Tensor categories arising from the Virasoro algebra.

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The Virasoro Lie algebra and its representations:

The Virasoro algebra has generators $\{L_n\}_{n\in\mathbb{Z}}, C$ with C central and commutator

$$[L_n, L_m] = (n-m)L_{n+m} + \frac{n^3-n}{12}\delta_{n,-m}C$$

Verma modules and irreducible quotients:

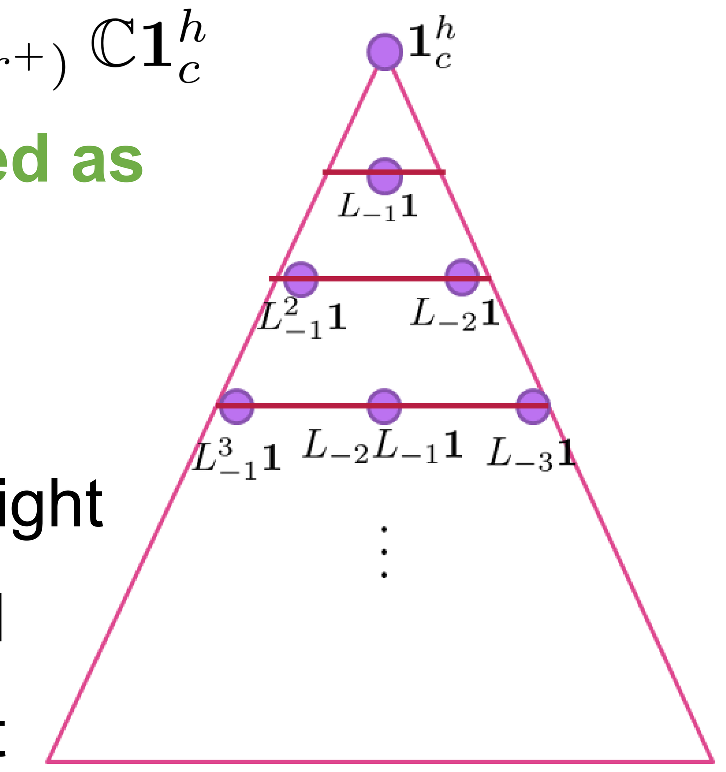
Let $Vir^+ := \bigoplus_{n\geq 0}\mathbb{C}L_n \oplus \mathbb{C}C$ and for $c, h \in \mathbb{C}$ let $\mathbb{C}\mathbf{1}_c^h$ be the Vir^+ - module such that

$$L_n\mathbf{1}_c^h = 0, n > 0$$
$$L_0\mathbf{1}_c^h = h\mathbf{1}_c^h$$
$$C\mathbf{1}_c^h = c\mathbf{1}_c^h$$

The Verma module of central charge c and conformal weight h is the induced module

$$M(c,h) := U(Vir)\otimes_{U(Vir^+)}\mathbb{C}\mathbf{1}_c^h$$

and it can be represented as



The irreducible lowest weight module $L(c,h)$, is defined as the irreducible quotient of the Verma by its maximal submodule

$$L(c,h) := M(c,h)/J(c,h)$$

The Virasoro Vertex operator algebras:

$M_c = M(c,0)/ < L_{-1}\mathbf{1} >$ is a VOA for any central charge $c \in \mathbb{C}$. Vermas and irreducibles are VOA modules for M_c. (Frenkel and Zhu, 92).

MAIN RESULT:

The category of finite length generalized M_c-modules with composition factors non- Verma irreducibles $L(c,h_i)$ has a braided tensor category structure.

Moreover, for generic central charges

$$c = 13 - 6t - 6t^{-1},\ t \notin \mathbb{Q}$$

this braided tensor category is rigid.

Choosing the candidate category to apply the logarithmic tensor product of Huang, Lepowsky and Zhang:

We define O_c^{fin} to be the category of finite length generalized M_c - modules with composition factors isomorphic to $L(c,h)$

O_c^{fin} is closed under taking submodules, direct sums, quotients and contragredient duals (all necessary to apply the tensor product theory)

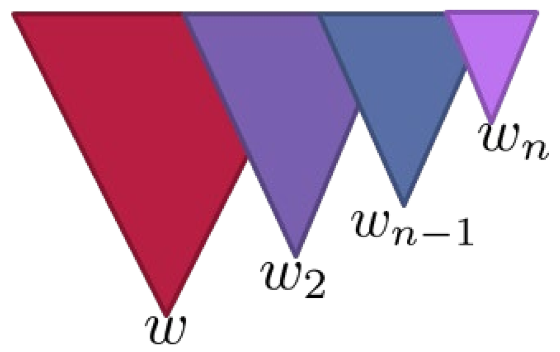
We define C_1^l to be the category of lower bounded generalized M_c- modules which are C_1 -cofinite.

C_1^l is closed under $P(z)$- tensor product (Miyamoto, 2014)

Theorem: These categories coincide. Namely,

$$O_c^{fin} = C_1^l$$

Sketch of proof:



The inclusion $O_c^{fin} \subset C_1^l$ is straightforward.

Given a C_1-cofinite lower bounded module we can build successive lowest weight sumodules that must also be C_1-cofinite. This produces a resolution with irreducibles isomorphic to $L(c,h)$ non-Vermas and we obtain $C_1^l \subset O_c^{fin}$.

Therefore $C_1^l = O_c^{fin}$