Basics on h-principle :

$$X \xrightarrow{\pi} B$$

$k \geq 0 \qquad X^{(k)} \xrightarrow{\pi} B$

$$X^{(k)} = \{ j^k_x s \mid s : Op(x) \longrightarrow X \}, \quad x \in B$$

$$j^1_x s = (x, s(x), s_{*_x} : T_x B \to T_{s(x)} X)$$

$$\Gamma X^{(k)} \supseteq \Gamma_{hol} X^{(k)} \qquad F = j^k s \quad \text{for} \quad s \in \Gamma X$$

Relation of order k : $\mathcal{R} \subseteq X^{(k)}$

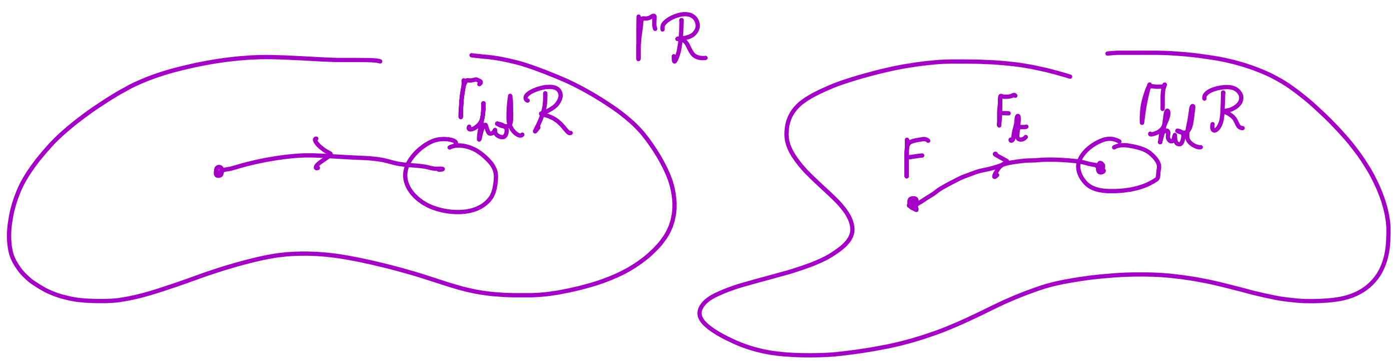
$$\Gamma \mathcal{R} = \{ F \in \Gamma X^{(k)} \mid F(B) \subseteq \mathcal{R} \}$$

$$\Gamma_{hol} \mathcal{R} = \{ j^k s \in \Gamma_{hol} X^{(k)} \mid j^k s(B) \subset \mathcal{R} \}$$

1-1 $\updownarrow$

$$Sol(\mathcal{R}) = \{ s \in \Gamma X \mid j^k s \in \Gamma_{hol} \mathcal{R} \}$$

Def: $\mathcal{R}$ satisfies the h-principle if f.s. can be deformed (among f.s.) to a hol. section of $\mathcal{R}$.



parametric h-principle

if $\pi_0(\Gamma_{hol} \mathcal{R}) \longrightarrow \pi_0(\Gamma \mathcal{R})$ bijection

$\forall$ connected comp. bijection between the π_k's

Examples: • $\mathcal{R}_{imm/sub} = \{ j^1_x f \mid df(x) \text{ inj.}/\text{surj.} \}$

$f : \mathcal{O}_p(x) \subseteq M \to N$

$X = M \times N \xrightarrow{\pi} M$

a sol. is an $\begin{cases} \text{immersion} \\ \text{submersion} \end{cases}$ $M \to N$

formal sol. $\begin{array}{ccc} TM & \xrightarrow{F} & TN \\ \downarrow & & \downarrow \\ M & \xrightarrow{f} & N \end{array}$ fibrewise inj./surj.

• $\mathcal{R}_{sympl} = \{ j^1_x \alpha \in T^*M^{(1)} \mid d\alpha(x) \text{ non-deg.} \}$

a sol. is an exact sympl. form

formal sol. $\longleftrightarrow$ non-deg. 2-forms

$\updownarrow$

acs on TM

• $\mathcal{R}_{contact} = \{ j^1_x \alpha \in T^*M^{(1)} \mid \alpha(x) \wedge d\alpha(x)^n \text{ vol. on } T_xM \}$

a sol. is a contact form

formal sol. $(\alpha, \beta) \in \Omega^1(M) \times \Omega^2(M)$

$\alpha \wedge \beta^n$ vol. on M

Gromov '69: Any open, invariant relation sat. the h-p. on any open man.

a) a subset of $X^{(k)}$

any isotopy of B $(\varphi_t : B \to B)$ lifts to isotopy of X

$\overline{}^{(k)} \quad {}^{(k)} \quad {}^{(k)}$

$\varphi_t^{(k)} : X^{(k)} \to X^{(k)}$

$\qquad\qquad\quad \downarrow \qquad\quad \downarrow$

$\varphi_t \qquad\quad B \;\to\; B$

$\varphi_t^{(k)}(\mathcal{R}) = \mathcal{R}.$

H-p for contact str.

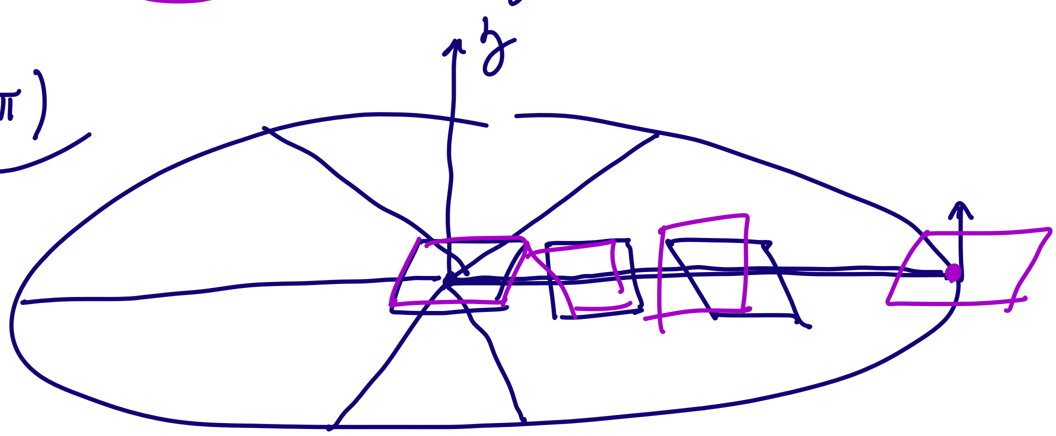
M^{2n+1} $\quad \xi = \ker \alpha \quad$ $\alpha \wedge d\alpha^n$ n.v.

Examples: $\mathbb{R}^3$ $\quad (x, y, z) \quad (r, \theta, z)$

tight $\quad \alpha_{std} = dz + r^2 d\theta = dz + \frac{1}{2}(x\,dy - y\,dx)$

$\alpha_{OT} = \cos r \, dz + r \sin r \, d\theta$

$D^2(\pi)$



$(\xi_{OT})_z = \ker \alpha_{OT}(z) = T_z D^2(\pi) \quad \forall z \in \partial D^2(\pi).$

$D^2(\pi)$ is an overtwisted disc.

ξ_{OT} ______________ contact str.

For contact str. on closed man. being homotopic $(\xi_t) \Rightarrow$ isotopic (Gray's thm)

Thm (BEM '14): M^{2n+1}, ACM closed, (α, β) formal contact str. on M, genuine on $Op(A)$. Then (α, β) is homot. to genuine ctct str. rel. to A.

Def: (M, ξ) is OT if it admits an OT model, i.e. an embedding

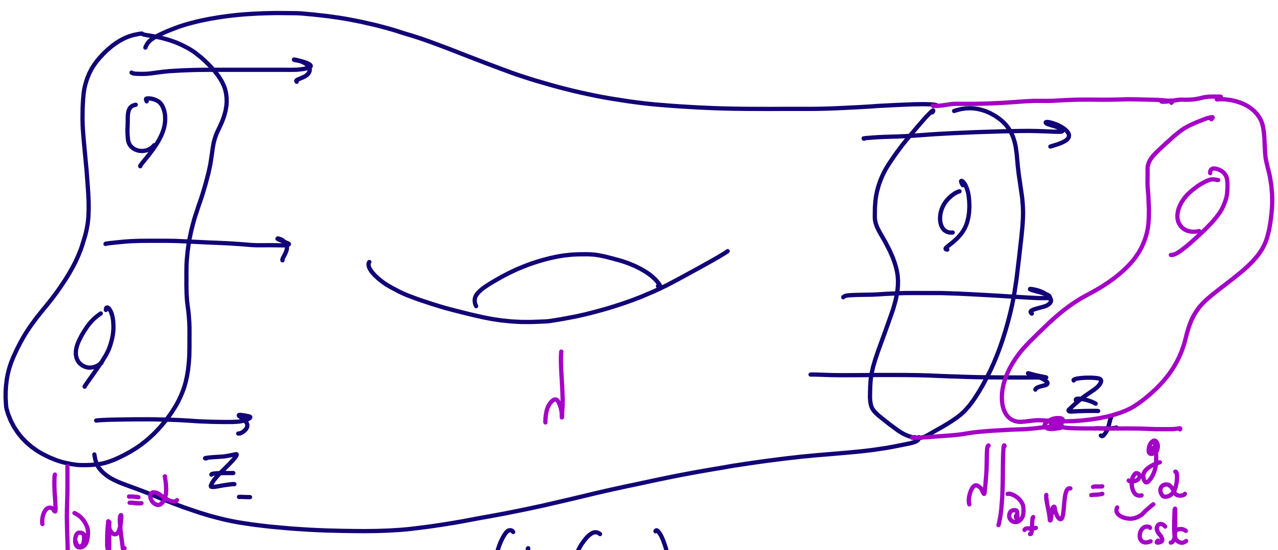
$$\left(Op_{\mathbb{R}^3}(D^2(\pi)) \times D^{2n-2}(R), \ \mathrm{Ker}(\alpha_{OT} + \lambda) \right)$$

into (M, ξ).

$$\lambda = \frac{1}{2} \sum_{i=1}^{n-1} (x_i \, dy_i - y_i \, dx_i)$$

Thm: (BEM) Full h-principle for OT ctct str.
(Full = parametric & relative.)

Symplectic cobordisms:



$\partial_- W \cong M_\xi$ $\quad (W, \omega) \quad$ $\partial_+ W \cong M_\xi$

$$\omega|_{Op(\partial_\pm W)} = d\lambda_\pm$$

$$i(Z_\pm)\omega = \lambda_\pm$$

$\xi_\pm = \ker \alpha_\pm$ contact str.

Liouville cob. $\omega = d\lambda$ Z

Thm: (EM) $W^{2n>2}$ $(\partial_\pm W, \xi_\pm)$ Ω non deg.

- Ω compatible with $\xi_\pm$
- $\partial_+ W \neq \emptyset$
- ξ_- OT
- ξ_+ OT if $n=2$

Then $\exists$ λ Liouville ($d\lambda = \omega$ sympl.) compatible ...

H-p for lcs

Thm (B., Meigniez) M^{2n} closed, η closed NON-EXACT 1-form, ω_a non-deg. $\eta_0 \in H^1(M;\mathbb{Z})$

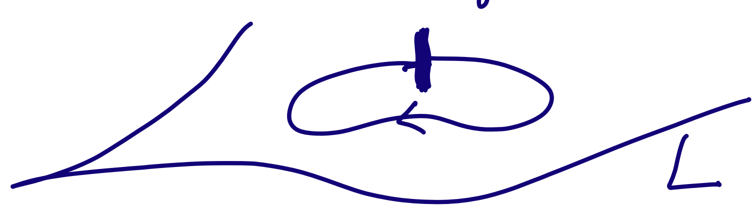
$\Downarrow$

$\exists$ λ $d_\eta \lambda$ lcs $c\eta_0$

Observation: M^{2n+1}, $T\mathcal{F}^{2n} = \ker \alpha$ $\alpha \wedge d\alpha = 0$

$\iff$ $d\alpha = \eta \wedge \alpha$ some 1-form η

$d_{\mathcal{F}} \eta = 0$ $\int_\gamma \eta_1 = \ln(h_\gamma'(0))$



$$\alpha + t\,\lambda = \alpha_t$$

($t \in \mathbb{R}$)

$$\underbrace{\alpha_t \wedge d\alpha_t^{\,n}}_{} = t^n \underbrace{\alpha \wedge (d_\eta \lambda)^n}_{\neq 0} + O(t^{n+1})$$

$\neq 0 \iff d_\eta \lambda$ lcs along leaves