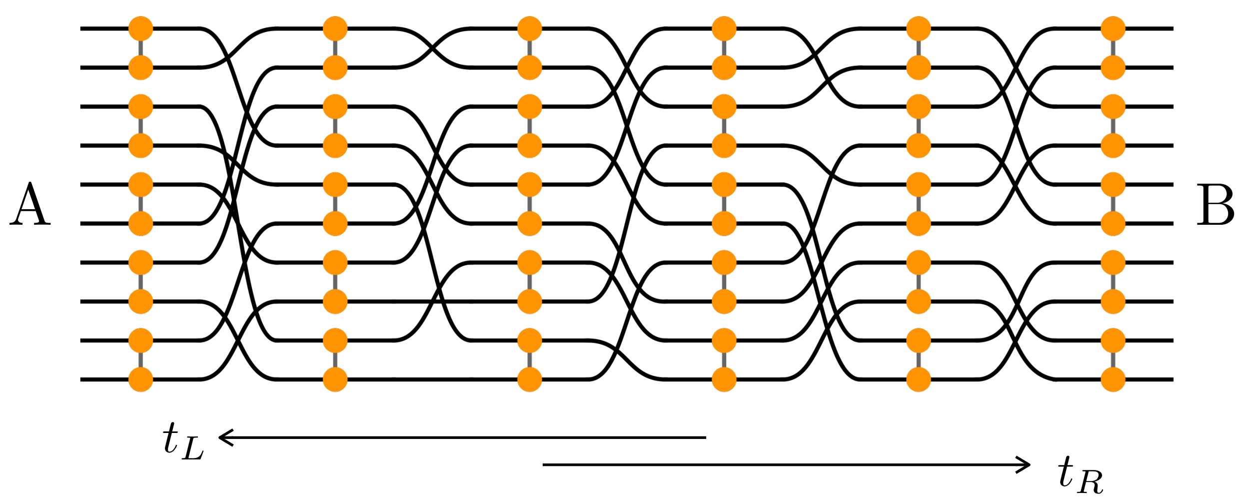
- The bulk geometry reflects the minimal circuit preparing the state.



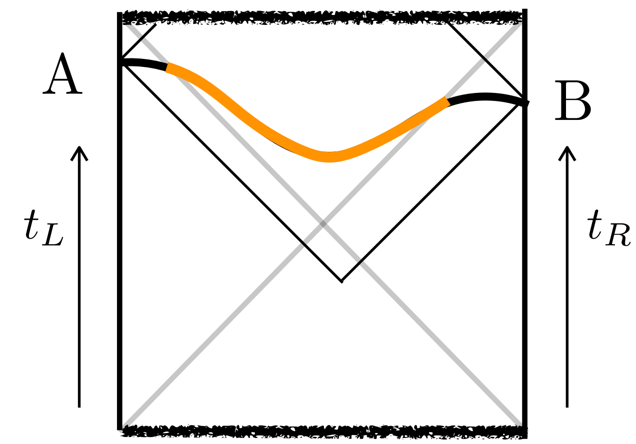


- As we apply unitary time evolution to the circuit, the state gets more complex, the minimal circuit gets longer, and the Einstein-Rosen bridge also gets longer.

Time-evolved thermofield Double



(a) Quantum circuit



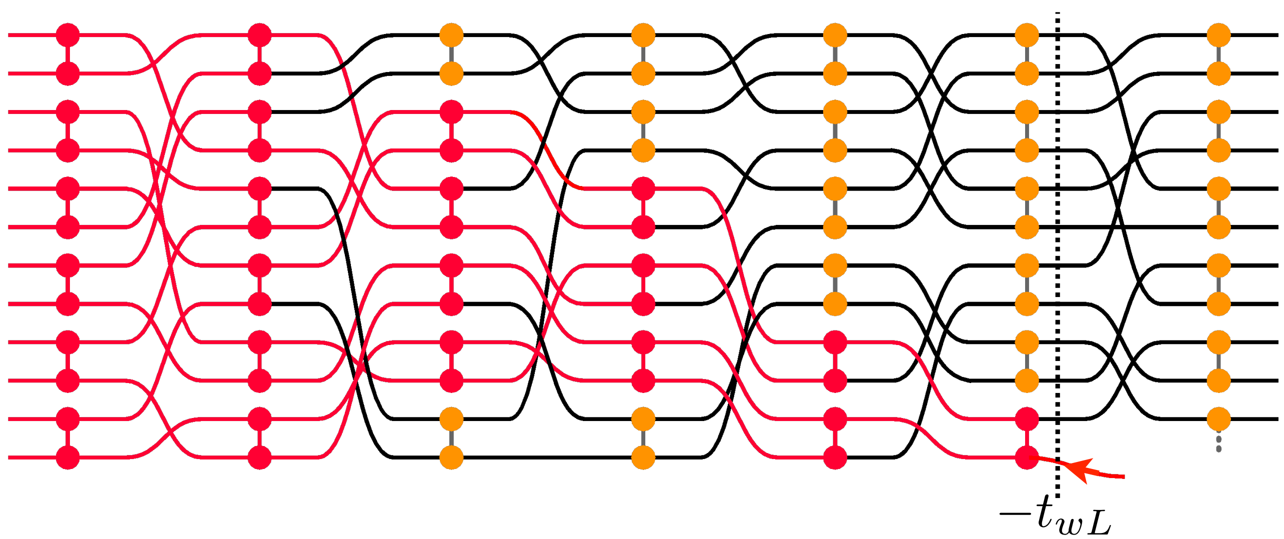
(b) Wormhole geometry

- The orange gates can be undone from either side. We call them healthy gates.
- The number of healthy gates per unit time is constant.

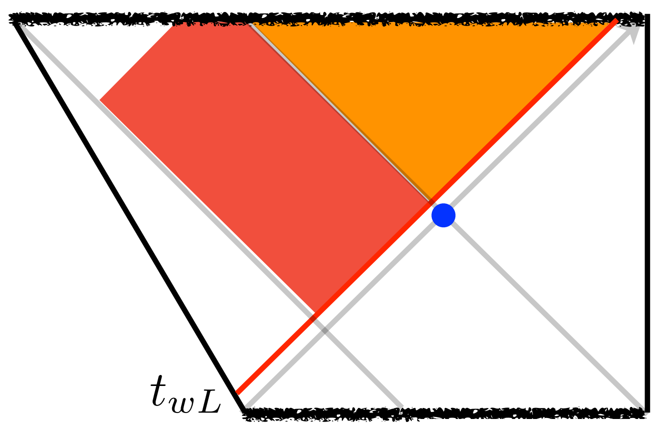
Entanglement ⟹ *Shared quantum circuit*

P. Hayden, J. Preskill   arXiv:0708.4025v2
L. Susskind, Y.Z.   arXiv:1408.2823
A. Brown, L. Susskind, Y.Z. arXiv:1608.02612
Y. Z., arXiv:1711.03135

Perturbed thermofield double



(a)

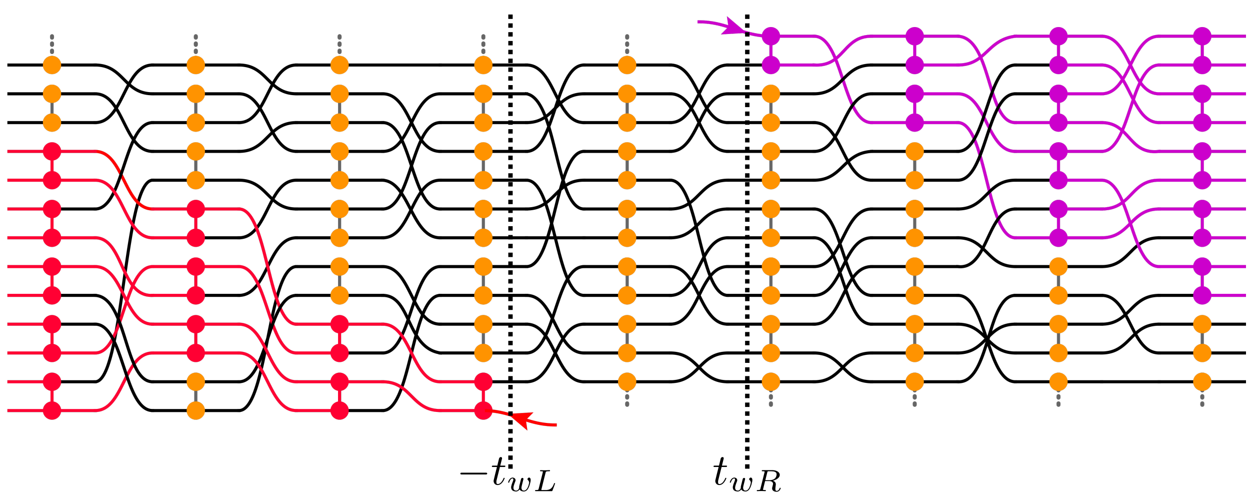


(b)

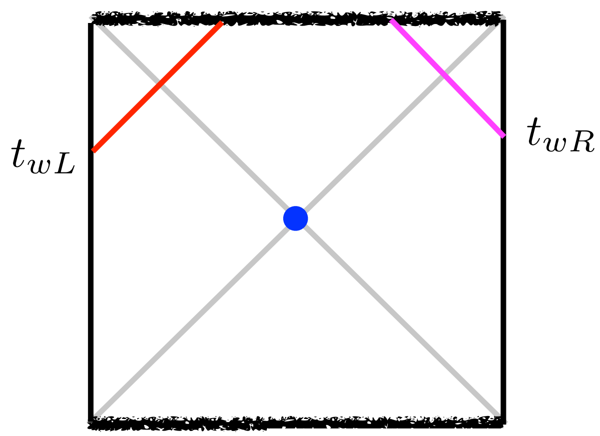
- Epidemic model: The red arrow represents the extra qubit due to Alice's perturbation. Any qubits that interact directly or indirectly with the perturbation will get perturbed relative to the original circuit describing the thermofield double state.

- The gates that can be undone by both Alice and Bob (healthy gates) are stored in the orange region.

Quantum circuit with perturbations from both sides



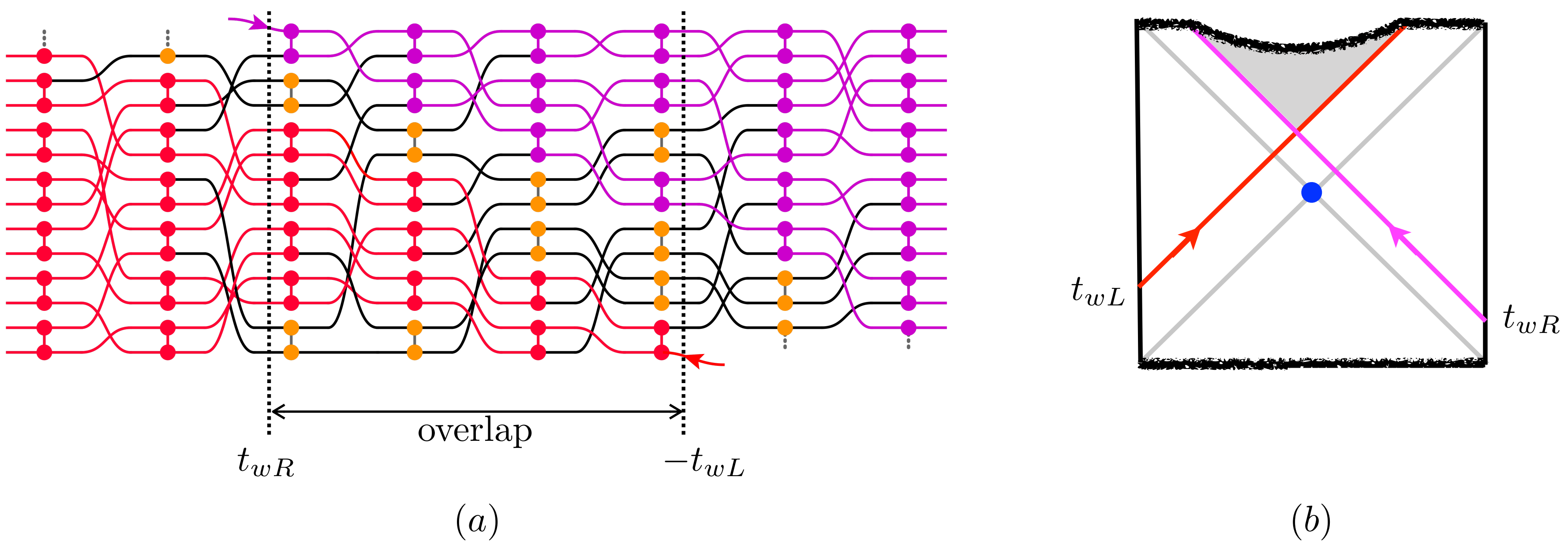
(a)



(b)

- When $t_{wL} + t_{wR} > 0$, the two perturbations do not have overlap in the quantum circuit. Correspondingly, the signals sent into the bulk hit the singularity before they have a chance to meet in the wormhole.

A meeting in the interior of the wormhole

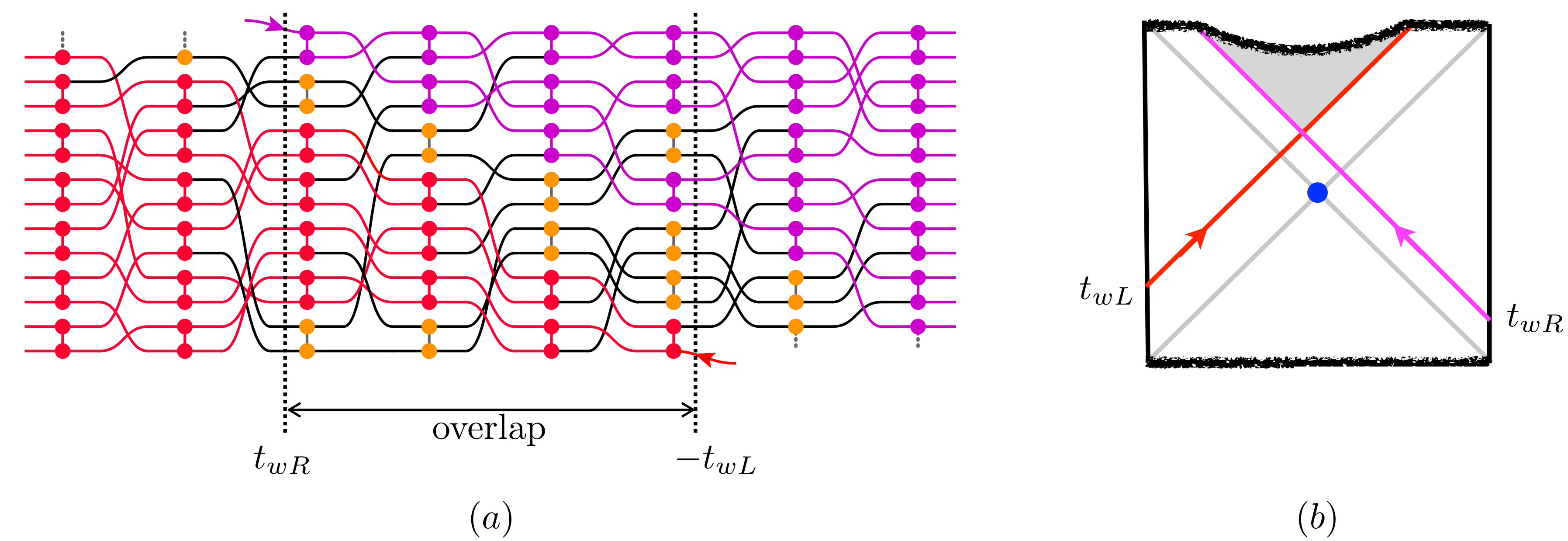


- When $t_{wL} + t_{wR} < 0$, the two perturbations have overlap in the quantum circuit. Correspondingly, the signals collide inside the wormhole. The larger the overlap is, the stronger the collision is.

- Can diagnose the collision by a particular six-point function.

Felix Haehl and Y.Z.   arXiv:2104.02736

Properties of a quantum circuit with overlapping perturbations and the spacetime geometry after the collision



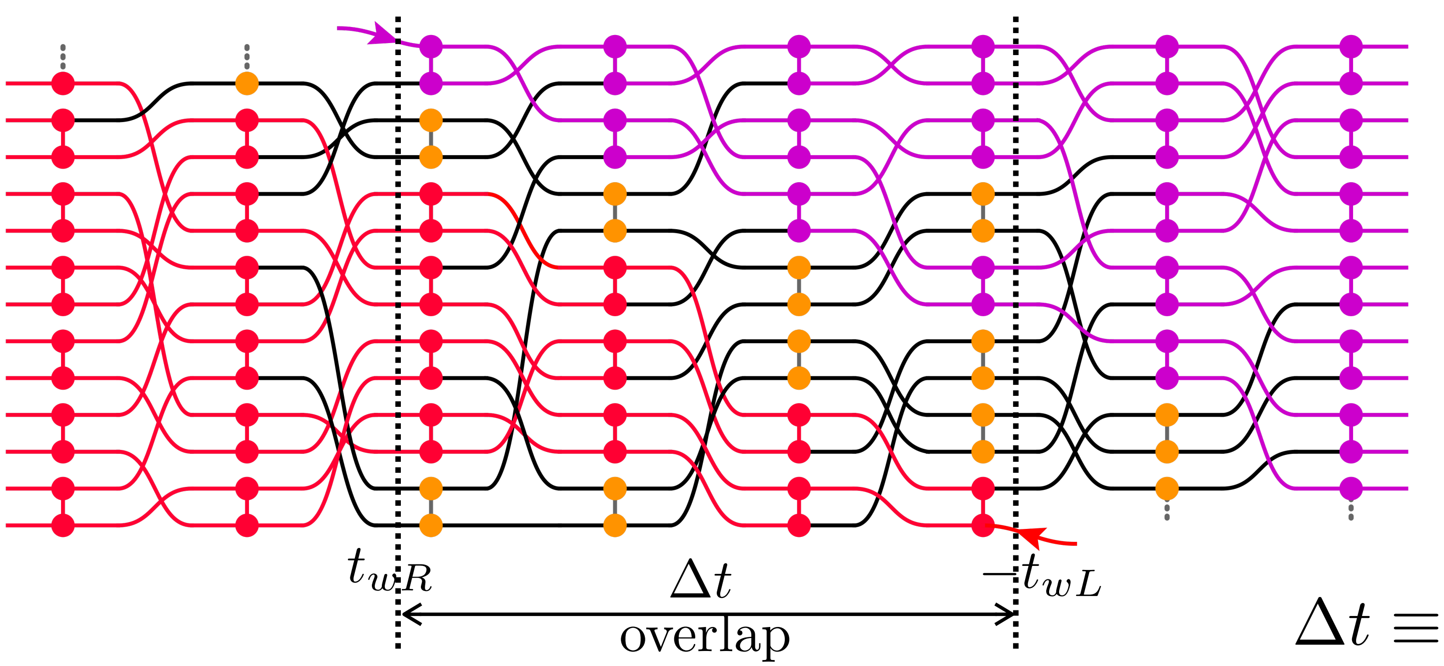
- The healthy gates in the overlap region in the quantum circuit can be undone from both sides. They are meaningless for one boundary system alone.

- These healthy gates are stored in the post-collision region in the shared interior.

Felix Haehl, Alex Streicher, Y.Z.   arXiv:2105.12755

Estimating the number of healthy gates in the overlap region

- $\mathcal{F}_6'(t_1, t_2) \equiv 1 - \frac{n_\infty[\psi_1(-t_1)\rho^{\frac{1}{2}}\psi_2(t_2)] - n_\infty[\rho^{\frac{1}{2}}]}{n_{max} - n_\infty[\rho^{\frac{1}{2}}]}$
- $\mathcal{F}_6'$ can be evaluated in Schwarzian theory
- $\mathcal{F}_6'$ is the probability of a gate being healthy with two perturbations present



$\Delta t \equiv -t_{wL} - t_{wR}$

$$\frac{N_{\text{healthy}}}{S} = \int_{t_{wR}}^{-t_{wL}} dt\ \mathcal{F}_6'(t - t_{wR}, -t_{wL} - t) = \frac{N_{\text{healthy}}}{S}(\Delta t)$$

Computing the spacetime volume in the post-collision region

- Post-collision region: A larger black hole forms

$$\frac{\tilde{r}_h^2}{r_h^2} = 1 + \frac{2\delta S_1}{S} + \frac{2\delta S_2}{S} + \frac{4\delta S_1 \delta S_2}{S^2} \cosh^2\left(\frac{\pi}{\beta}\Delta t\right)$$

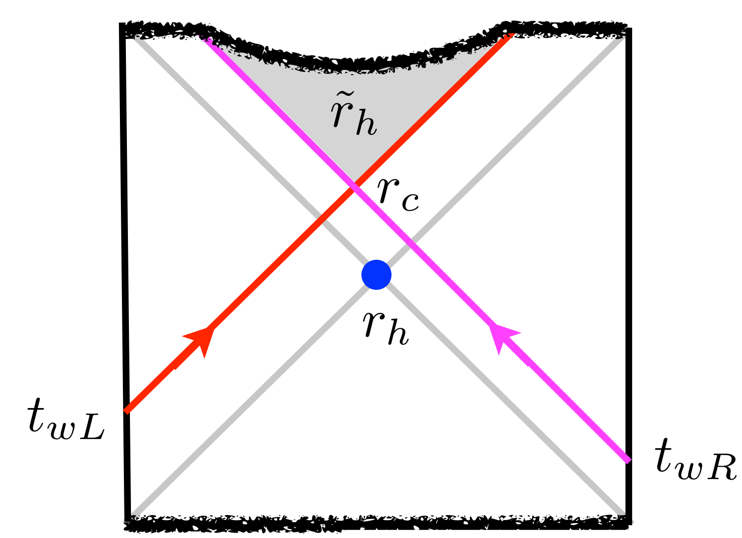
$$\Delta t \equiv -t_{wL} - t_{wR}$$

T. Dray, G. 't Hooft 1985
S. Shenker, D. Stanford arXiv:1312.3296

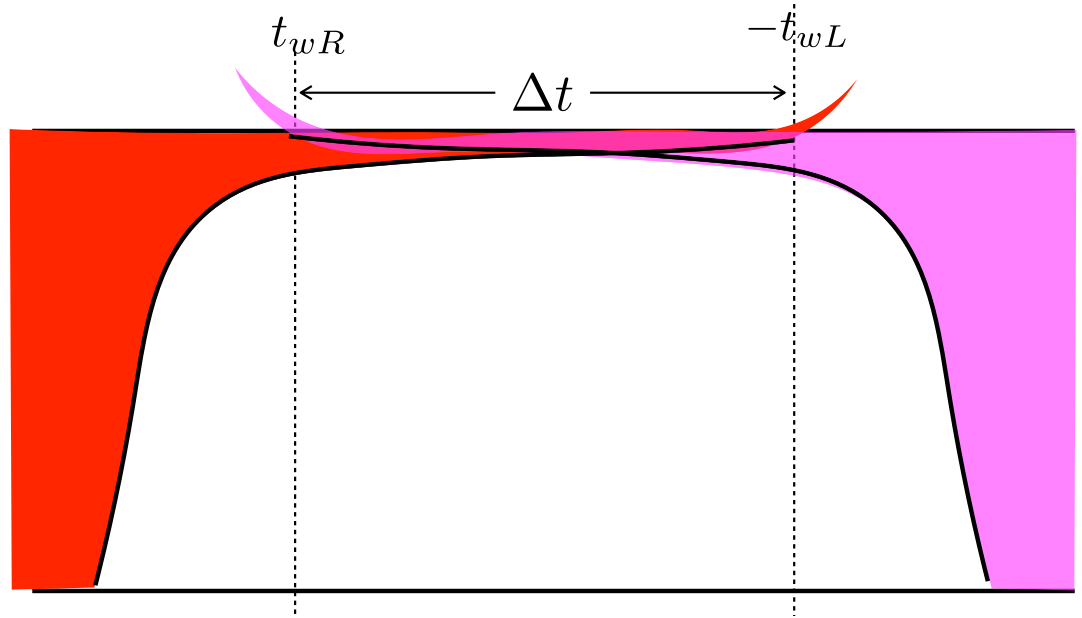
- Spacetime volume of post-collision region in BTZ black hole

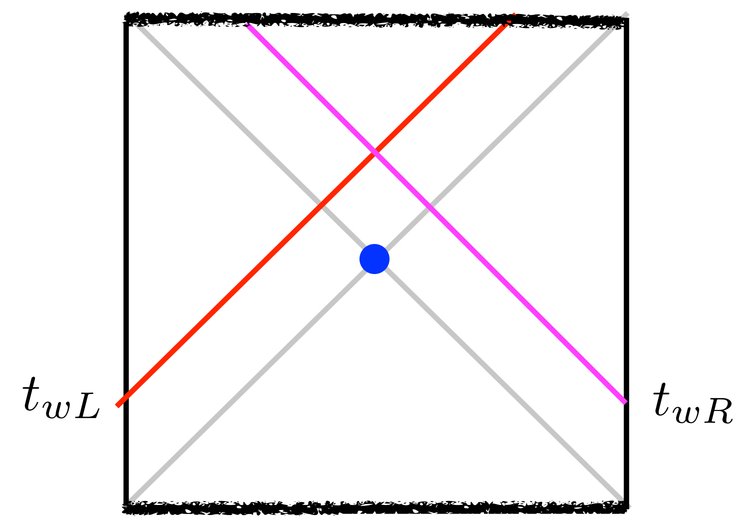
$$V = 2\pi \tilde{r}_h l^2 \left(\frac{1}{2}\log\frac{\tilde{r}_h + r_c}{\tilde{r}_h - r_c} - \frac{r_c}{\tilde{r}_h}\right)$$

$$= V(\Delta t)$$



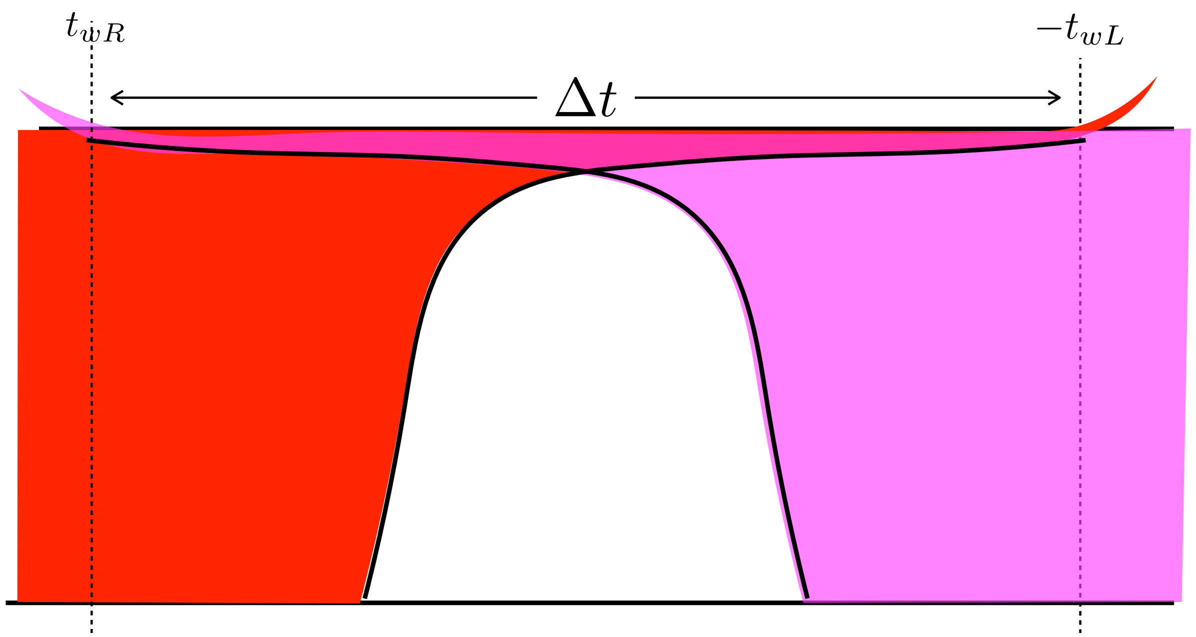
1. Early time $\frac{\beta}{2\pi} \ll \Delta t \ll t_*$

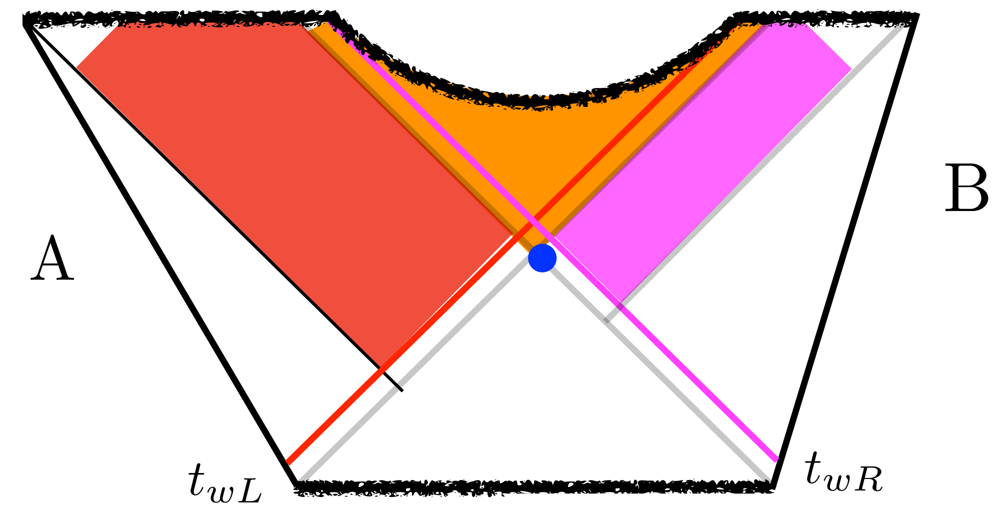




- The number of healthy gates in the overlap region grows linear in Δt
- Can ignore non-linear effect. As the collision point moves toward the bifurcating surface, the post collision region grows.

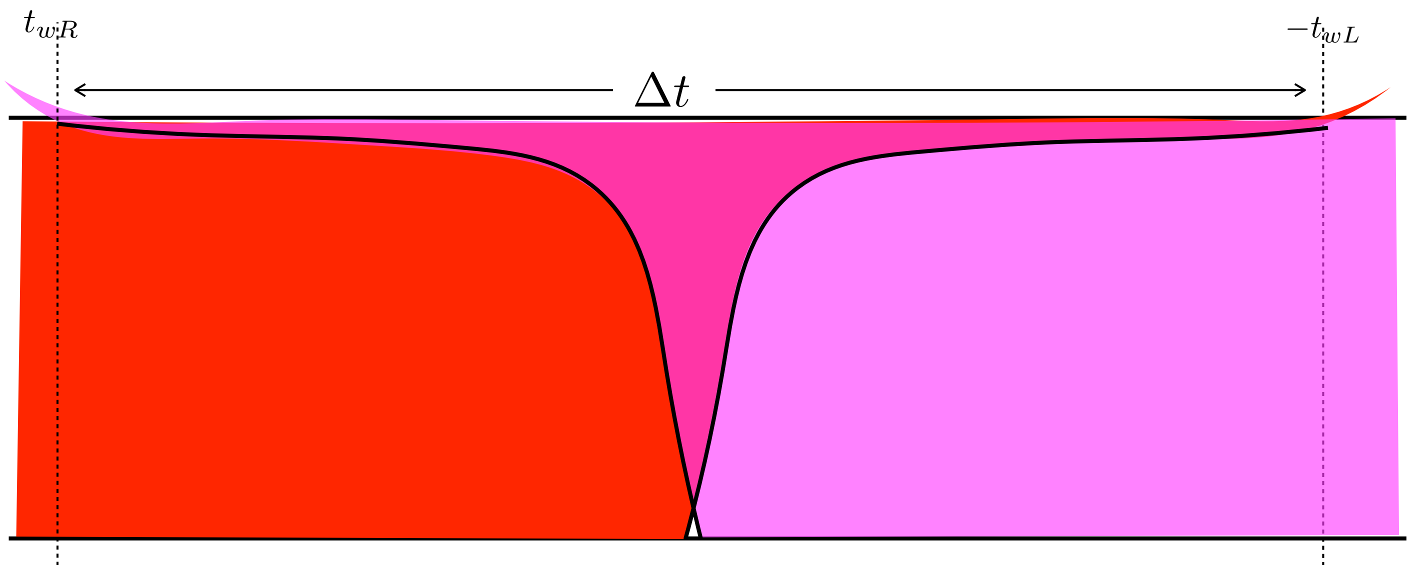
2. Intermediate time $t_* \ll \Delta t \ll 2t_*$

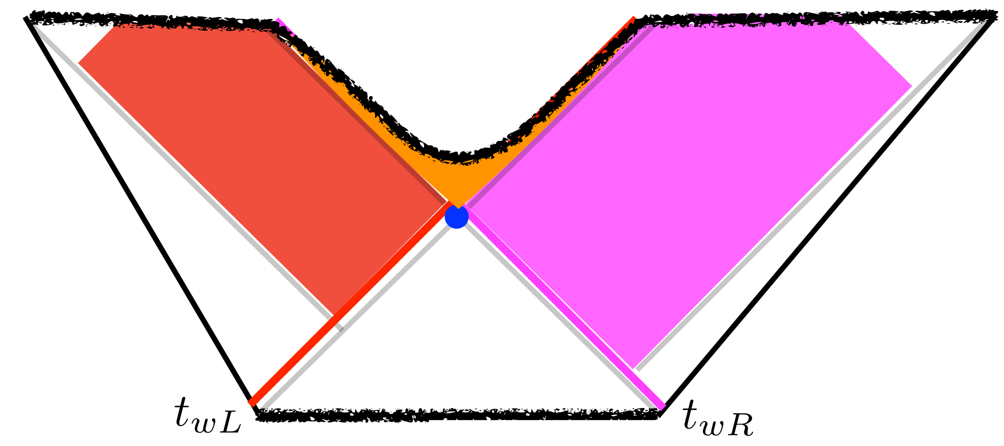




- The number of healthy gates decreases linearly in Δt
- The collision point stays close to the bifurcating surface. Singularity bends down due to backreaction. The post-collision region shrinks.

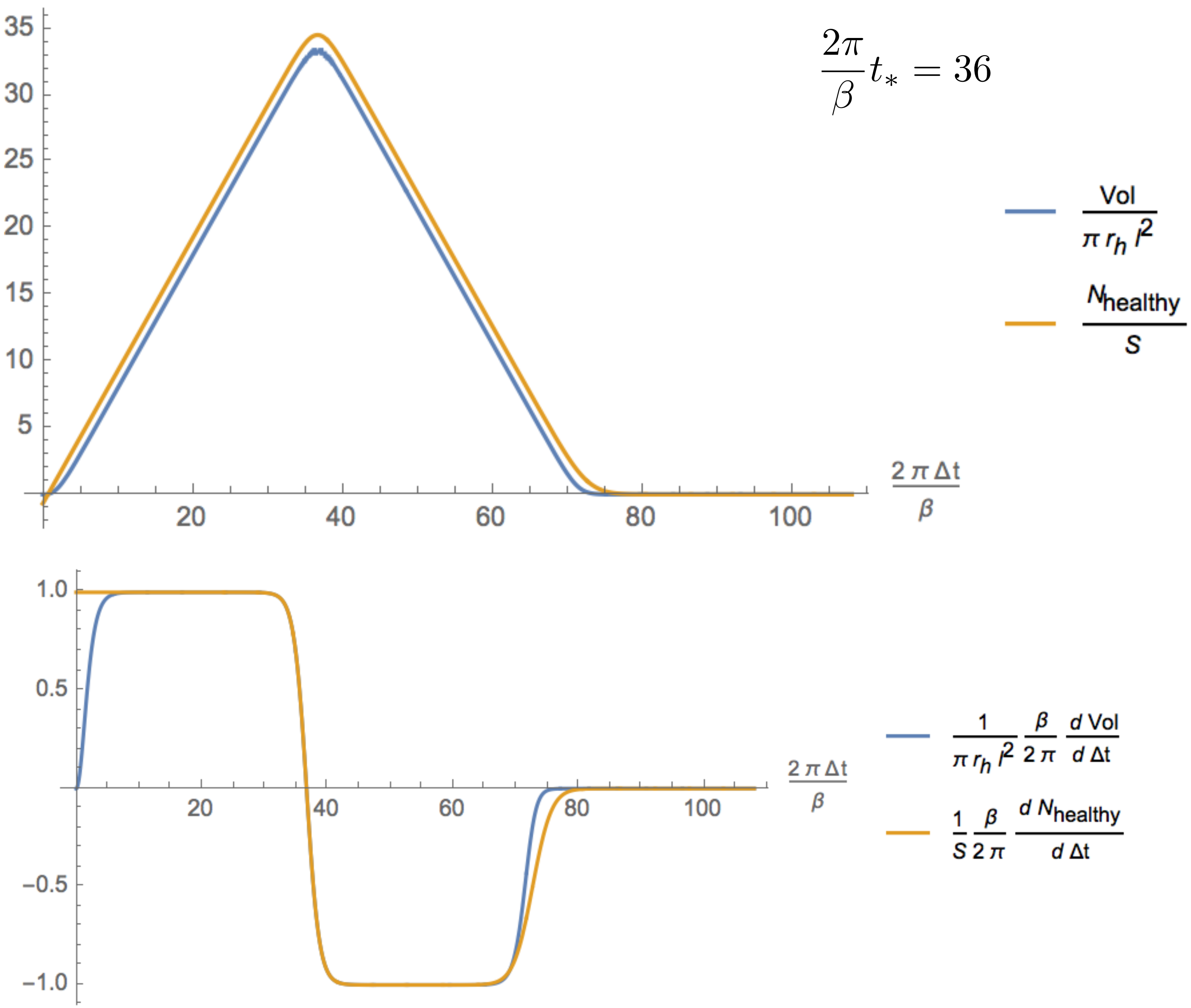
3. Late time $\Delta t \gg 2t_*$





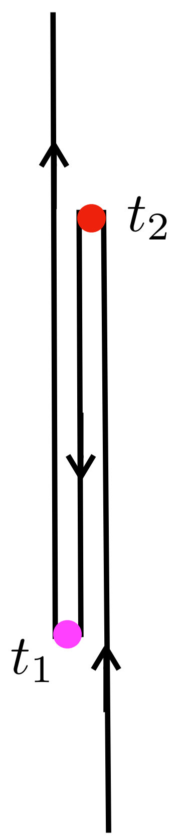
- The number of healthy gates is exponentially small
- A large black hole forms. The collision happens exponentially close to the singularity.
- Little is known about collision at this high energy. There might be significant corrections from stringy effect.

Comparison of the total number of healthy gates in the overlap region $N_{\text{healthy}}(\Delta t)$ and the spacetime volume of post collision region $V(\Delta t)$

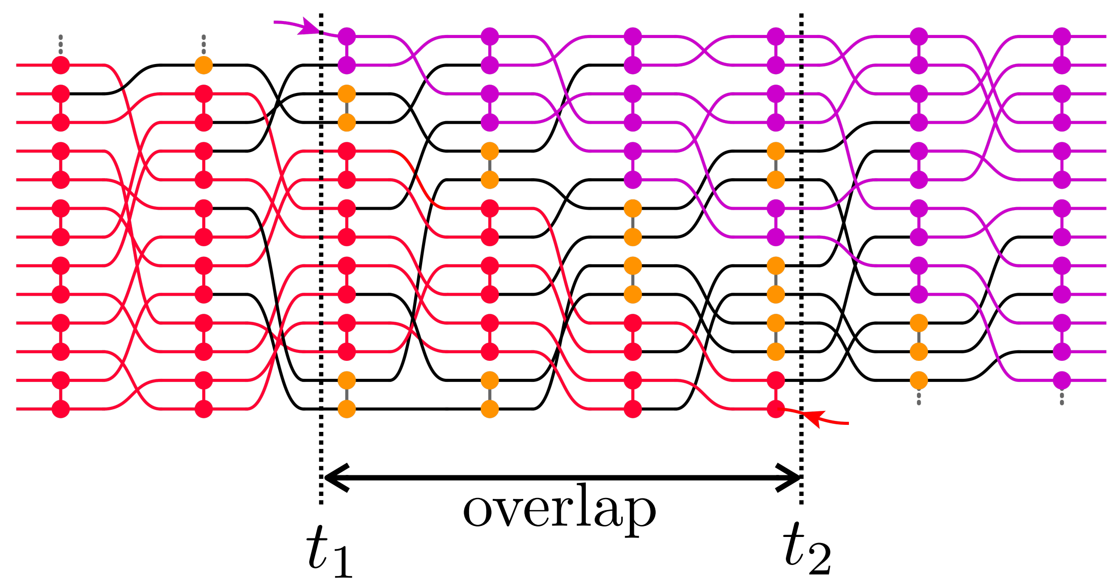


Comparison with scattering of two objects coming from the same boundary

$$V_R(t_1)W_R(t_2)\,|\mathrm{TFD}\rangle \qquad t_2 > t_1 \qquad t_2 - t_1 \equiv \Delta t$$

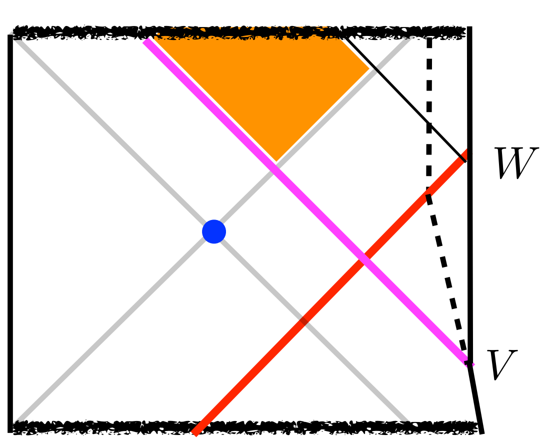


Two directions of the quantum circuit: future and past time evolutions.

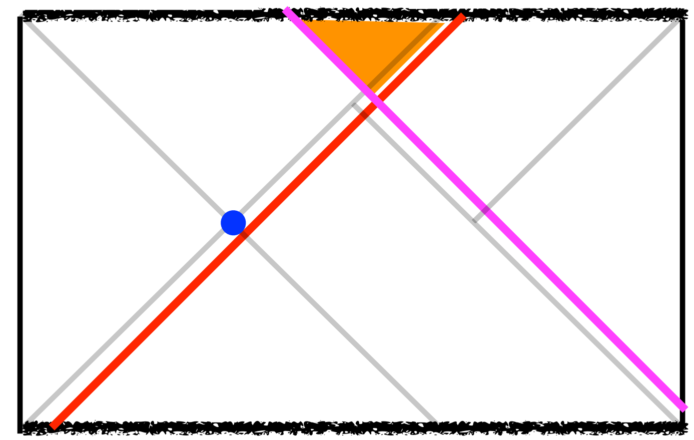


The quantum circuit picture is similar: Two perturbations grow in opposite directions in one quantum circuit. The number of health gates as a function of Δt should be similar.

Bulk picture



$\Delta t < t_*$



$\Delta t > t_*$

The bulk picture is very similar for $\Delta t > t_*$: The healthy gates are stored in the post-collision region.

One sees some universal features when the collision is strong enough such that it is behind a black hole horizon: The shrinking of post-collision region has to do with the fact that there are no more healthy gates in the quantum circuit.

Essentially the same boundary mechanism with the collision between two signals coming from different non-interacting boundaries.

Beyond ER = EPR

- A boundary mechanism with which matter gravitationally interact: perturbations overlapping in the quantum circuit.

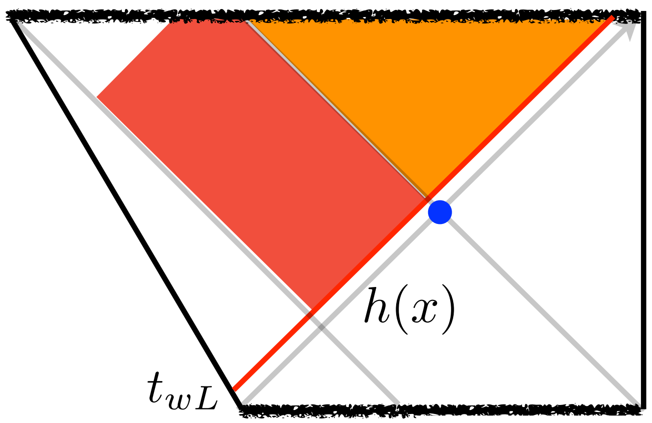
- One can apply this picture to one-sided black holes, e.g., black holes with end of world brane.

- A more ambitious goal: can we derive gravitational interactions (approximately) from knowing properties of the quantum circuit?

  What's the right quantity to ask about?

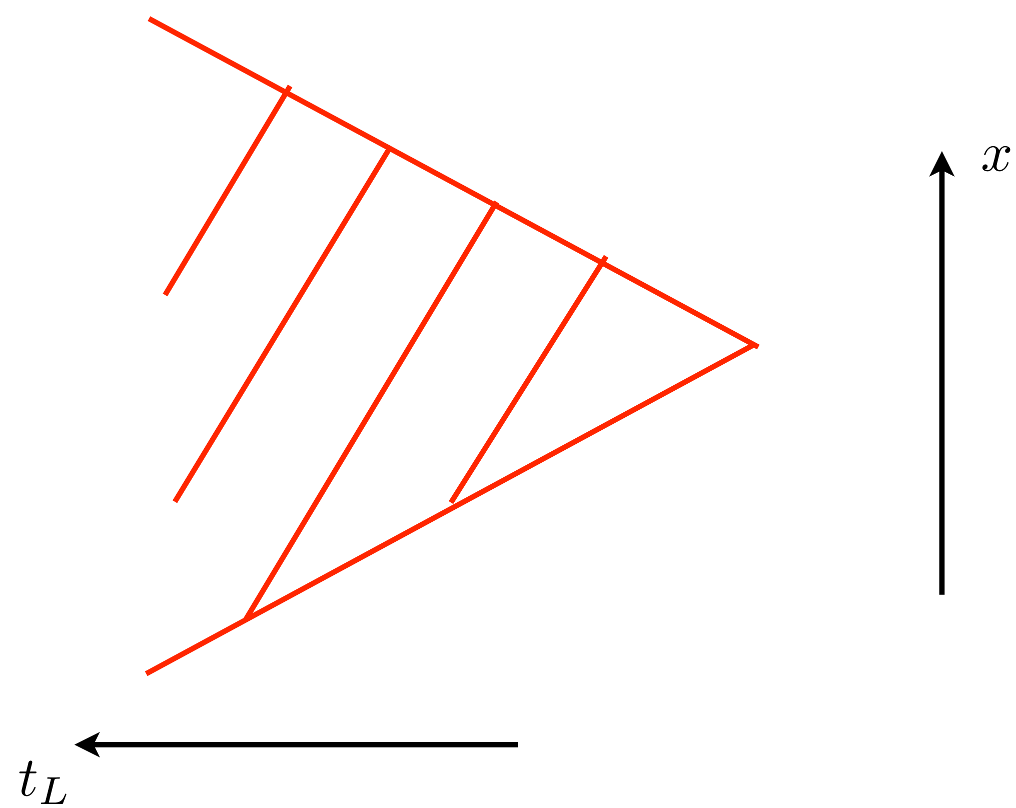
D. Roberts, D. Stanford, L. Susskind, arXiv:1409.8180

Localized shockwaves

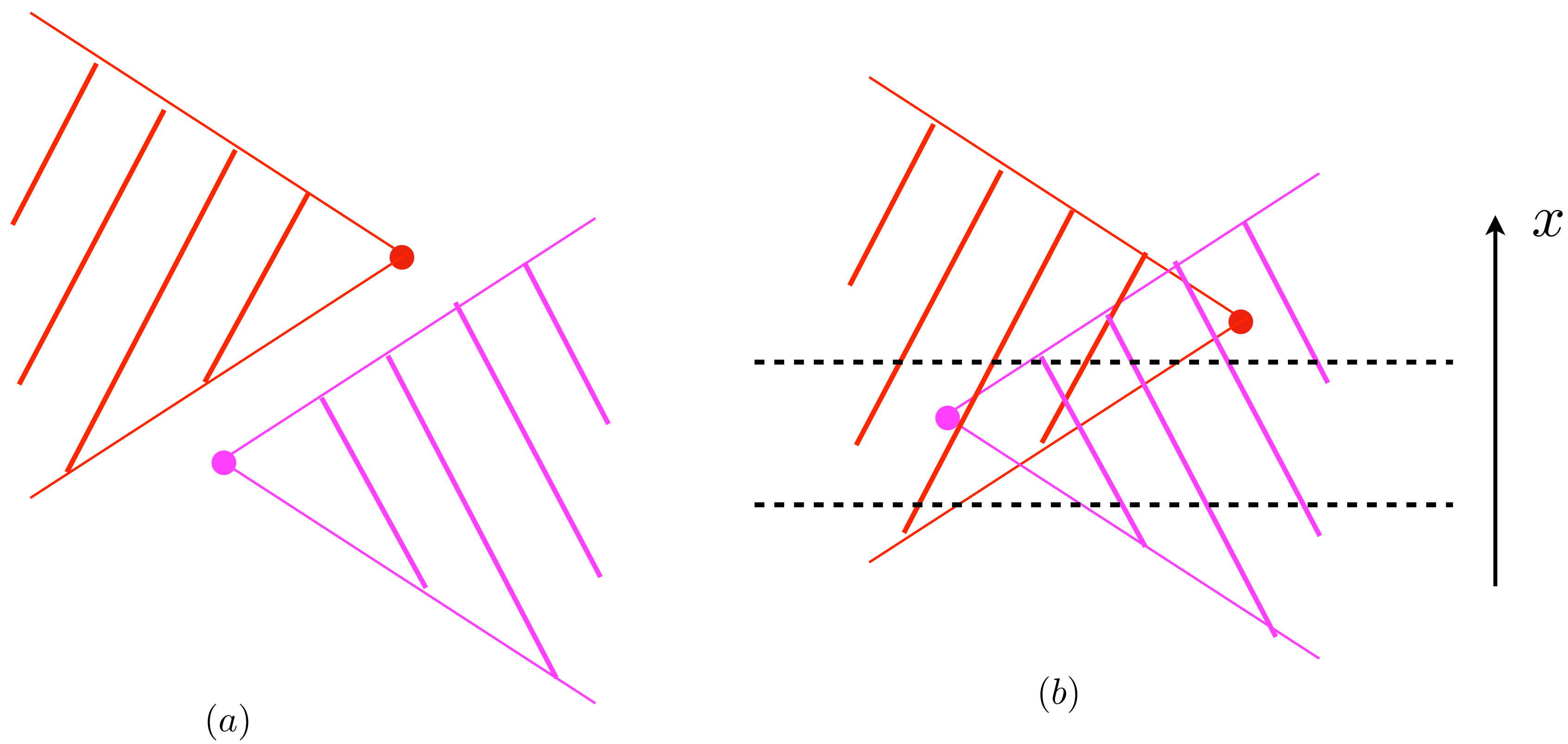


$$h(x) = \frac{e^{\frac{2\pi}{\beta}(-t_{wL}-t_*)-\mu|x-x_{wL}|}}{|x-x_{wL}|^{\frac{D-3}{2}}}$$

$$t_{wL} \to t_{wL}(x) = t_{wL} + \frac{|x-x_{wL}|}{v_B}$$



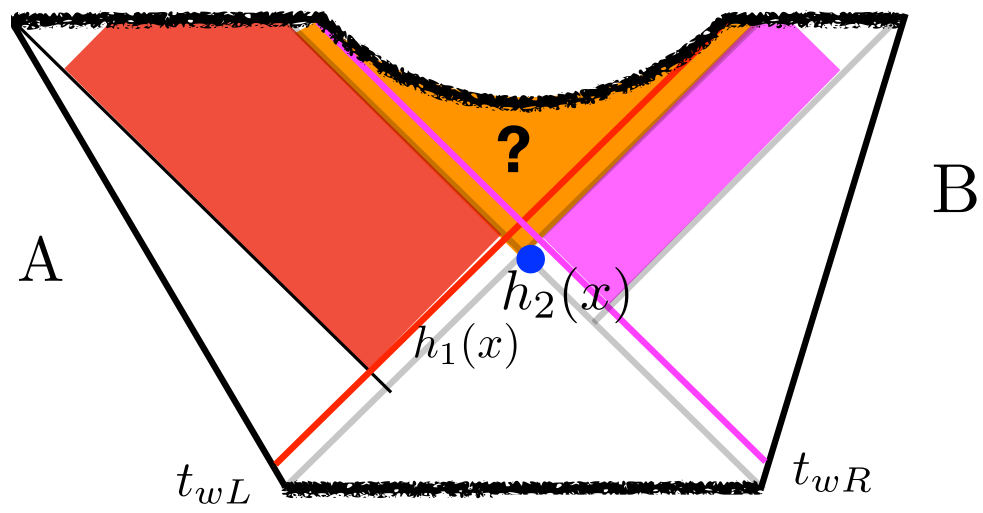
Collisions of localized shockwaves



One should be able to tell the difference between the two cases from boundary six-point functions.

$$\Delta t \to \Delta t(x) = -t_{wL} - \frac{|x - x_{wL}|}{v_B} - t_R - \frac{|x - x_{wR}|}{v_B}$$

The metric in the post collision region is in general unknown.



- $$h_1(x)h_2(x) \approx \frac{\delta S_1}{c}\frac{\delta S_2}{c}e^{\Delta t(x)} \approx e^{\Delta t(x) - 2t_*}$$
  characterizes the amount of overlap in the circuit at location x.

- On the bulk side, $h_1(x)h_2(x)$ controls the non-linear effect in GR.

Consider when $\Delta t(x) > t_*$, intermediate time and late time regime

- From the circuit model, the total number of healthy gates in the quantum circuit at location x is a function of $h_1(x)h_2(x)$

$$\int_{t_{wR}}^{-t_{wL}} dt \left(\frac{1}{1+\frac{\delta S_1}{c}e^{t-t_{wR}}}\right)\left(\frac{1}{1+\frac{\delta S_2}{c}e^{-t_{wL}-t}}\right) = \frac{\Delta t - \log\left(1+\frac{\delta S_1}{c}e^{\Delta t}\right) - \log\left(1+\frac{\delta S_2}{c}e^{\Delta t}\right)}{1-\frac{\delta S_1 \delta S_2}{c^2}e^{\Delta t}}$$

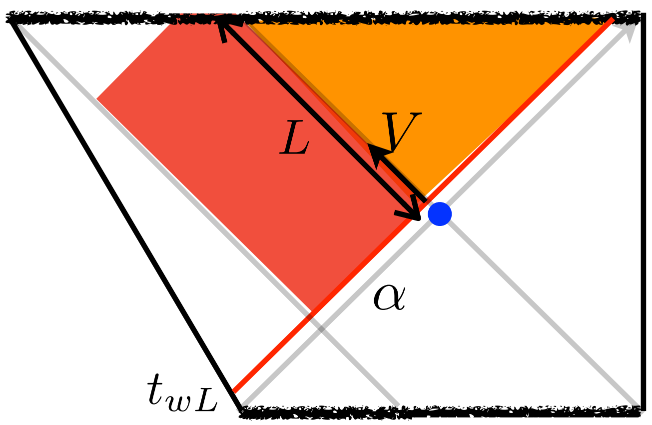
$$\approx \frac{1}{1-h_1(x)h_2(x)}\log\left(\frac{1}{h_1(x)h_2(x)}\right)$$

$$\approx \begin{cases} 2t_* - \Delta t & h_1(x)h_2(x) < 1 \\ (\Delta t - 2t_*)e^{-(\Delta t - 2t_*)} & h_1(x)h_2(x) > 1 \end{cases}$$

- Implications from quantum circuit picture: the singularity bends down and post-collision region becomes small at transverse locations x where $h_1(x)h_2(x)$ becomes of order 1.

It is reasonable because this is when the non-linear effect of GR becomes important.

Spherically symmetric case revisited



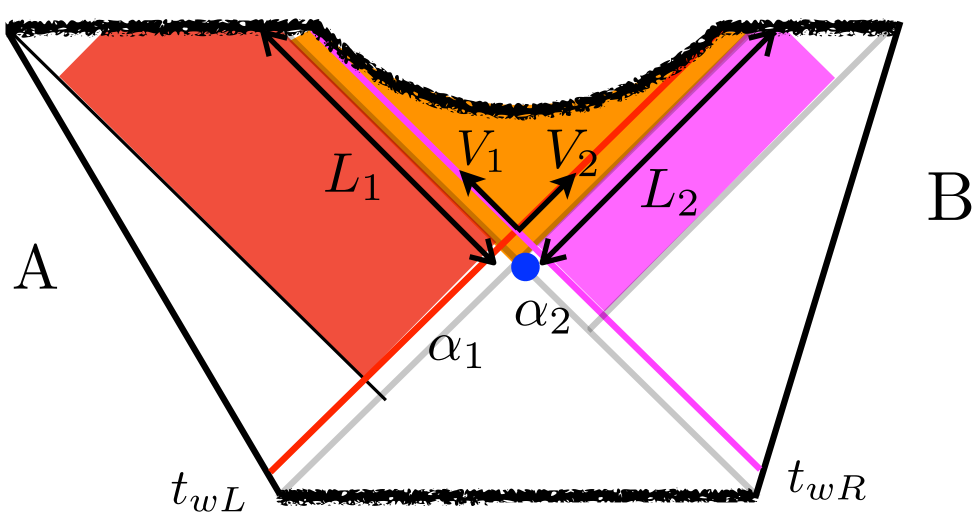
Raychaudhuri equation:

$$\dot{\theta} = -\frac{\theta^2}{D-2} - \sigma^2 + \omega^2 - R_{ab}V^aV^b + \dot{V}^a_{;a}$$

$$R_{ab}V^aV^b = 8\pi G T_{ab}V^aV^b \qquad T_{uu} = \frac{\alpha}{4\pi G}\delta(u)$$

$$\theta_0 = -C\alpha$$

$$\dot{\theta} \leq -\frac{\theta^2}{D-2} \implies L < -\frac{D-2}{\theta_0} = \frac{D-2}{C\alpha}$$



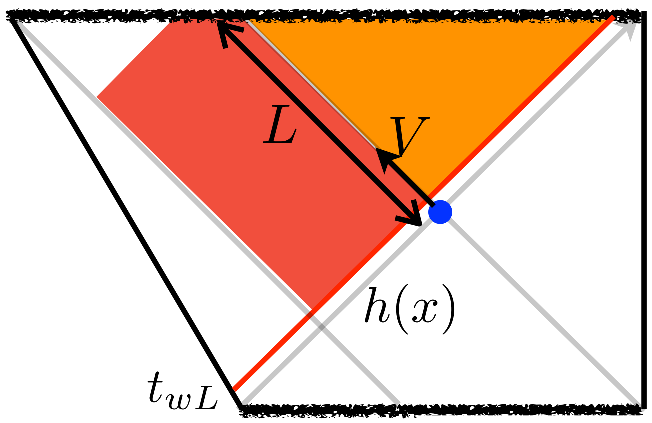
$$V_1 \cdot V_2 = -1$$

$$L_1 L_2 < \frac{l^2}{\alpha_1 \alpha_2}$$

$$\alpha_1 \alpha_2 = \frac{\delta S_1 \delta S_2}{S^2} e^{\Delta t}$$

Jump of expansion across localized shockwave

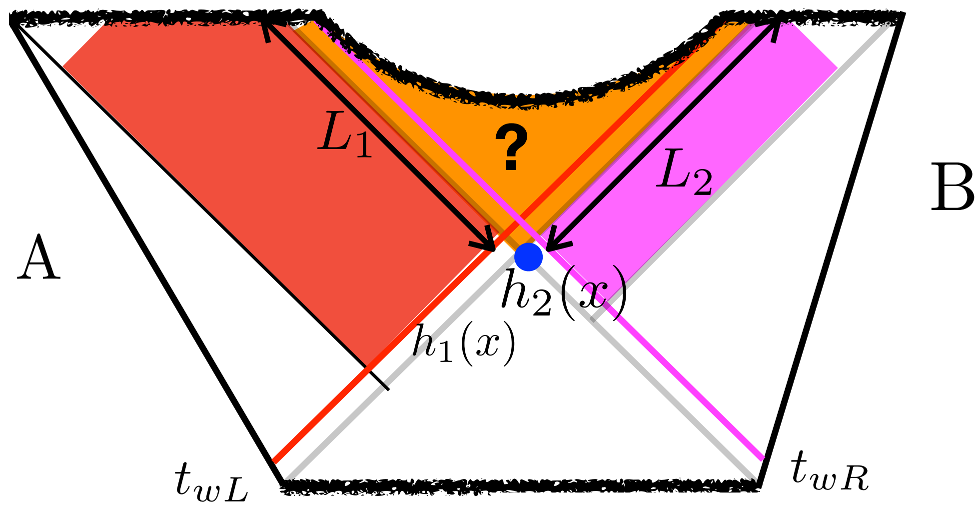
Again consider a family of radial null lines.



- With localized source at x_{wL}, the stress energy tensor vanishes for x away from x_{wL}.
- Radial null lines are no longer geodesics.

$$\dot{\theta} = -\frac{\theta^2}{D-2} - \sigma^2 + \omega^2 - R_{ab}V^a V^b + \dot{V}^a_{;a} \qquad \dot{V}^a_{;a} \propto h(x)\delta(u)$$

$$L \leq \frac{1}{Ch(x)}$$



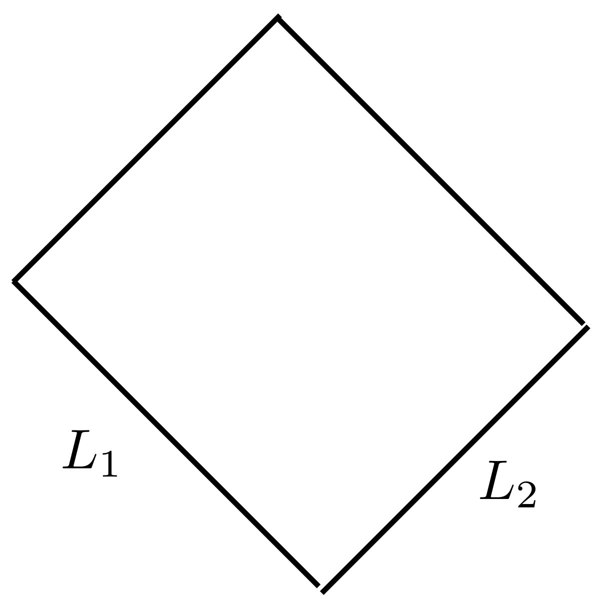
$$V_1 \cdot V_2 = -1$$

$$L_1 L_2 \leq \frac{l^2}{h_1(x) h_2(x)}$$

$$h_1(x) h_2(x) \approx e^{\Delta t(x) - 2t_*}$$

Naive estimate:

AdS$_2$



$$ds^2 = -\frac{4l^2 dU dV}{(1+UV)^2}$$

$$A = 2l^2 \log\left(1 + \frac{L_1 L_2}{2l^2}\right) \approx \begin{cases} 2l^2 \log \frac{L_1 L_2}{2l^2} & L_1 L_2 > l^2 \\ L_1 L_2 & L_1 L_2 < l^2 \end{cases}$$

$$\log\left(1 + \frac{1}{h_1(x) h_2(x)}\right) \approx \begin{cases} \log\left(\frac{1}{h_1(x) h_2(x)}\right) \approx 2t_* - \Delta t & h_1(x) h_2(x) < 1 \\ \frac{1}{h_1(x) h_2(x)} \approx e^{-(\Delta t - 2t_*)} & h_1(x) h_2(x) > 1 \end{cases}$$

Future directions

- Better bulk calculation / estimation
- Multiple shockwaves and connections with earlier work
- Quantify the circuit picture by six-point functions or other definition of size
- The formation of black holes

Unanswered questions

- The role of singularity: the existence of spacelike singularity prevents the meeting of signals that are sent in too late. What about collision near the inner horizon for charged black holes?

  Y. Lensky, X-L. Qi, arXiv:2012.15798

- How to make this picture of black hole interior as unitary quantum circuit more rigorous? Similarity / difference with tensor network in the exterior?

- I had a fixed Hamiltonian in mind in the context of AdS/CFT. How to incorporate different states of baby universes?

- We have been trying to understand matter propagating in the interior in terms of quantum circuit. How about firewalls?

Thank you.