On indecomposable and logarithmic modules for affine vertex algebras

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Znanstveni centar izvrsnosti
za kvantne i kompleksne sustave te
reprezentacije Liejevih algebri

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Logarithmic modules for VOAs

- Let $(V, Y, \mathbf{1}, \omega)$ be a VOA. $Y(v,z) = \sum_{n\in\mathbb{Z}} v_n z^{-n-1}$.
- Let (M, Y_M) be any weak V–module.
- Let $Y_M(\omega, z) = \sum_{n\in\mathbb{Z}} L(n) z^{-n-2}$.
- Then M is a module for the Virasoro algebra.
- (M, Y_M) is called logarithmic if it admits a decomposition

$$M = \coprod_{h\in\mathbb{C}} M_h, M_h = \{v \in M, \ (L(0)-h)^m v = 0 \ \text{ for some } \ m\}.$$

- M has nilpotent rank m if

$$(L(0) - L(0)_{sem.s.})^m = 0, \ (L(0) - L(0)_{sem.s.})^{m-1} \neq 0.$$

Logarithmic modules for VOAs

- Recent interest in logarithmic modules for C_2–cofinite, non-rational VOAs of triplet type (works D. A, A. Milas, T. Abe, Arike, Nagatomo, Wood, Tsuchiya, Gannon, Creutzig, Ridout and physicists in framework of LCFT)
- Important connection with projective modules for quantum groups (several future talks at the conference).
- Logarithmic modules are related to twisted modules for non semi-simple VOA automorphism (Y.Z. Huang)
- Existence of log. modules are/were usually longstanding conjectures
- Methods: Zhu's algebra theory modular invariance approach, Kazhdan–Lusztig corredpondence, explicit lattice realization

A general method for construction of log-modules

- Let $v \in V$ be an even vector such that:

$$[v_n, v_m] = 0 \quad \forall n, m \in \mathbb{Z}, \tag{1}$$

$$L(n)v = \delta_{n,0} v \quad \forall\ n \in \mathbb{Z}_{\geq 0}. \tag{2}$$

Define

$$\Delta(v, z) = z^{v_0} \exp\left(\sum_{n=1}^{\infty} \frac{v_n}{-n}(-z)^{-n}\right). \tag{3}$$

A general method for construction of log-modules

Theorem (D.A, A.M, 2009)

Assume that V is a VOA and that $v \in V$ is an even vector which satisfies conditions (1) and (2). Let $\overline{V}$ be the vertex subalgebra of V such that $\overline{V} \subseteq \mathrm{Ker}_V v_0$.
Assume that (M, Y_M) is a weak V–module. Define the pair $(\widetilde{M}, \widetilde{Y}_{\widetilde{M}})$ such that

$$\widetilde{M} = M \quad \text{as a vector space},$$

$$\widetilde{Y}_{\widetilde{M}}(a, z) = Y_M(\Delta(v, x)a, z) \quad \text{for } a \in \overline{V}.$$

Then $(\widetilde{M}, \widetilde{Y}_{\widetilde{M}})$ is a weak $\overline{V}$–module.
Assume that $L(0)$ acts semisimply on M. Then $(\widetilde{M}, \widetilde{Y}_{\widetilde{M}})$ is logarithmic $\overline{V}$–module if and only if v_0 does not act semisimply on M.

Applications

- Let U be a VOA, M U–module, $V = U \oplus M$ extended VOA.
- $S : U \to M$ a screening operator,
- $\overline{V} = \text{Ker}(S : U \to M)$ important VOA.
- Log $\overline{V}$–modules can be constructed using previous theorem.

So far applied on

- Triplet vertex (super)algebras $W(p)$, $SW(p)$
- Logarithmic extensions of minimal models $W(p, p')$
- Affine VOA $L_k(\mathfrak{sl}_2)$
- We shall discuss new applications.

Affine VOAs

- $\mathfrak{g}$ simple Lie (super)algebra over $\mathbb{C}$.
- $\hat{\mathfrak{g}} = \mathfrak{g} \otimes \mathbb{C}[t, t^{-1}] + \mathbb{C}K$ the affine Kac–Moody Lie algebra.
- $V^k(\mathfrak{g})$ universal affine VOA of level k (k is not critical).
- As $\hat{\mathfrak{g}}$–module $V^k(\mathfrak{g}) = U(\hat{\mathfrak{g}}) \otimes_{U(\hat{\mathfrak{g}}_{\geq 0} + \mathbb{C}K)} \mathbb{C}.1$.
- $L_k(\mathfrak{g})$ simple quotient of $V^k(\mathfrak{g})$
- ω_{sug} Sugawara Virasoro vector in $V_k(\mathfrak{g})$ of central charge

$$c(sug) = \frac{k \mathrm{sdim} \mathfrak{g}}{k + h^{\vee}}.$$

The category KL_k

- Let KL^k be the subcategory of $\mathcal{O}_k$ consisting of modules M on which $\mathfrak{g}$–acts locally finite

  $=$ the category of ordinary modules for $V^k(\mathfrak{g})$ when $\mathfrak{g}$ is a simple Lie algebra
- Category KL_k: $L_k(\mathfrak{g})$–modules which are in KL^k.
- For $\mathfrak{g}$ Lie superalgebra, we introduce KL_k^{fin}, subcategory of KL_k consists of weight modules.
- Semi-simpilicty of KL_k and KL_k^{fin} (D.A-Kac-Moseneder-Papi-Perse)
- Investigate the fusion rules and associated fusion algebras for modules from KL_k. Tensor category of KL_k modules (Creutzig-Yang).

Semi-simplicity of KL_k

We prove the following results on complete reducibility result in KL_k

Theorem (AKMPP, 2018)

Assume that $\mathfrak{g}$ is a simple Lie algebra and $k \in \mathbb{C} \setminus \mathbb{Z}_{\geq 0}$. Then KL_k is a semi-simple category in the following cases:

- *k is a collapsing level.*
- *$W_k(\mathfrak{g}, \theta)$ is a rational vertex operator algebra.*

Theorem (AMP, 2021)

Assume that $\mathfrak{g}$ is a simple Lie superalgebra and $k \in \mathbb{C} \setminus \mathbb{Z}_{\geq 0}$. Then KL_k^{fin} is a semi-simple category in the following cases:

- *k is a collapsing level.*
- *$W_k(\mathfrak{g}, \theta)$ is a rational vertex operator superalgebra.*

Ex: KL_k^{fin} is not semi-simple category for $\mathfrak{g} = sl(n|1)$ and $k \in \mathbb{Z}_{\geq 1}$.

New approach: Realization as an inverse of QHR

- For f nilpotent element in $\mathfrak{g}$ and $k \in \mathbb{C}$ one associate the universal affine W-algebra $W^k(\mathfrak{g}, f)$ as $H_f(V^k(\mathfrak{g}))$ where H_f is quantum Hamiltonian reduction function
- (studied in vertex algebra setting by Kac, Wakimoto, Frenkel, Arakawa,+ new work by Creutzig, Genra, ...)
- $W_k(\mathfrak{g}, f)$ simple quotient of $W^k(\mathfrak{g}, f)$.
- Inverse of QHR means embedding of type

$$L_k(\mathfrak{g}) \hookrightarrow W_k(\mathfrak{g}, f) \otimes \mathcal{F}$$

  where $\mathcal{F}$ is a free field algebra
- Recent results by D.A, Ridout, Kawasetsu, Creutzig, Genra, Fechily for algebras of low rank.

Vertex algebra $\Pi(0)$

- Consider lattice $L = \mathbb{Z}c + \mathbb{Z}d, \quad \langle c, c\rangle = \langle d, d\rangle = 0, \ \langle c, d\rangle = 2,$
- Associated lattice vertex algebra $V_L = M(1) \otimes \mathbb{C}[L]$, where $M(1)$ is Heisenberg vertex algebra with generators $c(z)$ i $d(z)$; $\mathbb{C}[L]$ group algebra associated to L.
- $\Pi(0) = M(1) \otimes \mathbb{C}[\mathbb{Z}c] \subset V_L$.
- $\Pi(0)$ is a localisation of the $\beta\gamma$ VOA $\mathcal{S}$
- $\Pi(0)$ is a localisation of the Weyl VOA $= \beta\gamma$ system
- Set $\beta = e^c$, $\beta^{-1} := e^{-c}$ i $\beta^{-1}(n) := e^{-c}_{n-2}$. We have:

$$\beta^{-1}(z) = Y(\beta^{-1}, z) = \sum_{n\in\mathbb{Z}} \beta^{-1}(n) z^{-n+1}.$$

$$\beta^{-1}(z)\beta(z) = Id.$$

Realisation of $L_k(\mathfrak{sl}_2)$

Let $k+2 = \frac{u}{v}$, $(u,v)=1$, $d_{u,v} = 1 - \frac{6(u-v)^2}{uv}$.

Theorem (D.A, CMP, 2019)

Assume that $k \notin \mathbb{Z}_{\geq 0}$.

- *There is a non-trivial vertex algebra homomorphism*

$$\Phi : L_k(\mathfrak{sl}_2) \to L^{Vir}_{d_{u,v}} \otimes \Pi(0)$$

  such that

$$e \mapsto e^c, \ \ h \mapsto 2\mu$$
$$f \mapsto ((k+2)\omega_{u,v} - \nu(-1)^2 - (k+1)\nu(-2))e^{-c}$$

  where $\nu = \frac{1}{2}d - \frac{k}{4}c$, $\mu = \frac{1}{2}d + \frac{k}{4}c$.
- *For each admissible k such that $v \geq 3$, there exist logarithmic $L_k(\mathfrak{sl}_2)$–modules of nilpotent rank two.*

Realisation of $L_k(\mathfrak{sl}_3)$

- Based on new paper

[ACG] D.A, T.Creutzig, N. Genra, Relaxed and logarithmic modules of $\widehat{\mathfrak{sl}_3}$, arXiv:2110.15203 (last week)

- Let
$$BP^k = H_{f_\theta}(V^k(\mathfrak{sl}_3)) = W^k(\mathfrak{sl}_3, f_\theta)$$
be the universal Bershadsky-Polyakov vertex algebra of level k
- BP_k its simple quotient of BP^k.
- $\mathcal{S}$ $\beta\gamma$ VOA, $\Pi(0)$ as before.
- We prove that there exist embeddings
$$\Phi_0 : V^k(\mathfrak{sl}_3) \hookrightarrow BP^k \otimes \mathcal{S} \otimes \Pi(0),$$
$$\Phi_1 : L_k(\mathfrak{sl}_3) \hookrightarrow BP_k \otimes \mathcal{S} \otimes \Pi(0) \quad (k \notin \mathbb{Z}_{\geq 0}).$$
- Therefore in our case, tensoring with the free-field algebra $\mathcal{F} = \mathcal{S} \otimes \Pi(0)$ we invert QHR

Realisation of $L_k(\mathfrak{sl}_3)$

- BP^k generators $G^{\pm}, J, L^{BP}$.
- $V^k(\mathfrak{sl}_3)$ generators $e_i, f_i, h_j,\ i = 1,2,3,\ j = 1,2.$

The homomorphism $\Phi_0 : V^k(\mathfrak{sl}_3) \to BP^k \otimes \mathcal{S} \otimes \Pi(0)$ is uniquely determined by:

$$e_1 \mapsto -\gamma \mathrm{e}^{c}, \quad e_2 \mapsto \beta, \quad e_3 \mapsto \mathrm{e}^{c},$$

$$h_1 \mapsto -2J + :\gamma\beta: - \frac{2k+9}{6}c + \frac{1}{2}d, \quad h_2 \mapsto J - 2:\gamma\beta: + \frac{4k+9}{6}c + \frac{1}{2}d,$$

$$f_1 \mapsto G^+ - :\left(2J + \frac{8k+9}{6}c - \frac{1}{2}d\right)\beta \mathrm{e}^{-c} + (k+1)(\partial\beta)\mathrm{e}^{-c},$$

$$f_2 \mapsto G^- \mathrm{e}^{-c} + \left(J + \frac{4k+9}{6}c + \frac{1}{2}d\right)\gamma + k\partial\gamma - :\gamma^2\beta,$$

$$f_3 \mapsto G^+\gamma - G^-\beta\mathrm{e}^{-2c} + \ldots.(k+3)L^{BP}\ldots\ldots$$

On the proof of simplicity

- Note that $\mathfrak{b} = \mathrm{span}_{\mathbb{C}}\{e_1, e_2, e_3, \bar{h} = h_1 + 2h_2\} \subset \mathcal{S} \otimes \Pi(0)$.
- Consider the action of $\widehat{\mathfrak{b}}$ on $\mathcal{S} \otimes \Pi(0)$.
- We show that each $\widehat{\mathfrak{b}}$-singular vector in $\mathcal{S} \otimes \Pi(0)$ is of the form $e^{\ell c}$.
- Consider $\widetilde{L}(\mathfrak{sl}_3) \subset BP_k \otimes \mathcal{S} \otimes \Pi(0)$.
- If $\widetilde{L}(\mathfrak{sl}_3)$ is not simple, it contains a singular vector, which must be of the form
$$v = Z \otimes e^{\ell c}, \quad Z \in BP_k.$$
- Using the action of other generators, we get that $v = e^{\ell c}$.
- Such singular vector can exist only for $k \in \mathbb{Z}_{\geq 0}$.
- This proves that $\widetilde{L}(\mathfrak{sl}_3) = L(\mathfrak{sl}_3)$ for $k \notin \mathbb{Z}_{\geq 0}$.

Applications of realizations: Category KL^k

Theorem (ACG)

- *Each irreducible module M in KL^k (resp. KL_k when $k \notin \mathbb{Z}_{\geq 0}$) is realizaed as*
$$M = \left(L[x,y] \otimes \mathcal{S} \otimes \Pi(0)^{1/3}\right)^{int_{\mathfrak{sl}_3}}$$
- *$L[x,y]$ is highest weight C_1–cofinite BP^k–module (resp. BP_k module).*
- *$\Pi(0)^{1/3}$ is a SCE of $\Pi(0)$:*
$$\Pi(0)^{1/3} = \Pi(0) + \Pi(0)e^{\frac{c}{3}} + \Pi(0)e^{\frac{2c}{3}}.$$

Relaxed highest weight modules

- Consider VOA $\Pi(0)^{\otimes 2}$ generated by $c_i, d_i, e^{\pm c_i}$, $i = 1, 2$.
- FMS bosonization gives that $\mathcal{S} \otimes \Pi(0) \hookrightarrow \Pi(0)^{\otimes 2}$.
- $\Pi_{r,s}(\lambda_1, \lambda_2) := \Pi(0)^{\otimes 2}.e^{r_1 d_1/2 + r_2 d_2/2 + \lambda_1 c_1 + \lambda_2 c_2}$ is an irreducible $\Pi(0)^{\otimes 2}$–module.
- Let R be a $\mathbb{Z}_{\geq 0}$–graded BP^k–module (resp. BP_k–module if $k \notin \mathbb{Z}_{\geq 0}$) which is a weight module with respect to $(J(0), \widetilde{L}(0))$, where $\widetilde{L} = L^{BP} + \frac{DJ}{2}$.
- Then $R \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2)$ is a $\mathbb{Z}_{\geq 0}$–graded $V^k(\mathfrak{sl}_3)$–module (resp. $L_k(\mathfrak{sl}_3)$–module.
- $R \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2)$ has finite-dimensional weight spaces for $\widehat{\mathfrak{sl}_3}$ if and only if R_{top} is finite-dimensional.

Irreducibility criterion for relaxed highest weight modules

Theorem (ACG)

Assume that $(R \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2))_{top}$ *is an irreducible* $U(\mathfrak{sl}_3)$*–module. Then:*

- $R \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2)$ *is an irreducible* $V^k(\mathfrak{sl}_3)$*–module.*
- *If* $k \notin \mathbb{Z}_{\geq 0}$ *and* R *is an* BP_k*–module, then* $R \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2)$ *is an irreducible* $L_k(\mathfrak{sl}_3)$*–module.*
- *For each* $N \in \mathbb{Z}_{\geq 0}$*, there exist an irreducible relaxed* $V^k(\mathfrak{sl}_3)$*–module realized as*
$$W = L[x, y] \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2)$$
such W_{top} *is Gelfand-Tsetlin module with all* N*–dimensional weight spaces.*

Large relaxed highest weight module

- In [D.A, Kawasetsu-Ridout, 2020], we constructed family of irreducible relaxed BP^k–modules (resp. BP_k-modules under some assumptions) $R_M(\lambda)$ such that $R_M(\lambda)_{top}$ is infinite-dimensional.
- Then we have the following $V^k(\mathfrak{sl}_3)$–module (resp. $L_k(\mathfrak{sl}_3)$–module):

$$R_M(\lambda) \otimes \Pi_{-1,-1}(\lambda_1, \lambda_2)$$

  with all infinite–dimensional weight spaces.
- The irreducibility proof is reduced to the irreducibility of some non Gelfand-Tsetlin modules with infinite-dimensional weight spaces for $sl(3)$, which seems to be unknown.
- We believe that they are related to modules recently studied by Futorny, Zhao, Lu and collaborators.

Logarithmic modules for $L_k(\mathfrak{sl}_3)$: strategy

- We are looking for modules on which

(*) $L_{sug}(0)$ doesn't act semisimple, but cartan subalgebra $\mathfrak{h} \subset \mathfrak{sl}_3$ acts semi-simply.

- It seems that logarithmic $\Pi(0)$ modules don't contribute to log. modules satisfying (*).
- Since for $k \notin \mathbb{Z}$, $L_k(\mathfrak{sl}_3) \hookrightarrow BP_k \otimes \mathcal{S} \otimes \Pi(0)$, existence of logarithmic modules will follow from the existence of logarithmic $\mathcal{S}$ and BP_k–modules.
- $\beta\gamma$ VOA $\mathcal{S}$ has logarithmic modules $\mathcal{P}_s$, $s \in \mathbb{Z}$ which are projective in the category $\mathcal{F}$ introduced by Allan and Wood (2020).
- For a construction of log. BP_k–modules, we need to explore realization $BP_k \hookrightarrow \mathcal{Z}_k \otimes \Pi(0)$.

What we can prove for log $L_k(\mathfrak{sl}_3)$–modules so far:

Theorem (ACG)

- *The universal affine VOA $V^k(\mathfrak{sl}_3)$ has logarithmic modules of rank three.*
- *For each level $k \notin \mathbb{Z}_{\geq 0}$, $L_k(\mathfrak{sl}_3)$ has logarithmic modules of rank two.*

  Assume that $k = -3 + \frac{u}{v}$ be an admissible level such that u, v are coprime positive integers and $v \geq 4$.
- *Then there exists log. BP_k–module $\widetilde{\mathcal{V}_k}$ of $L^{BP}(0)$ nilpotent rank two.*
- *For each $r, s \in \mathbb{Z}$, $\lambda \in \mathbb{C}$:*

$$\widetilde{\mathcal{V}_k} \otimes \mathcal{P}_s \otimes \Pi(0).e^{\frac{r}{2}d_2 + \lambda c_2}$$

  is a logarithmic $L_k(\mathfrak{sl}_3)$–module of $L_{sug}(0)$–nilpotent rank three.

Realisation of BP_k

Let $\mathcal{Z}^k = W^k(\mathfrak{sl}_3, f_{pr})$ be the universal Zamolodchikov vertex algebra $\mathcal{Z}^k$ freely generated by fields T and W. Let $\mathcal{Z}_k$ be its simple quotient.

Theorem (D.A, K. Kawasetsu, D. Ridout 2020)

For $k \neq -3$, $2k+3 \notin \mathbb{Z}_{\geq 0}$, there is a non-trivial vertex algebra homomorphism $\phi : BP_k \to \mathcal{Z}_k \otimes \Pi(0)$, uniquely determined by

$$
\begin{aligned}
&G^+ \mapsto e^c, \qquad J \mapsto j, \qquad L \mapsto T + \omega_{\Pi(0)}, \\
&G^- \mapsto \left(W + \tfrac{1}{2}(k+2)(k+3)\partial T\right) \otimes e^{-c} + (k+3)T \otimes i_{-1}e^{-c} \\
&\qquad - \mathbf{1} \otimes \left(i_{-1}^3 + 3(k+2)i_{-2}i_{-1} + 2(k+2)^2 i_{-3}\right) e^{-c}.
\end{aligned} \tag{4}
$$

$$\omega_{\Pi(0)} = \frac{1}{2}c(-1)d(-1) + \frac{2k+3}{3}c(-2) - \frac{1}{2}d(-2)$$

$$i = d - j, \; j = (d-c)/2 + \frac{k+3}{3}c.$$

Screening operators for realization

Theorem (D.A, T. Creutzig, N. Genra, 2021)

There exist a highest weight $\mathcal{Z}^k$–module $\widetilde{L}$ with the highest weight vector v_0 such that

$$T_0 v_0 = \frac{4k+9}{3} v_0, \quad W_0 v_0 = -\frac{(k+3)(4k+9)(5k+12)}{27} v_0,$$

and that

$$V = \int (v_0 \otimes \mathrm{e}^{-\frac{2k+3}{6}c + \frac{1}{2}d})(z)dz : \mathcal{Z}^k \otimes \Pi(0) \to \widetilde{L} \otimes \Pi(0).\mathrm{e}^{-\frac{2k+3}{6}c + \frac{1}{2}d}$$

is a screening operator of $BP^k \hookrightarrow \mathcal{Z}^k \otimes \Pi(0)$.

- Question: Is $\widetilde{L}$ a $\mathcal{Z}_k$–module?

Construction of logarithmic BP_k–modules

- Let L be the irreducible quotient of $\widetilde{L}$.
- Assume that L is an irreducible module for the simple VOA $\mathcal{Z}_k$.
- Consider the extended vertex algebra

$$\mathcal{V}_k = \mathcal{Z}_k \otimes \Pi(0) \bigoplus Z \otimes \Pi(0).\mathrm{e}^{-\frac{2k+3}{6}c+\frac{1}{2}d}.$$

- New screening operator

$$Q = \int \mathcal{Y}(v_0 \otimes e^i, z)dz : \mathcal{Z}_k \otimes \Pi(0) \to L \otimes \Pi(0).\mathrm{e}^{-\frac{2k+3}{6}c+\frac{1}{2}d}.$$

- For a $\mathcal{V}_k$–module $\mathcal{M}$ we define

$$(\widetilde{\mathcal{M}}, \widetilde{Y}_{\mathcal{M}}(\cdot, z)) := (\mathcal{M}, Y_{\mathcal{M}}(\Delta(s,z)\cdot, z)),$$

where

$$s = v_0 \otimes \mathrm{e}^{-\frac{2k+3}{6}c+\frac{1}{2}d},$$

$$\Delta(s,z) = z^{s_0} \exp\left(\sum_{n=1}^{\infty} \frac{s(n)}{-n}(-z)^{-n}\right).$$

Construction of logarithmic BP_k–modules

Theorem (ACG)

Assume that Z is a $\mathcal{Z}_k$–module and $2k+3 \notin \mathbb{Z}_{\geq 0}$.

- *Let $\mathcal{M}$ be any $\mathcal{V}_k$–module. Define*
- *Assume that $\mathcal{M}$ is a $L^{BP}(0)$ semi–simple $\mathcal{V}$–module such that Q acts non-trivially on $\mathcal{M}$. Then $\widetilde{\mathcal{M}}$ is a logarithmic BP_k–module.*
- *$\widetilde{\mathcal{V}_k}$ is a logarithmic BP_k–module of $\widetilde{L}^{BP}(0)$ nilpotent rank two.*
- *In particular, BP_k has logarithmic modules of nilpotent rank two for each admissible k such that $k+3 = \frac{u}{v}$, $v \geq 4$.*

- With suitable modification we can get log modules for admissible modules of levels with denominator $v = 3$.
- We expect that even beyond admissible levels, L is also Z_k–module, so we will have logarithmic BP_k–modules in that case too.

More applications: Affine analog of Feigin-Tipunin algebra

- Note that for $2k+3 \notin \mathbb{Z}_{\geq 0}$:

$$L_k(\mathfrak{sl}_3) \hookrightarrow \mathcal{Z}_k \otimes \mathcal{S} \otimes \Pi(0)^{\otimes 2}.$$

- We can transfer "classical" logarithmic VOAs to logarithmic affine VOAs:

  Ex. Take Feigin-Tipunin algebra $W_{A_2}(p)$ of central charge $(1,p)$ models for $\mathcal{Z}_k$ (higher rank analog of triplet VOA). Then

$$\mathcal{V}_{A_2}^{(p)} = (W_{A_2}(p) \otimes \mathcal{S} \otimes \Pi(0)^{1/2} \otimes \Pi(0)^{1/3})^{int_{\mathfrak{sl}_3}}$$

  is a logarithmic VOA associated to $L_k(\mathfrak{sl}_3)$ for $k = -3 + 1/p$.

- Admits $\mathfrak{sl}_3$–action inherited from $W_{A_2}(p)$. Applications to VOA tensor category theory.

Literature

- D.A, Realizations of simple affine vertex algebras and their modules: the cases $\widehat{sl(2)}$ and $\widehat{osp(1,2)}$, Communications in Mathematical Physics (2019)
- D. A., T. Creutzig, N. Genra, Relaxed and logarithmic modules of $\widehat{\mathfrak{sl}_3}$, arXiv:2110.15203 [math.RT]
- D.A, K Kawasetsu, D. Ridout, A realisation of the Bershadsky–Polyakov algebras and their relaxed modules, Lett Math Phys 111, 38 (2021).
- + some related work

Thank you