UC SANTA BARBARA Department of Physics

A No-Replica Trick for the Free Energy

Sergio Hernández-Cuenca

based on w.i.p. with

Ven Chandrasekaran, Netta Engelhardt and Sebastian Fischetti

Gravitational Emergence in AdS/CFT

BIRS

October 28, 2021

Introduction

BACKGROUND

- The Euclidean GPI is a mysterious object in quantum gravity

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  - von Neumann entropies (unitary Page curve)
  - gravitational correlators (factorization problem $\overline{Z(B^m)} \neq \overline{Z(B)}^m$)
  - free energies [Engelhardt-Fischetti-Maloney]

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  - free energies [Engelhardt-Fischetti-Maloney]
- Annealed vs quenched free energies:

$$F_a = -\frac{1}{\beta} \log \overline{Z} \qquad \text{vs} \qquad F_q = -\frac{1}{\beta} \overline{\log Z}$$

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- Not so fast: continuation to no replicas is ill-defined

The General Prescription

An LM-Style No-Replica Trick

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  [Lewkowycz-Maldacena]

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  [Lewkowycz-Maldacena]
- Localize the path integral to gravitational saddle points:

$$\overline{\log Z} = \lim_{m\to 0} \frac{1}{m}(e^{-I[M_m]} - 1) = \lim_{m\to 0} \frac{1}{m}(e^{-mI[\hat{M}_m]} - 1) = -I[\hat{M}_0]$$

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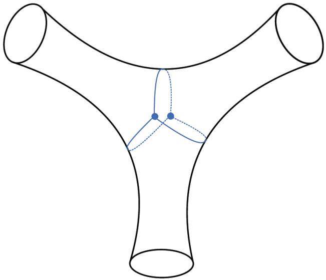
   ...bring the CHEF recipe!

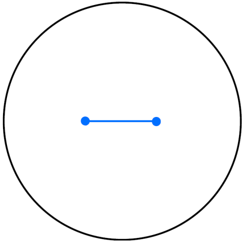
2. Saddle points in the $m \to 0$ limit give quenched generating functionals in quantum gravity... who are these creatures?

A Story about JT

The Quotient Geometry

- Example: replica wormhole M_3 and $\mathbb{Z}_3$ orbifold $\hat{M}_3$ [East Coast]





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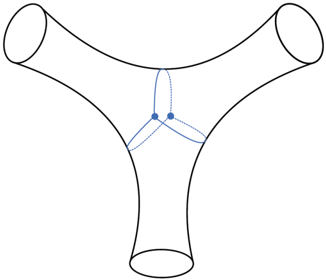

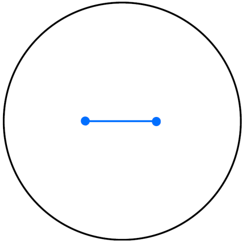

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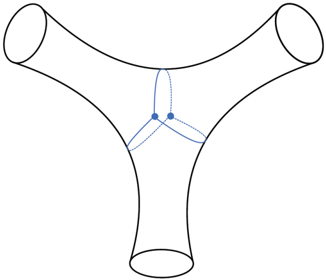

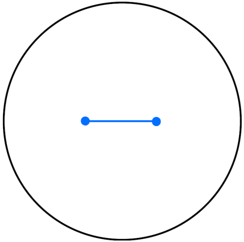

- In general, $\hat{M}_m$ is conformal to a Poincaré disk with two conical defects of opening angle $2\pi/m$
- Wormhole throat sizes relate to proper distance between defects

JT AND BOUNDARY CONDITIONS

- JT action:

$$I = -\frac{S_0}{4\pi}\left[\int_M R + 2\int_{\partial M} K\right] - \frac{1}{2}\int_M \Phi(R+2) - \int_{\partial M} \Phi(K-1)$$

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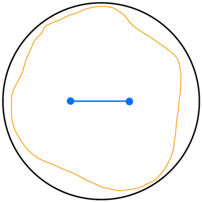
- Boundary conditions:

Cutoff boundaries identified with level sets of the dilaton $\Phi|_{\partial M} = 1/\delta$. Limit $\delta \to 0$ taken with fixed ratio $L_{\partial M}/\Phi|_{\partial M} = \beta$



AN INTERACTING SCHWARZIAN THEORY

- Near-boundary metric:

$$g = \left(\frac{1}{(1-\xi)^2} + h_a^{(m)}(\phi) + \mathcal{O}\left(1-\xi\right) \right) \left(d\xi^2 + d\phi^2 \right)$$

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- Wiggle $\phi : \mathbb{S}_\beta \to \partial M \cong \mathbb{S}$ defined by $g|_{\partial M} = du^2/\delta^2$

$$-\int_{\partial M} \Phi(K-1) = \int_{\mathbb{S}_\beta} du \left(\left\{ \tan\frac{\phi}{2}, u \right\} + \frac{1}{2}(1 + 3h_a^{(m)}(\phi))\ \phi'(u)^2 \right)$$

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- Wiggle equation of motion:

$$\left(\frac{1}{\phi'} \left(\frac{\phi''}{\phi'} \right)' - 3h_a^{(m)}(\phi)\phi' \right)' + \frac{3}{2} \left(h_a^{(m)}(\phi)\right)'\phi' = 0$$

The JT Hellscape

- Explicit JT wiggle action for all replica $m \in \mathbb{R}_{\geq 0}$ and moduli a:

$$I_a^{(m)}(\beta) = \frac{8}{\beta} \operatorname{arcsinh}^2 \sqrt{\sin^2 \frac{\pi}{m} \cosh^2 \frac{a}{2} - 1}$$

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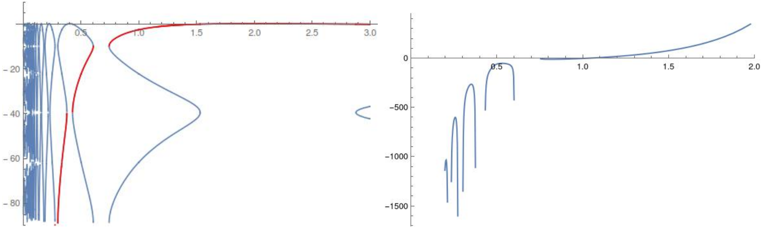
- Action and stability analysis...



... and the modulus is not stabilized

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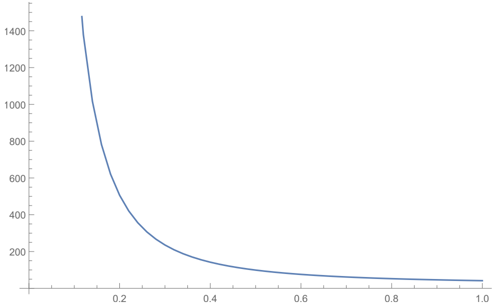
A Silver Lining

- Modulus saddles appear for $m = 2$ as sources are turned on:



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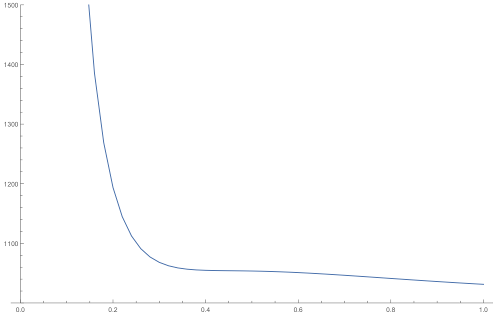

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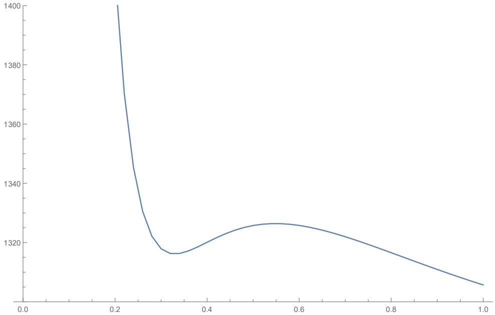

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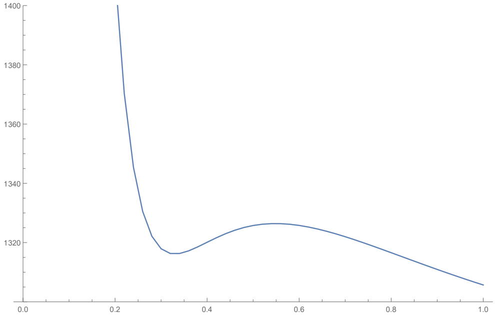

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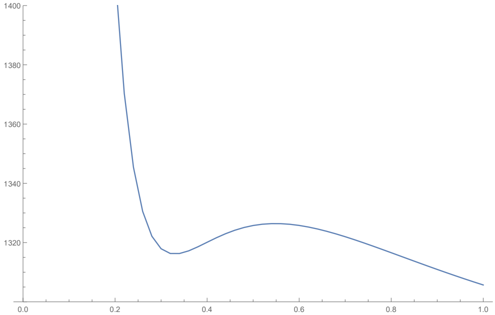

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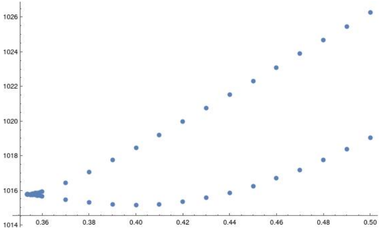
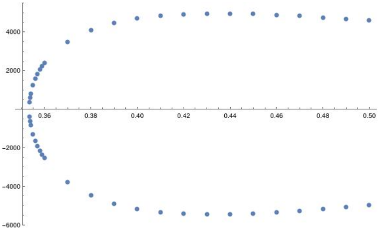
Yes!

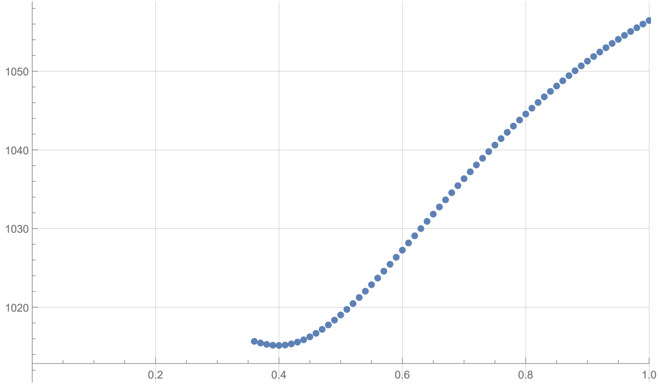
The Little Saddle that Could

- Pair of stable/unstable branches of solutions exist for $m < 1$!
- The little wormhole can be made to dominate over the disk one
- Action and stability analysis for $m = .75$:





The Journey just Began



Little saddle spotted at $m = .75$

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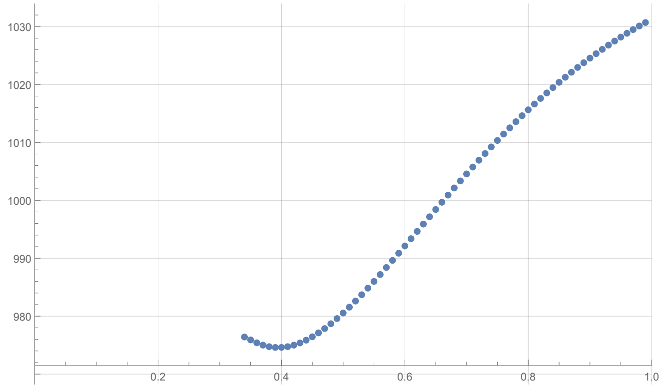

Little saddle spotted at $m = .7$

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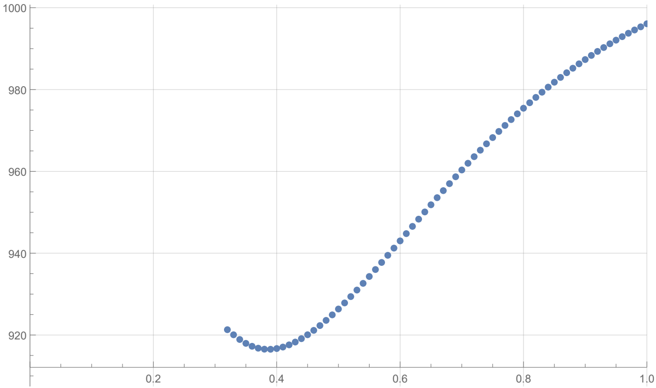

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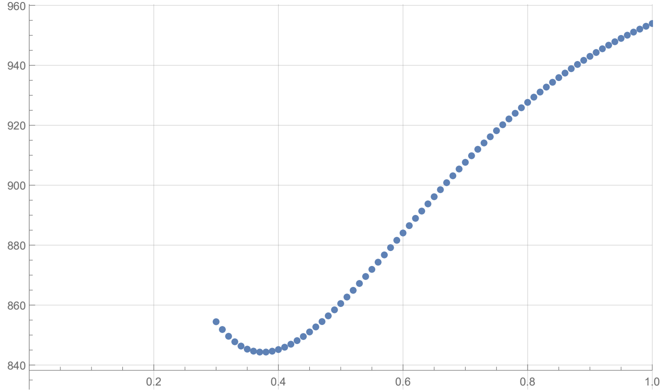

Little saddle spotted at $m = .6$

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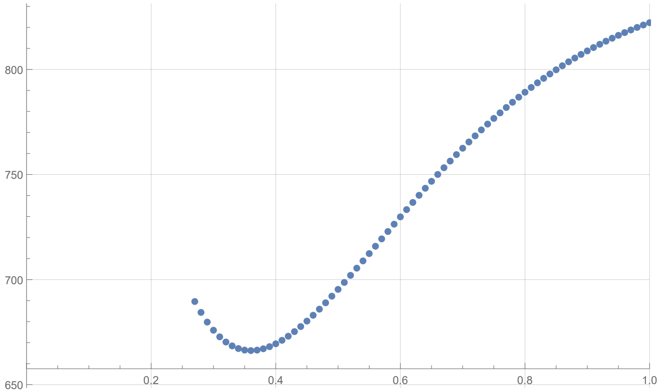

Little saddle spotted at $m = .55$

Discussion

Outlook

- Will the little saddle make it to $m \to 0$?
- What properties does the resulting generating functional have?
- How does it differ from the annealed result?
- What is the effect on scalar correlation functions?

Open Questions and Future Directions

- Is there a simple diagnostic for when quenched $m \to 0$ saddle points will differ from annealed ones?
- Is there any correlation between dominance of replica wormholes for $m \in \mathbb{Z}_+$ and for $0 < m < 1$?
- Are there any universal features about $m \to 0$ saddle points and quenched generating functionals in quantum gravity?
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Thank you for listening!