Homology constraints in Double Holography and the Semi-Classical Approximation

BIRS workshop on Gravitational Emergence in AdS/CFT

based on 2105.01130, 2104.02801; 2010.00018, 2006.04851 (w/ Chen, Myers, Reyes, Sandor)

Dominik Neuenfeld

October 2021

Perimeter Institute

Motivation

Novel way of computing entropies:

$$S_{\text{island}}(A) = \min_I \operatorname{ext} \left\{ \frac{\text{Area}(\partial I)}{4G_N} + S_{\text{vN}}(A \cup I) \right\}$$
$$S_{\text{vN}}(A) = -\text{tr}(\rho_A \log \rho_A)$$

[Almheiri, Engelhardt, Marolf, Maxfield; Penington; Almheiri, Maldacena, Mahajan, Zhao]

Important for Black Holes!

The island I is encoded in the state of region A

Semi-classical gravity as a theory of

- local (up to gauge constraints) degrees of freedom on a
- dynamical spacetime

fails as a LEEFT.

Motivation

What is semi-classical gravity?

- Regime of validity?
- Why use the island formula? Hilbert space picture?

How does semi-classical gravity emerge from quantum gravity?

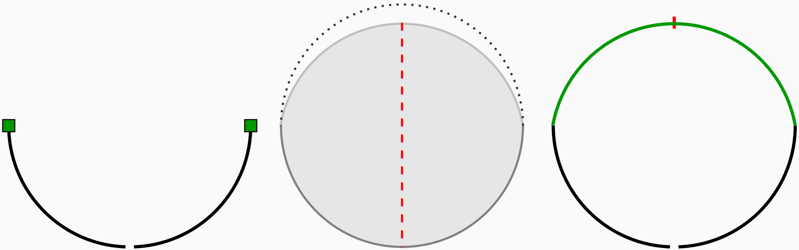
- In 2d: Ensemble average [Saad, Shenker, Stanford]
- In higher d: Ensemble average? Coarse graining? [Pollack, Rozali, Sully, Wakeham; Marolf, Maxfield; ...]
- Role of computational complexity? [Harlow, Hayden; Kim, Tang, Preskill; Brown, Gharibyan, Penington, Susskind]

Use double holography to make progress

Three triality frames

Simple model: Rindler AdS with ETW brane

($t = 0$ slices)



boundary description

(AdS/BCFT)

TFD state

$\frac{c_{bdr}}{c_{amb}} \gg 1$

bulk description

Rindler AdS

$T \rightarrow T_c$

brane description

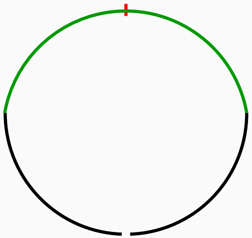
Topological black hole

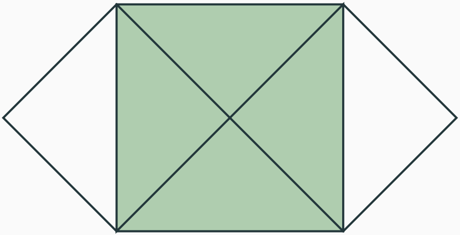
coupled to bath;

Hartle-Hawking state

Computing entropies in brane perspective

Goal: Identify semi-classical gravity with brane perspective.





What are the rules for computing "observables" in the (semi-classical) brane perspective, e.g., von Neumann entropy?

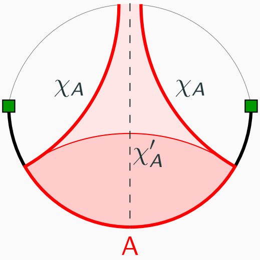
- RT should hold in the bath region (holographic theory).
- Require local brane theory.

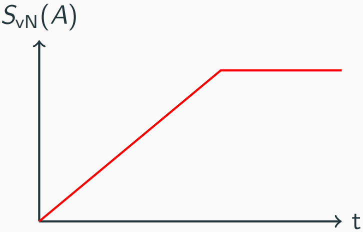
The Page curve

Computing entropies in holographic theories is easy.

Ryu-Takayanagi in AdS/BCFT: [Takayanagi; Fujita, Takayanagi, Tonni]

$$S_{\mathrm{vN}}(A) = \frac{\mathrm{Area}(\chi_A)}{4G_N} + \ldots, \qquad \partial\Sigma_A \quad = A \cup \chi_A \cup X, \qquad X \subset \text{ETW brane}$$





"Island RT surface" can be interpreted as giving island entropy in brane perspective!

Contradiction

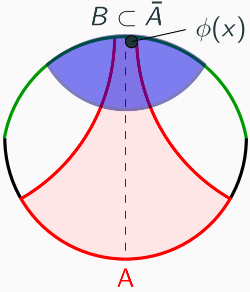
Assume

1. Local theory in brane perspective
2. Isometry from bulk to brane perspective
3. Standard rules for EW reconstruction

then, **island RT surface** is not consistent.

Argument

- $\phi(x)$ is in $\mathrm{EW}(A)$
- $\phi(x)$ is **not** in $\overline{\mathrm{EW}(A)} = \mathrm{EW}(\bar{A})$
- $\phi(x)$ can be reconstructed from $B \subset \bar{A}$ via HKLL.



HKLL [Hamilton, Kabat, Lifschytz, Lowe]

- Use data at the brane.
- Solve bulk equations of motion inwards.

Homology Constraints for Double Holography

Way out:

Homology Constraint for RT depends on duality frame

$$S_{\mathrm{vN}}(A) = \frac{\mathrm{Area}(\chi_A)}{4G_N}, \quad \text{with} \quad \begin{cases} \partial\Sigma_A = A \cup \chi_A \cup X & \text{(bdry perspective)} \\ \partial\Sigma_A = A \cup \chi_A & \text{(brane perspective)} \end{cases}$$

Two different notions of von Neumann entropy of region $A \subset$ bath:

1. $S_{\mathrm{vN}}^{\mathrm{boundary}}(A)$ saturates at t_P.
2. $S_{\mathrm{vN}}^{\mathrm{brane}}(A) \sim t$ agrees with semi-classical computation.

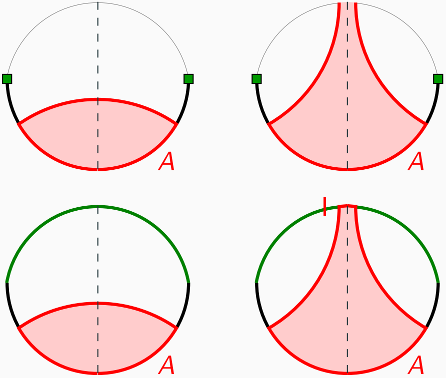
Relation between boundary and brane perspective

How to compute $S_{\mathrm{vN}}^{\mathrm{boundary}}(A)$ within **brane perspective**?

$\rightarrow$ Island formula

Boundary perspective:
$S_{\mathrm{vN}}(A)$

Brane perspective:
$S_{\mathrm{island}}(A)$



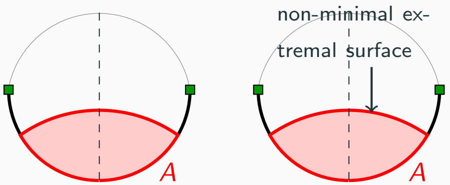
$S_{\mathrm{vN}}^{\mathrm{boundary}}(A)$ is **dual to** $S_{\mathrm{island}}(A)$ in brane perspective.

Relation between boundary and brane perspective

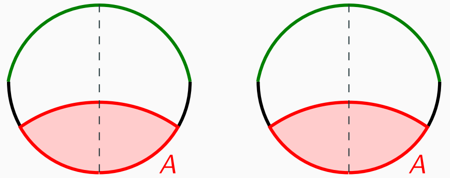
How to compute $S_{\text{vN}}^{\text{brane}}(A)$ within **boundary perspective**?
$\rightarrow$ Coarse graining

Boundary perspective:
$S_{\text{simple}}(A) = \max_{\rho'} S_{\text{vN}}(\rho')$
s.t. $\langle \mathcal{O}_s(t) \rangle_{J_s,\rho'} = \langle \mathcal{O}_s(t) \rangle_{J_s,\rho_A}$

[Engelhardt, Wall; Engelhardt, Penington, Shabbazi-Moghaddam]

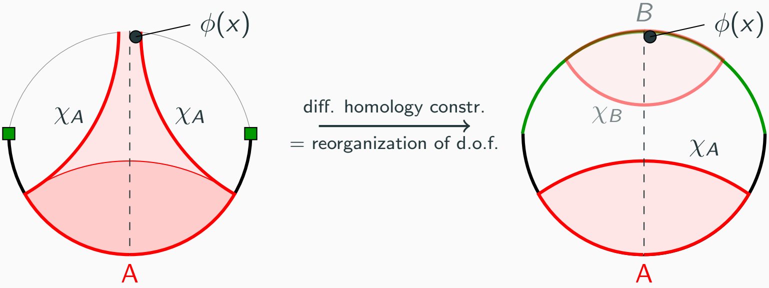


Brane perspective:
$S_{\text{vN}}(A)$



$S_{\text{vN}}^{\text{brane}}(A)$ is **dual to** $S_{\text{simple}}(A)$ in boundary perspective.

Relation between boundary and brane perspective



Boundary Perspective
≈ Quantum Gravity

Brane Perspective
≈ Semi-Classical Gravity

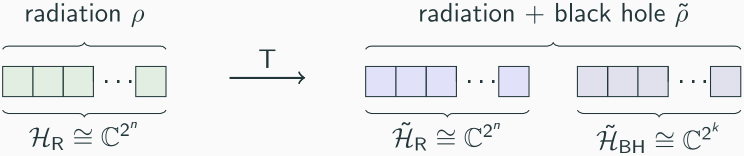
- **(Converse) Python's lunch:** Simple observables map trivially, Complex observables get rearranged.
- In the semi-classical picture, information about island is not contained in bath!
- Rather low energy **duality** than coarse graining.

Simple Toy Model

$$\overbrace{\underbrace{\square\square\square\cdots\square}_{\mathcal{H}_{\mathsf{R}} \cong \mathbb{C}^{2^n}}}^{\text{radiation } \rho} \quad \xrightarrow{\mathsf{T}} \quad \overbrace{\underbrace{\square\square\square\cdots\square}_{\tilde{\mathcal{H}}_{\mathsf{R}} \cong \mathbb{C}^{2^n}} \quad \underbrace{\square\square\square\cdots\square}_{\tilde{\mathcal{H}}_{\mathsf{BH}} \cong \mathbb{C}^{2^k}}}^{\text{radiation } + \text{ black hole } \tilde{\rho}}$$

Evaporated black hole

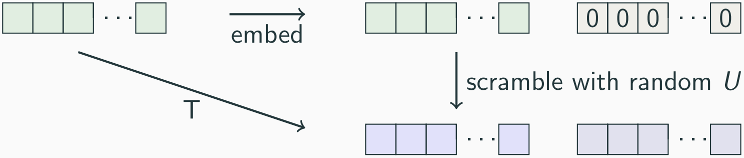
Simple Toy Model



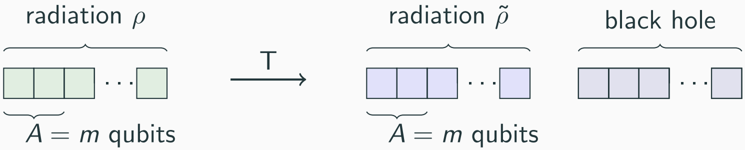
Consider

1. random, pure state $|\psi\rangle_{\mathsf{R}} \in \mathcal{H}_{\mathsf{R}}$
2. $n \leq k$

Define isometry T by a two-step procedure

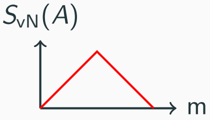


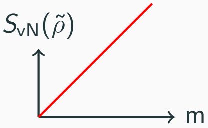
Simple Toy Model



$$S_{\mathrm{vN}}(\rho_A) = \begin{cases} m, & \text{if } m < \frac{n}{2} \\ n - m, & \text{if } \frac{n}{2} \le m \le n \end{cases}$$

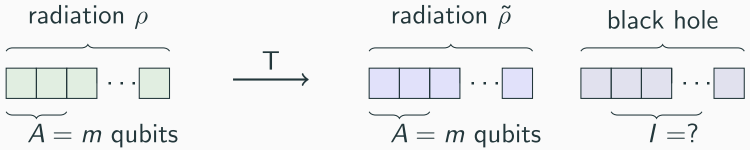
$$S_{\mathrm{vN}}(\tilde{\rho}_A) = m, \text{if } m < n$$





1. Two different notions of von Neumann entropy.
2. Simple (i.e., few qubit) observables agree.

Simple Toy Model



Define:

$$S_{\text{island}}(\tilde{\rho}_A) \equiv \min_{I \subset BH} S_{\text{island}}(\tilde{\rho}_{A \cup I})$$

For radiation subregions A in the "tilde" system:

$$S_{\text{island}}(\tilde{\rho}_A) = \begin{cases} m, & \text{if } m < \frac{n}{2} \\ n - m, & \text{if } \frac{n}{2} \leq m \leq n \end{cases} = S_{\text{vN}}(\rho_A)$$

- Reproduces von Neumann entropy in un-tilded system.
- No intrinsic meaning in tilded system.

Final Comments

- In double holography, semi-classical brane perspective corresponds to a non-local reorganization of boundary d.o.f.
- In the semi-classical description we do not have access to the island from the bath.
- Semi-classical description is good for "simple" observables, can access complex questions through non-local dual question.

- More dictionary entries, e.g., how to compute $S_{\text{gen}}(B)$ from boundary perspective?
- Can we use double holography / toy model to understand how pair creation at a horizon in the boundary perspective?
- Carry over lessons from double holography to more general situations/"real life" quantum gravity?