

# Overview of Some Results in Energy Market Modelling and Clean Energy Vision \*

*Anatoliy Swishchuk*

University of Calgary  
Calgary, Alberta, Canada

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## Outline of Presentation

- *Introduction*
- *Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market*
- *Variance and Volatility Swaps in Energy Markets*
- *Weather Derivatives in Energy Markets*
- *Pricing Crude Oil Options using Lévy Processes*
- *Energy Market Contracts with Delayed and Jumped Volatilities*
- *Energy Switching and Carbon Pricing*
- *A Vision to Transition to 100% Wind, Water & Solar Energy in Canada*

## Introduction: Abstract

The talk overviews my recent results in energy market modelling, including:

- [option pricing formula](#) for a mean-reversion asset,
- [variance and volatility swaps](#) in energy markets,
- applications of [weather derivatives](#) in energy markets,
- [pricing crude oil options](#) using Levy processes,
- energy contracts modelling with [delayed and jumped volatilities](#), and some latest results on
- [energy-switching](#) and [carbon pricing](#).

I will also talk about

- the [clean renewable energy](#) prospective and a vision to transition to 100% wind, water & solar energy in [Canada](#), and, in particular, in [Vancouver](#) and [Calgary](#).

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## **A Brief Overview**

## A Brief Overview

- Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market (Sw., A.: *J. Numer. Appl. Math.*, V.1(96), 2008, 216-233)
- Variance and Volatility Swaps in Energy Markets (Sw., A.: *The J. Energy Markets*, V. 6, N.1, Spring 2013, 33-50)
- Weather Derivatives in Energy Markets (Sw., A. & Cui, Kaijie: *The J. Energy Markets*, V.8, N.1, March 2015, 59-76)
- Pricing Crude Oil Options using Lévy Processes (Shahmoradi, Akbar & Sw., A.: *The J. Energy Markets*, V.9, N 1, March 2016, 47-64)

## A Brief Overview

- Energy Market Contracts with Delayed and Jumped Volatilities  
(*Handbook of Energy Finance: Theories, Practices and Simulations*, World Scientific, 2019)
- Energy Switching and Carbon Pricing  
(Arrigoni, A., Goutte, S., Lu, W. & Sw., A.: *J. Energy Markets*, 2019, Submitted)
- A Vision to Transition to 100% Wind, Water & Solar Energy in Canada, and, in particular, in Vancouver and Calgary

# Overview

## Overview

- **Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market**

**(Sw., A.: *J. Numer. Appl. Math.*, V.1(96), 2008, 216-233)**

Some commodity prices, like oil and gas, exhibit the [mean reversion](#), unlike stock price. It means that they tend over time to return to some long-term mean. [Black's model \(1976\)](#) and [Schwartz's model \(1997\)](#) have become a standard approach to the problem of pricing options on commodities.





Mean-Reversion

We presented explicit option pricing formula for a mean-reverting asset in energy market.

## Overview

- **Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market**

(Sw., A.: *J. Numer. Appl. Math.*, V.1(96), 2008, 216-233)

In this paper we considered a risky asset  $S_t$  following the mean-reverting stochastic process given by the following stochastic differential equation

$$dS_t = a(L - S_t)dt + \sigma S_t dW_t,$$

where  $W$  is a standard Wiener process,  $\sigma > 0$  is the volatility, the constant  $L$  is called the 'long-term mean' of the process, to which it reverts over time, and  $a > 0$  measures the 'strength' of mean reversion.

## Overview

- **Explicit Option Pricing Formula for a Mean-Reverting Asset in Energy Market**

(*J. Numer. Appl. Math.*, V.1(96), 2008, 216-233)

This mean-reverting model is a one-factor version of the two-factor model made popular in the context of energy modelling by Pilipovic (1997). We call it *continuous-time GARCH* or *inhomogeneous geometric Brownian motion* model.

Using a change of time method we find an explicit solution of this equation and using this solution we are able to find the *option pricing formula* under risk-neutral measure.

## Overview

- **Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market**

*(J. Numer. Appl. Math., V.1(96), 2008, 216-233)*

$$C_T^* = e^{-(r+a^*)T} S(0) N(y_+) - e^{-rT} K N(y_-) \\ + L^* e^{-(r+a^*)T} [(e^{a^*T} - 1) - \int_0^{y_0} z F_T^*(dz)],$$

where

$$y_+ := \sigma\sqrt{T} - y_0 \quad \text{and} \quad y_- := -y_0,$$

$$a^* := a + \lambda\sigma, \quad L^* := \frac{aL}{a + \lambda\sigma},$$

## Overview

### •Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market

(*J. Numer. Appl. Math.*, V.1(96), 2008, 216-233)

$y_0$  is the solution of the following equation

$$y_0 = \frac{\ln\left(\frac{K}{S(0)}\right) + \left(\frac{\sigma^2}{2} + a^*\right)T}{\sigma\sqrt{T}} - \frac{\ln\left(1 + \frac{a^*L^*}{S(0)} \int_0^T e^{a^*s} e^{-\sigma y_0 \sqrt{s} + \frac{\sigma^2 s}{2}} ds\right)}{\sigma\sqrt{T}},$$

and  $F_T^*(dz)$  is the probability distribution  $F_T(dz)$  as above, but instead of  $a$  we have to take  $a^* = a + \lambda\sigma$ ,  $\lambda$  is a market price of risk.

Remark: When  $L^* = 0$  and  $a^* = -r$ , then the explicit option pricing formula is the well-known [Black-Scholes formula](#)!

## Overview

- **Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market**

*(J. Numer. Appl. Math., V.1(96), 2008, 216-233)*

Numerical Example: [AECO Natural GAS Index \(1 May 1998-30 April 1999\)](#). We shall calculate the value of a European call option on the price of a daily natural gas contract. To apply our formula for calculating this value we need to calibrate the parameters  $a$ ,  $L$ ,  $\sigma$  and  $\lambda$ . These parameters may be obtained from futures prices for the AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999 (see Bos, Ware and Pavlov (2002), p.340). The parameters pertaining to the option are the following:

## Overview

- **Explicit Option Pricing Formula for a Mean-Reverting Risk-Neutral Asset in Energy Market**  
(*J. Numer. Appl. Math.*, V.1(96), 2008, 216-233)

| Price and Option Process Parameters |        |          |        |           |      |     |
|-------------------------------------|--------|----------|--------|-----------|------|-----|
| $T$                                 | $a$    | $\sigma$ | $L$    | $\lambda$ | $r$  | $K$ |
| 6<br>months                         | 4.6488 | 1.5116   | 2.7264 | 0.1885    | 0.05 | 3   |

From this table we can calculate the values for  $a^*$  and  $L^*$  :

$$a^* = a + \lambda\sigma = 4.9337,$$

and

$$L^* = \frac{aL}{a + \lambda\sigma} = 2.5690.$$

For the value of  $S_0$  we can take  $S_0 \in [1, 6]$ .

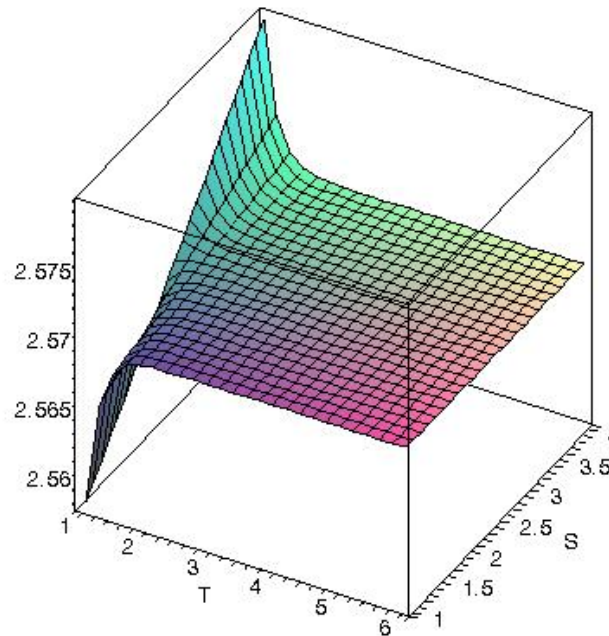
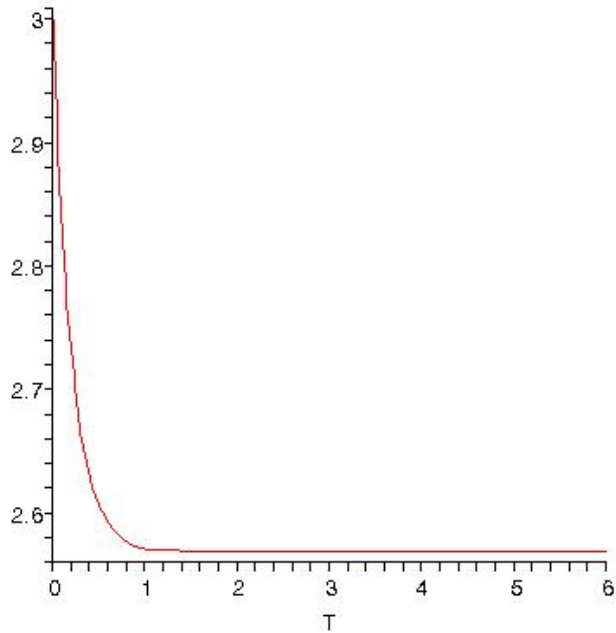


Fig. 1. Dependence of  $ES_t$  on  $T$  (AECO Natural Gas Index (1 May 1998-30 April 1999))

Fig. 2. Dependence of  $ES_t$  on  $S_0$  and  $T$  (AECO Natural Gas Index (1 May 1998-30 April 1999))



## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets*, V. 6, N.1, Spring 2013, 33-50)

We calculated variance and volatility swaps in energy market.

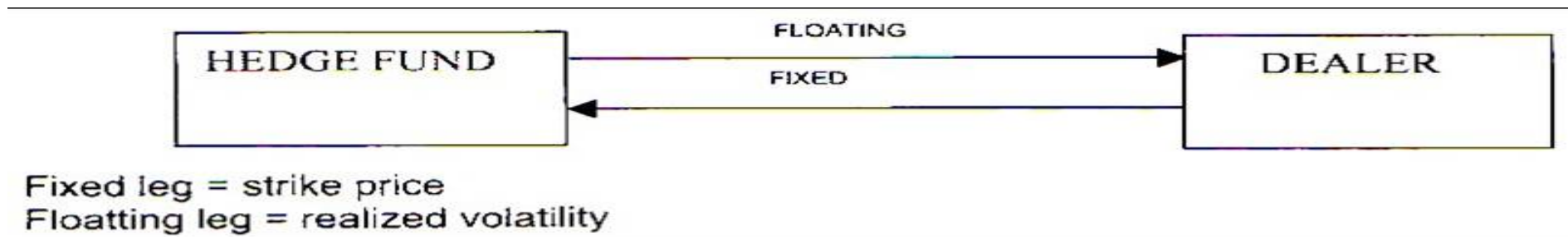


Fig. 1.Hedge Fund+Dealer

## SCENARIOS

A – The volatility increases:



B – The volatility decreases:

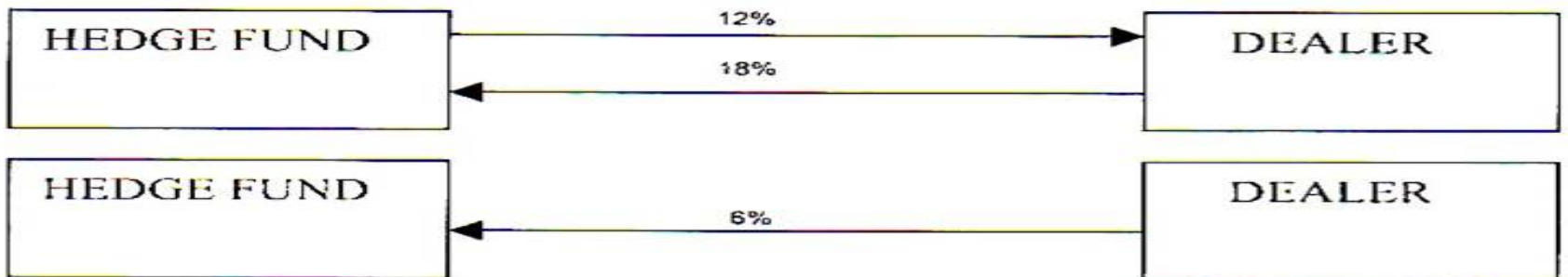


Fig. 2. Scenarios: A-volatility increases and B-volatility decreases

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets*, V. 6, N.1, Spring 2013, 33-50)

Variance swaps are quite common in commodity, e.g., in energy market, and they are commonly traded. We consider Ornstein-Uhlenbeck process for commodity asset with stochastic volatility following continuous-time GARCH model or Pilipovic (1998) one-factor model. The classical stochastic process for the spot dynamics of commodity prices is given by the Schwartz' model (1997). It is defined as the exponential of an Ornstein-Uhlenbeck (OU) process, and has become the standard model for energy prices possessing mean-reverting features.

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

Our focus on energy commodities derives from **two reasons**:

- 1) energy is the most important commodity sector, and crude oil and natural gas constitute the largest components of the two most widely tracked commodity indices: the Standard & Poors Goldman Sachs Commodity Index ( **S & P GSCI**) and the Dow Jones-AIG Commodity Index ( **DJ-AIGCI**);
- 2) existence of a liquid options market: **crude oil and natural gas** indeed have the deepest and most liquid options markets among all commodities.

The idea is to use **variance (or volatility) swaps on futures contracts**.

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets*, V. 6, N.1, Spring 2013, 33-50)

Variance risk premia in energy commodities, crude oil and natural gas, has been considered by [A. Trolle and E. Schwartz \(2009\)](#).

The same methodology as in Trolle & Schwartz (2009) was used by [Carr & Wu \(2009\)](#) in their study of equity variance risk premia.

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

The **S & P GSCI** is comprised of 24 commodities with the weight of each commodity determined by their relative levels of world production over the past five years. The **DJ-AIGCI** is comprised of 19 commodities with the weight of each component determined by liquidity and world production values, with liquidity being the dominant factor. Crude oil and natural gas are the largest components in both indices. In 2007, their weight were 51.30% and 6.71%, respectively, in the S & P GSCI and 13.88% and 11.03%, respectively, in the DJ-AIGCI.

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

The Chicago Board Options Exchange (CBOE) recently introduced a [Crude Oil Volatility Index](#) (ticker symbol [OVX](#)). This index also measures the conditional risk-neutral expectation of crude oil variance, but is computed from a cross-section of listed options on the United States Oil Fund (USO), which tracks the price of WTI as closely as possible.

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

The CBOE Crude Oil ETF Volatility Index (Oil VIX, Ticker - OVX) measures the market's expectation of 30-day volatility of crude oil prices by applying the VIX methodology to [United States Oil Fund, LP](#) (Ticker - [USO](#)) options spanning a wide range of strike prices (see Figures below. Courtesy-CBOE: <http://www.cboe.com/micro/oilvix/introduction.aspx>).

We have to notice that crude oil and natural gas trade in units of 1,000 barrels and 10,000 British thermal units (mmBtu), respectively. Usually, prices are quoted as US dollars and cents per barrel or mmBtu.



## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets*, V. 6, N.1, Spring 2013, 33-50)

In this paper, we considered a risky asset in energy market with stochastic variance following a mean-reverting stochastic process satisfying the following SDE ( [continuous-time GARCH\(1,1\) model](#)):

$$d\sigma^2(t) = a(L - \sigma^2(t))dt + \gamma\sigma^2(t)dW_t,$$

where  $a$  is a speed of mean reversion,  $L$  is the mean reverting level (or equilibrium level),  $\gamma$  is the volatility of volatility  $\sigma(t)$ ,  $W_t$  is a standard Wiener process.

## Overview

- **Variance and Volatility Swaps in Energy Markets**  
(*The J. Energy Markets, V. 6, N.1, Spring 2013, 33-50*)

Using a [change of time method](#) we found an explicit solution of this equation, and using this solution we are able to find the [variance and volatility swaps](#) pricing formula under the physical measure. Then, using the same argument, we find the option pricing formula under risk-neutral measure. We applied Brockhaus-Long (2000) approximation to find the value of volatility swap. A numerical example for the AECO Natural Gas Index for the period 1 May 1998 to 30 April 1999 is presented.

## Overview

- **Weather Derivatives in Energy Markets**

(Sw., A. & Cui, Kaijie: *The J. Energy Markets*, V.8, N.1, March 2015, 59-76)



## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

We used [future contracts written on temperature](#) to demonstrate the hedging strategies for commodities as an application of weather derivatives.

Our focus was on the [dynamic hedging strategy](#) of energy futures using temperature futures and constructing the [hedge ratio](#).

## Overview

- **Weather Derivatives in Energy Markets**

(Sw. & Cui, Kaijie: *The J. Energy Markets*, V.8, N.1, March 2015, 59-76)

The [weather derivatives market](#), in which contracts written on weather indices was firstly appeared over-the-counter (OTC) in July 1996 between Aquila Energy and Consolidated Edison Co. from United States. After that, companies accustomed to trading weather contracts based on electricity and gas prices in order to hedge their price risks realized by weather during the end of 1990s and the beginning of 2000s. Consequently, the market grew rapidly and expanded to other industries and to Europe and Japan.

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

Reported from [Weather Risk Management Association \(WRMA\)](#), an industry body that represents the weather market, recently, the total notional value of the global weather risk market has reached \$11.8 billion in 2014. With geographic expansion, the OTC market boosted nearly 30% in 2014. In this article, we concentrated on the market of temperature derivatives found at the [Chicago Mercantile Exchange \(CME\)](#), which is one of the largest weather derivatives trading platforms. Up to now, the CME has weather futures and options traded based on a range of weather indices for 47 cities from United States, Canada, Europe, Australia and Asia.

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

As a common sense, weather affects different entities in different ways. In order to hedge these different types of risks, weather derivatives are written on different types of weather variables or weather indices. The most commonly used weather variable is the temperature. Widely used temperature indices include **cumulative average temperature (CAT)**, **heating degree days (HDD)** and **cooling degree days (CDD)**. They are originated from the energy industry, and designed to correlate well with the local demands for heating or cooling.

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

CAT is defined as the sum of the daily average temperature over the period  $[\tau_1, \tau_2]$  of the contract, the index  $CAT := \sum_{t=\tau_1}^{\tau_2} T(t) = \int_{\tau_1}^{\tau_2} T(t) dt$ , where  $T(t)$  is the daily average temperature. It is mainly used in Europe and Canada. In winter, HDD are used to measure the demand for heating, i.e. they are a measure of how cold the weather is and usually used in United States, Europe, Canada and Australia. In contrast, CDD are used in summer to measure the demand of energy used for cooling and a measure of how hot the weather is. They are usually used in United States, Canada and Australia.



## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

The definitions for HDD and CDD are given by  $\text{HDD} := \max(T(t) - c, 0)$  and  $\text{CDD} := \max(c - T(t), 0)$ , where the constant  $c$  denotes the threshold, say  $65^\circ F (18^\circ C)$ . Since most air conditioners are switched on when temperatures are above or below  $c$ .

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

With respect to our model, consider the weather index  $T(t)$ , which is the daily average temperature (DAT). We suppose the DAT has a generalization of the Ornstein-Uhlenbeck dynamics

$$dT(t) = ds(t) + k(T(t) - s(t))dt + \sigma(t)dL(t),$$

where  $L(t)$  is a Lévy process (jump-diffusion),  $s(t)$  is the seasonal mean level and  $k$  is the speed in which the temperature reverts to  $s(t)$ .  $\sigma(t)$  is assumed to be a measurable and bounded function represents the seasonal volatility of temperature.

In the simplest case,  $L(t) = W(t)$ -a standard Wiener process.

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

This model was firstly introduced by Dornier and Queruel (2000) with Brownian motion as the random noise. Benth and Saltyte-Benth (2005) has successfully applied this model with generalized hyperbolic Lévy process to the Norwegian temperature data. We applied this model to our [Canadian temperature data](#) (Sw. & Cui (2013)).

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

We define the temperature futures prices written on **CAT**, **CDD** and **HDD**, which constitute the three main classes of futures products at CME market. Consider the price dynamic of future written on CAT over specific time period  $[\tau_1, \tau_2]$ , with  $\tau_1 < \tau_2$ . Firstly, assume the daily average temperature follows stochastic differential equation with  $L(t)$  being Lévy process and a constant continuously compounding interest rate  $r$ .

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

The future price  $F_{CAT}(t, \tau_1, \tau_2)$  at time  $0 \leq t \leq \tau_1$  based on CAT under risk-neutral probability measure  $Q$  is:

$$F_{CAT}(t, \tau_1, \tau_2) = E^Q \left[ \int_{\tau_1}^{\tau_2} T(s) ds \mid \mathcal{F}_t \right],$$

where  $Q$  is the risk-neutral measure (specified through Esscher transform) and  $\mathcal{F}_t$  is  $\sigma$ -algebra generated by  $L(t)$ .

## Overview

- **Weather Derivatives in Energy Markets**

(*The J. Energy Markets*, V.8, N.1, March 2015, 59-76)

Similarly, the risk-neutral CDD and HDD future prices are defined as:

$$F_{CDD}(t, \tau_1, \tau_2) = E^Q \left[ \int_{\tau_1}^{\tau_2} \max(T(s) - c, 0) ds | \mathcal{F}_t \right],$$

and

$$F_{HDD}(t, \tau_1, \tau_2) = E^Q \left[ \int_{\tau_1}^{\tau_2} \max(c - T(t), 0) ds | \mathcal{F}_t \right],$$

The relationship between futures prices of CAT, CDD and HDD is defined as

$$F_{CAT}(t, \tau_1, \tau_2) + F_{HDD}(t, \tau_1, \tau_2) = c(\tau_2 - \tau_1) - F_{CDD}(t, \tau_1, \tau_2).$$

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

Our focus will be on the **dynamic hedging strategy** of energy futures using temperature futures. In the spirit of Broadie and Jain (2008), consider a portfolio at time  $t$  containing one unit of energy (e.g. heating oil) future  $F_E$  and  $\beta_t$  ( $\beta_t$  is the hedge ratio for energy future  $F_E$ ) units of weather futures  $F_W$ , both with maturity (delivery) at time  $T$ . Assume the portfolio has value  $\Pi(t)$  at time  $t$ , a constant risk-free interest rate  $r$ , then

$$\Pi(t) = e^{-r(T-t)} [F_E(t) + \beta_t F_W(t)]. \quad (1)$$

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

The portfolio is self-financing, so the change in this portfolio in a small amount of time  $dt$  is given by

$$d\Pi(t) = r\Pi(t)dt + e^{-r(T-t)}[dF_E(t) + \beta_t dF_W(t)]. \quad (2)$$



## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

Hence, in order to dynamically hedge the energy future  $F_E$  with maturity  $T$ , the stochastic component of portfolio vanishes, the **hedge ratio**  $\beta_t$  could be defined as

$$\beta_t = -\frac{dF_E(t)}{dF_W(t)}, \quad (3)$$

with an assumption that  $dF_W(t) \neq 0$ . Therefore, from the last equation, to hedge an energy futures, we are required to hold  $\beta_t$  units of temperature future at time  $t$ .

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

The data used to calibrate the energy future consist of daily generic observations of WTI light, sweet crude oil futures prices (these data are obtained from Bloomberg financial service) with delivery periods in the first two front months. The [WTI crude oil futures](#) data used in calibration cover the CME exchange daily settlement prices ranging from January 2nd, 2001 to December 31st, 2010, resulting in 2508 record for each future contracts set (this choice of data set is consistent with that in Swishchuk and Cui (2013), which is [10 years of temperature data](#) from January 1st, 2001 to December 31st, 2010 in Calgary, AB, Canada).

## Overview

- **Weather Derivatives in Energy Markets**

(*The J. Energy Markets*, V.8, N.1, March 2015, 59-76)

Table below presents the [estimation results](#) for the energy model applied to the WTI crude oil future price data. The last two parameters  $\xi_1$  and  $\xi_2$  are the diagonal entries of matrix  $H := \text{Var}(\epsilon_t)$  with random noise  $\epsilon_t$ .

| Parameter  | $\mu$  | $\sigma_E$ | $\kappa_E$ | $\theta$ | $\xi_1$ | $\xi_2$ |
|------------|--------|------------|------------|----------|---------|---------|
| Estimation | 3.9187 | 0.0215     | 0.0025     | 0.2009   | 0.0003  | 0.0123  |

## Overview

- **Weather Derivatives in Energy Markets**

(*The J. Energy Markets*, V.8, N.1, March 2015, 59-76)

For the temperature market, we follow the calibration procedure described in Sw. and Cui (2013) to get the parameter set  $\Theta_W = \{\kappa_W, \sigma_W\}$ . For illustration purpose, we choose the estimated parameters in Calgary as the ones under the temperature market to calculate the hedge ratio. Recall the calibration results for Calgary in Swishchuk and Cui (2013), we could get the parameter set  $\Theta_W = \{\kappa_W, \sigma_W\}$  in Calgary as follows:

$$\kappa_W = -0.2411$$

and annual seasonal volatility

$$\sigma_W = 4.424 + 1.633 \cos(0.0167t) + 0.1912 \sin(0.0167t).$$

## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

To calculate the **correlation parameter  $\rho$** , we use the correlation between the filtered log-spot price and daily average temperature as a natural approximation to  $\rho$ . By taking all the daily average temperature on the dates with future prices available, and calculating the correlation coefficient between log-spot prices and average temperature of these days over 10 years (from January 2nd, 2001 to December 31st, 2010), we have the correlation  **$\rho = 0.1058$** . This correlation indicates a positive correlation between the log-spot price of crude oil and daily average temperature.

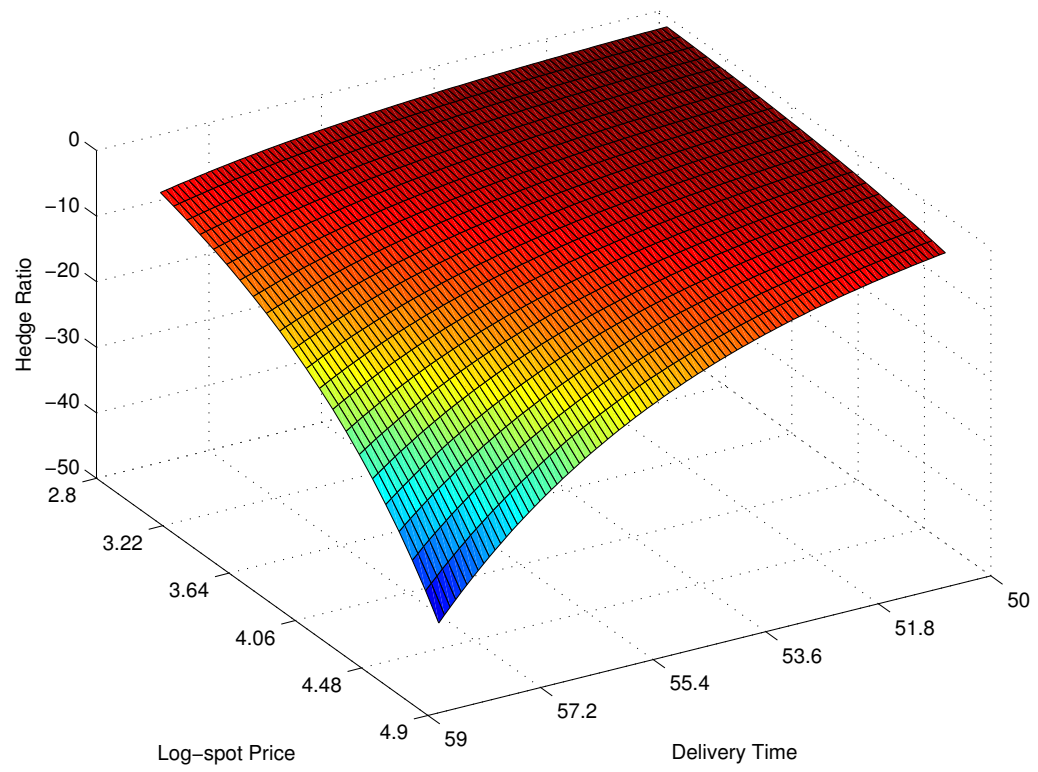
## Overview

- **Weather Derivatives in Energy Markets**

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

With the calibrated parameters in energy model and temperature model, we could then calculate the **dynamic hedge ratio**  $\beta_t$  explicitly.

In the Figure below, we plot the initial hedge ratio  $\beta_0$  along the crude oil future delivery time (in days) and initial log-spot price dimensions.



## Overview

### ●Weather Derivatives in Energy Markets

*(The J. Energy Markets, V.8, N.1, March 2015, 59-76)*

From this Figure, we could find that if one hold a **crude oil futures**, initially he need to short some CAT futures in the portfolio depending on the spot price of the crude oil and the time to delivery (trade termination) length. Basically **the number of temperature futures one need to hold will be more with longer time to delivery and higher spot price of the crude oil.**



## Overview

- **Pricing Crude Oil Options using Lévy Processes**  
(Shahmoradi, Akbar & Sw., A.: *The J. Energy Markets*, V.9, N 1, March 2016, 47-64)

Crude oil prices exhibit significant volatility over time and the distribution of returns on crude oil prices show fat tails and skewness, and they barely follow normal distribution.

## Overview

- **Pricing Crude Oil Options using Lévy Processes**  
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64*)

This was the reason we used **Normal Inverse Gaussian Process (NIG)**, **Jump Diffusion Process (JD)**, and **Variance-Gamma Process (VG)** as three Lévy processes that do not have these drawbacks and their tails carry heavier mass than normal distribution. Our results indicate that all these three Levy processes have very good out of sample results for near at the money options than others.

## Overview

- **Pricing Crude Oil Options using Lévy Processes**  
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64*)

The volatility of crude oil prices is very important for policy makers, crude oil producers and refineries. We used most recent data through April 2016 from crude oil futures and options markets to model dynamics of crude oil prices. Our results indicate that **crude oil prices show significant jumps** that are very frequent. **Crude oil price returns show skew** as well. These findings are consistent across all three models we used in this research.

## Overview

- **Pricing Crude Oil Options using Lévy Processes**  
(*The J. Energy Markets, V.9, N 1, March 2016, 47-64*)

In the case of **JDM**, the volatility of size of the **jumps is bigger than volatility of the diffusion part**. The **VG** process results in slightly **smaller volatility than JDM**. The mean of the jump component size implied by JDM, and skew parameter of VG process both indicate existence of right-skew in crude oil price returns, but the **NIG process** implies that the density of returns are skewed to the left.

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, World Scientific, 2019)*

In this paper we concentrated on [stochastic modelling and pricing of energy markets' contracts for stochastic volatilities with delay and jumps](#). Our model of stochastic volatility exhibits jumps and also past-dependence: the behaviour of a stock price right after a given time  $t$  not only depends on the situation at  $t$ , but also on the whole past (history) of the process  $S(t)$  up to time  $t$ .

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, World Scientific, 2019)*

The basic products in these markets are spot, futures and forward contracts and options written on these. [We study forwards and swaps](#). A numerical examples is presented for stochastic volatility with delay using the [Henry Hub daily natural gas data \(1997-20011\)](#).

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

Let the stochastic process  $S(t)$  be denoted as (er call it **Geometric Models** with Stochastic Delayed and Jumped Volatility):

$$\ln S(t) = \ln \Lambda(t) + \sum_{i=1}^m X_i(t) + \sum_{j=1}^n Y_j(t),$$

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

where for  $i = 1, \dots, m$

$$dX_i(t) = (\mu_i(t) - \alpha_i(t)X_i(t))dt + \sigma_i(t, X_i(t + \theta))dB(t),$$

and for  $j = 1, \dots, n$

$$dY_j(t) = (\delta_j(t) - \beta_j(t)Y_j(t))dt + \eta_j(t, Y_j(t + \theta))dI_j(t).$$



## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

Here,  $\theta \in [-\tau, 0]$ ,  $\tau > 0$ , is the delay, and on the interval  $[-\tau, 0]$ ,  $X_i(t) = \phi_i(t)$  and  $Y_j(t) = \psi_j(t)$ , where  $\phi_i(t)$  and  $\psi_j(t)$  are deterministic functions,  $i = 1, \dots, m$  and  $j = 1, \dots, n$ .

We remark that two factors  $X_i(t)$ ,  $i = 1, \dots, m$ , and  $Y_j(t)$ ,  $j = 1, \dots, n$ , represent the long- and short-term fluctuations of the spot dynamics which may be correlated. We suppose that jumps components  $I_j$  are independent, which is an obvious restriction of generality.

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

(*Handbook of Energy Finance: Theories, Practices and Simulations*, 2019)

The deterministic seasonal price level is modelled by the function  $\Lambda(t)$ , ( **seasonal function**) which is assumed to be continuously differentiable.

The coefficients  $\mu_i, \alpha_i, \delta_j \beta_j$  are all continuous functions. We suppose that volatilities  $\sigma_{ik}(t)$  and  $\eta_j(t)$  are **stochastic volatilities with delay and jumps**.

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

(*Handbook of Energy Finance: Theories, Practices and Simulations*, 2019)

We consider two cases in this situation:

$$\begin{aligned} \frac{d\sigma_i^2(t, X_i(t+\theta))}{dt} &= \gamma_i^1 V_i + \frac{\alpha}{\tau} \left[ \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta)) dB(u) \right. \\ &+ \left. \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta)) d\tilde{N}_1(t) \right]^2 \\ &- (a_i + b_i) \sigma_i^2(t, X_i(t+\theta)) \end{aligned}$$

and

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

$$\begin{aligned} \frac{d\eta_j^2(t, Y_j(t+\theta))}{dt} &= \gamma_j^2 W_i + \frac{\alpha}{\tau} \left[ \int_{t-\tau}^t \eta_j(u, X_j(u+\theta)) dB_1(u) \right. \\ &+ \left. \int_{t-\tau}^t \sigma_i(u, X_i(u+\theta)) d\tilde{N}_2(t) \right]^2 \\ &- (c_j + d_j) \eta_j^2(t, X_i(t+\theta)) \end{aligned}$$

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

Here,  $B(t)$  and  $B_1(t)$  are two independent Brownian motions and  $\tilde{N}_1(t)$  and  $\tilde{N}_2(t)$  are two independent compensated Poisson processes with intensities  $\lambda_1$  and  $\lambda_2$ , independent of  $B(t)$  and  $B_1(t)$ .

We note, that in [Benth *et al.*, (2008)] it was considered **only deterministic**  $\sigma_i(t)$  and  $\eta_j(t)$ .

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

(*Handbook of Energy Finance: Theories, Practices and Simulations, 2019*)

A simple model for the **spot price** is considered ( $\tau$  is the delay parameter.):

$$\ln S(t) = X(t),$$

where

$$dX(t) = \gamma(k - X(t))dt + \sigma(t, X(t))dB(t),$$

and

$$\frac{\sigma^2(t, X(t))}{dt} = [\alpha + \beta \int_{t-\tau}^t \sigma(s, X(s))dB(s)]^2 + c\sigma^2(t, X(t)).$$

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

The model for  $\sigma^2(t, X(t))$  above is the same as the model for stochastic volatility with delay that we considered in [[Kazmerchuk, Swishchuk and Wu, 2005](#)].

## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

Discrete scheme is implemented:  $l = 2 = \lfloor \frac{\tau}{\Delta} \rfloor$ , where  $\Delta$  is the size of the mesh of the discrete-time grid,  $\lfloor \cdot \rfloor$  is the floor function.

Estimated Parameters are (Courtesy- [\[Otunuga and Ladde, 2014\]](#)):

| $\gamma$ | $k$    | $\tau$ | $\alpha$ | $\beta$ | $c$   |
|----------|--------|--------|----------|---------|-------|
| 1.8943   | 1.5627 | 0.008  | 0.433    | - 0.07  | - 1.5 |



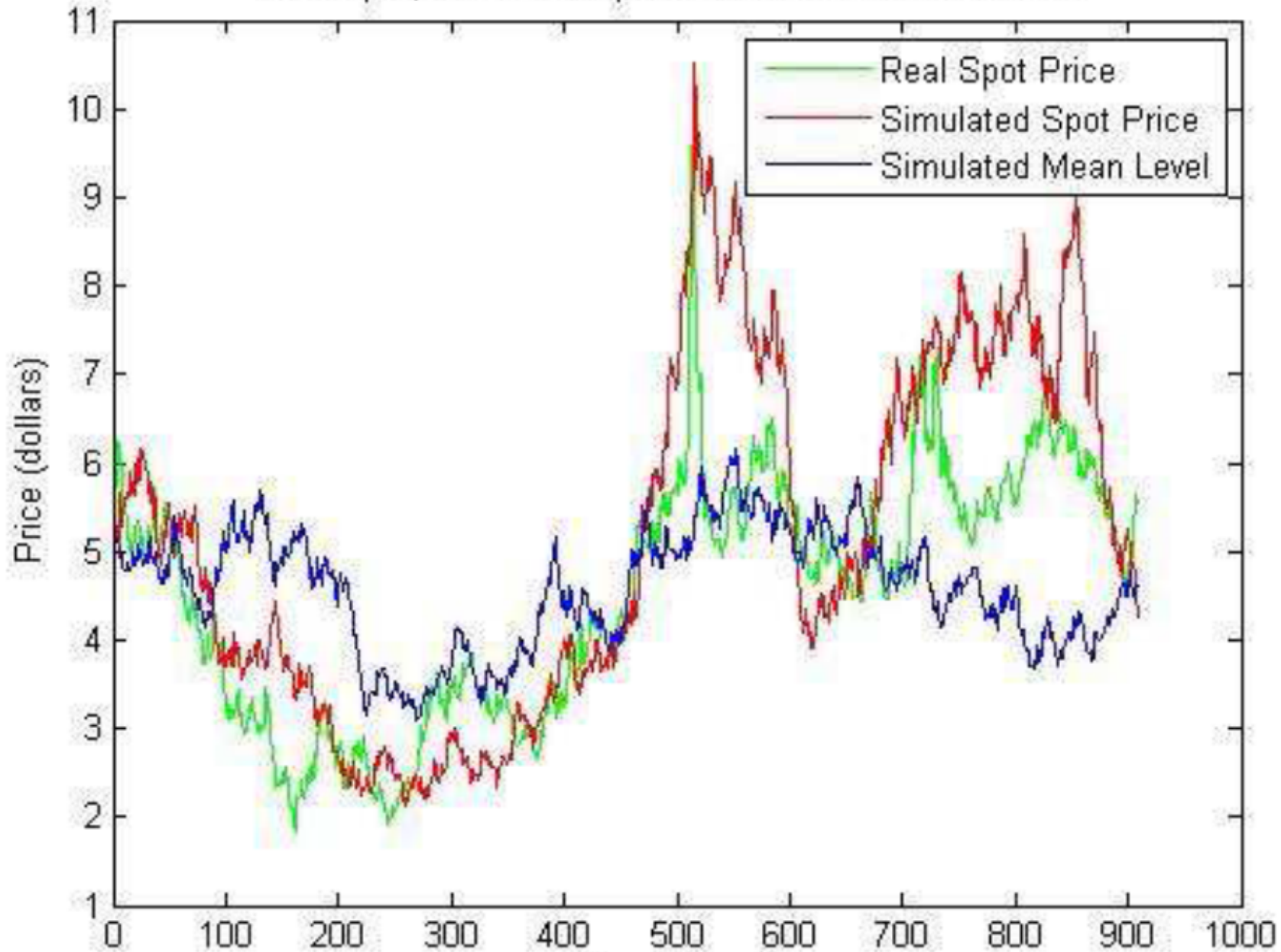
## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

Graphs below, Figure 2, includes Real, Simulated Spot Prices and Simulated Expected Spot Price ( [Henry Hub Daily Natural Gas Data](#) Set (02/01/2001-09/30/2004)):

Real Spot, Simulated Spot and Simulated Mean Prices



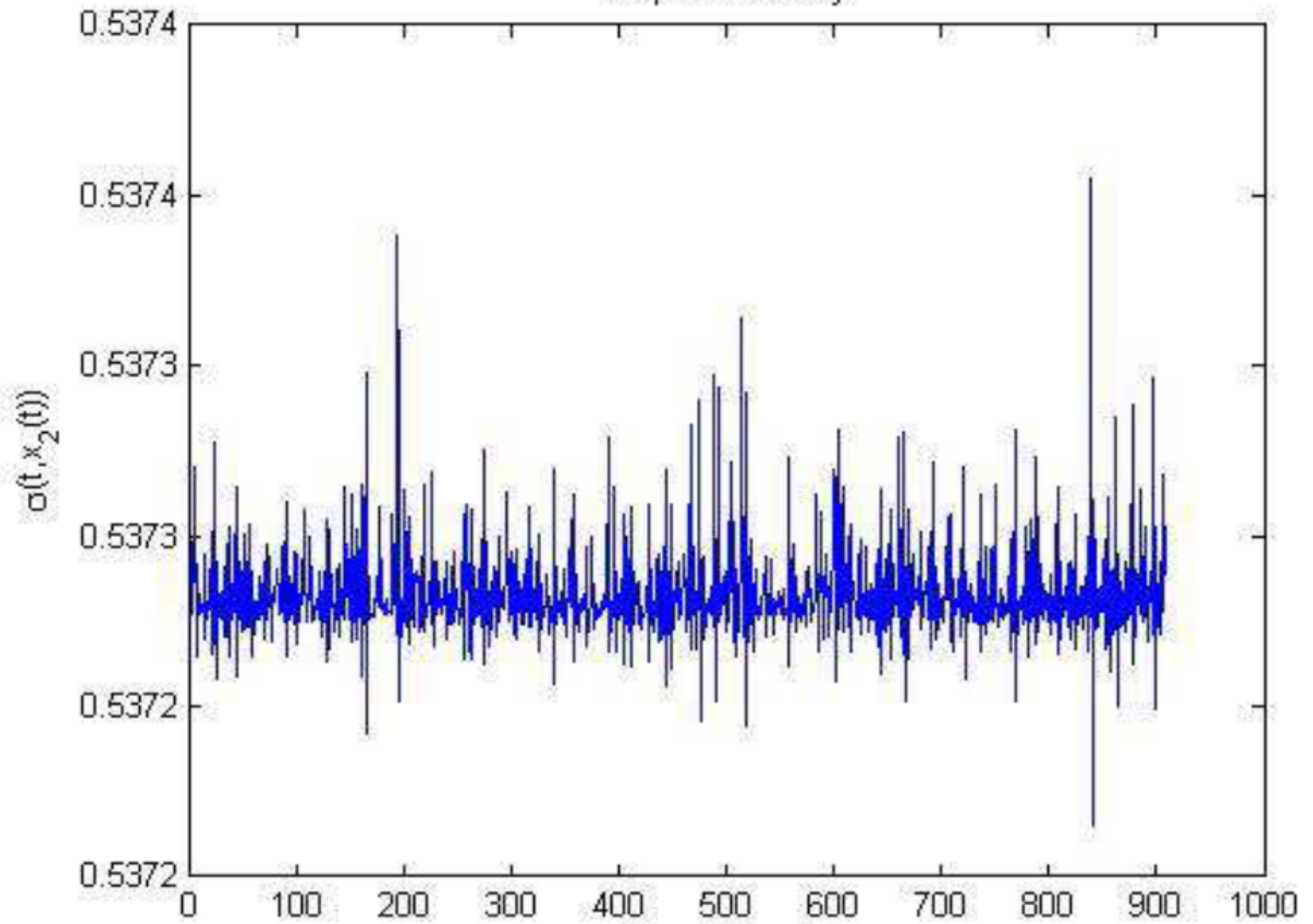
## Overview

- **Energy Market Contracts with Delayed and Jumped Volatilities**

*(Handbook of Energy Finance: Theories, Practices and Simulations, 2019)*

Graph below, Figure 3, shows **simulated  $\sigma(t, X(t))$**  from Henry Hub Daily Natural Gas Data Set (02/01/2001-09/30/2004).

Graph of volatility



## Overview

### •Energy-switching and Carbon Pricing

( Arrigoni, A., Goutte, S., Lu, W. & Sw., A.: 'Energy-switching using Lévy Processes-An Application to Canadian and North American Data', *J. Energy Markets*, 2019, submitted)

The [Paris agreement in 2016](#) marks a global effort to limit the increase in temperature. In that spirit, the Federal Government of Canada introduced a carbon tax to [reduce greenhouse gas emissions](#).

The main goal of this paper is to define the correct approach to [carbon pricing](#).

## Overview

- **Energy-switching and Carbon Pricing**

( Arrigoni, A., Goutte, S., Lu, W. & Sw., A.: 'Energy-switching using Lévy Processes-An Application to Canadian and North American Data', *J. Energy Markets*, 2019, submitted)

Following the method, introduced by Goutte and Chevalier (2015), we define the **carbon price** as the necessary tax to incite electricity producers to switch from coal to natural gas.

## Overview

### •Energy-switching and Carbon Pricing

( Arrigoni, A., Goutte, S., Lu, W. & Sw., A.: 'Energy-switching using Lévy Processes-An Application to Canadian and North American Data', *J. Energy Markets*, 2019, submitted)

The novelty of this paper is that we use this method for [Alberta and North America](#). In addition, we consider the case of switching from natural gas to wind as a potential [new approach to carbon pricing](#).

More details: [Poster presented by Weiliang Lu](#) (today, 14:40-15:15pm).

## Overview

- **A Vision to Transition to 100% Wind, Water & Solar Energy in [Canada](#)**

A group of U.S. civil engineering has calculated that [Canada could be completely powered by renewable energy](#), if we just decide to do it.

They say that would save [\\$110.1 billion on health care](#) costs every year and prevent [9,884 annual air pollution deaths](#).

Their research is available at [thesolutionsproject.org](http://thesolutionsproject.org).



## Overview

- **A Vision to Transition to 100% Wind, Water & Solar Energy in Canada**

# 2050

## PROJECTED ENERGY MIX



Residential rooftop solar  
**5.3%**



Solar plants  
**6.9%**



Concentrating solar  
plants  
**9.8%**



Onshore wind  
**27.5%**



Offshore wind  
**22.9%**

Commercial &  
government rooftop solar  
**9.1%**



Wave devices  
**2.2%**



Geothermal  
**1.7%**



Hydroelectric  
**14.5%**



Tidal Turbines  
**0.2%**



## A Vision to Transition to 100% Wind, Water & Solar Energy in Canada



### 40-Year Jobs Created

Number of jobs where a person is employed for 40 consecutive years

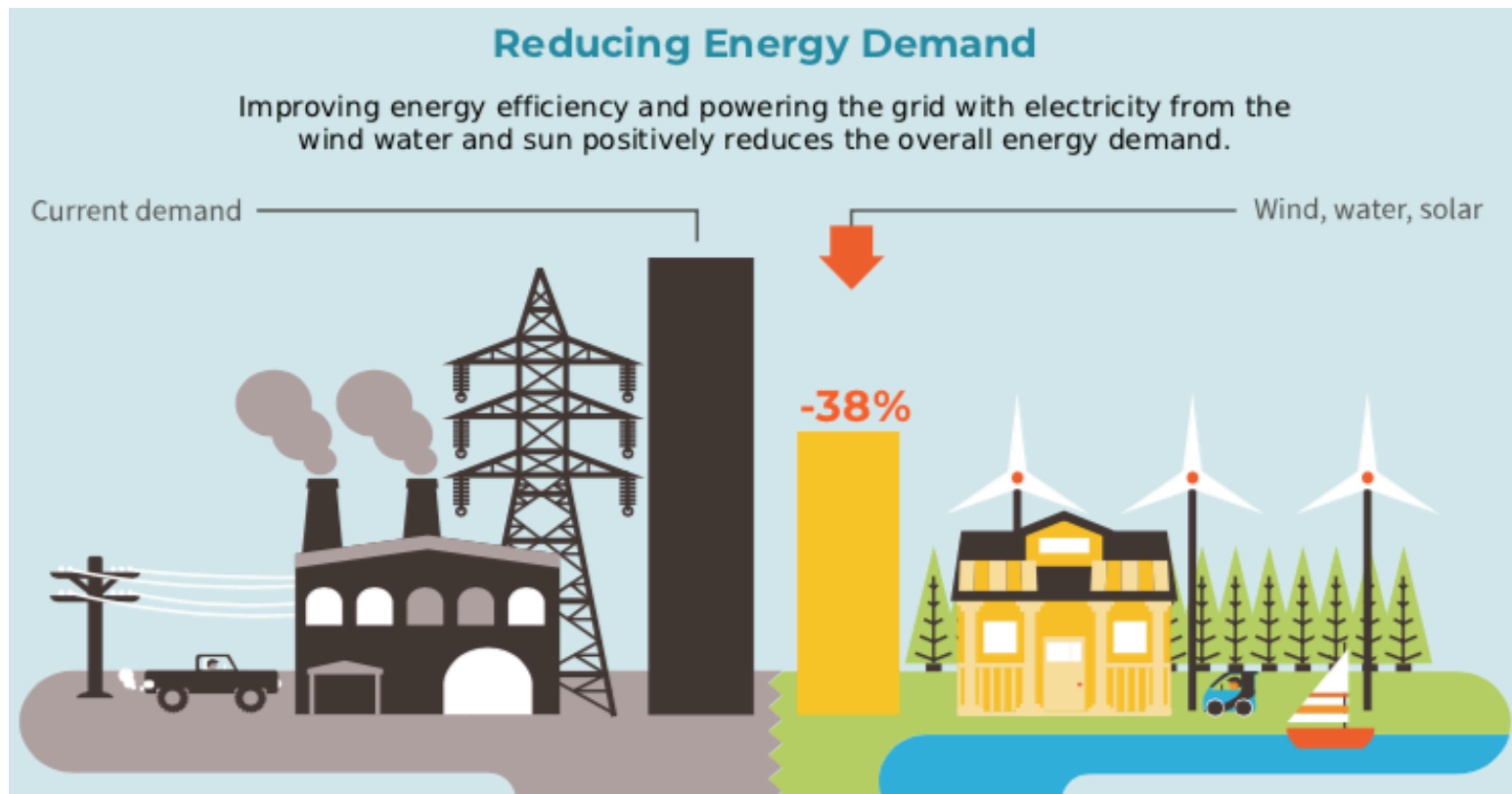


Construction jobs: **315,138**



Operation jobs: **367,889**

# A Vision to Transition to 100% Wind, Water & Solar Energy in Canada



**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:  
Health Cost Savings**

## Health Cost Savings

Avoided health costs per year:



**3.94%** of  
country GDP

**Lives lost to air pollution that we could  
save each year:**

**9,884**

The transition pays for itself in as little as  
**4.1 years** from air pollution and climate  
cost savings alone

**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:  
Land Usage**

## Land Usage

Percentage of Canada Land Needed for All  
New Wind, Water & Solar Generators

**0.03%**

Footprint Area

**0.25%**

Spacing Area





**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:  
Average Energy Costs in 2050**

## Average Energy Costs in 2050



**7.96¢**  
/kWh\*



**9.71¢**  
/kWh\*

- Fossil Fuels & Nuclear Energy
- Wind, Water & Solar

\*Health and climate externality costs of fossil fuels are another 5.7¢/kWh

**A Vision to Transition to 100% Wind, Water & Solar Energy in Canada:  
Money in Your Pocket**

## Money in your Pocket



Energy cost savings per person:

**\$40.70**



Energy, health, and climate  
cost savings per person:

**\$10,618**

# **A Vision to Transition to 100% Wind, Water & Solar Energy in Vancouver**

# 100% VANCOUVER

A vision for the transition to 100% wind, water & solar energy



Residential rooftop solar  
8%



Solar plants  
9%



Concentrating solar plants  
0%



Onshore wind  
30%



Offshore wind  
8%

# 2050

PROJECTED  
ENERGY MIX



Commercial &  
government rooftop solar  
8%



Wave devices  
1%



Geothermal  
1%



Hydroelectric  
35%



Tidal Turbines  
0%



## 40-Year Jobs Created

Number of jobs where a person is employed for 40 consecutive years



Construction jobs: **1,338**



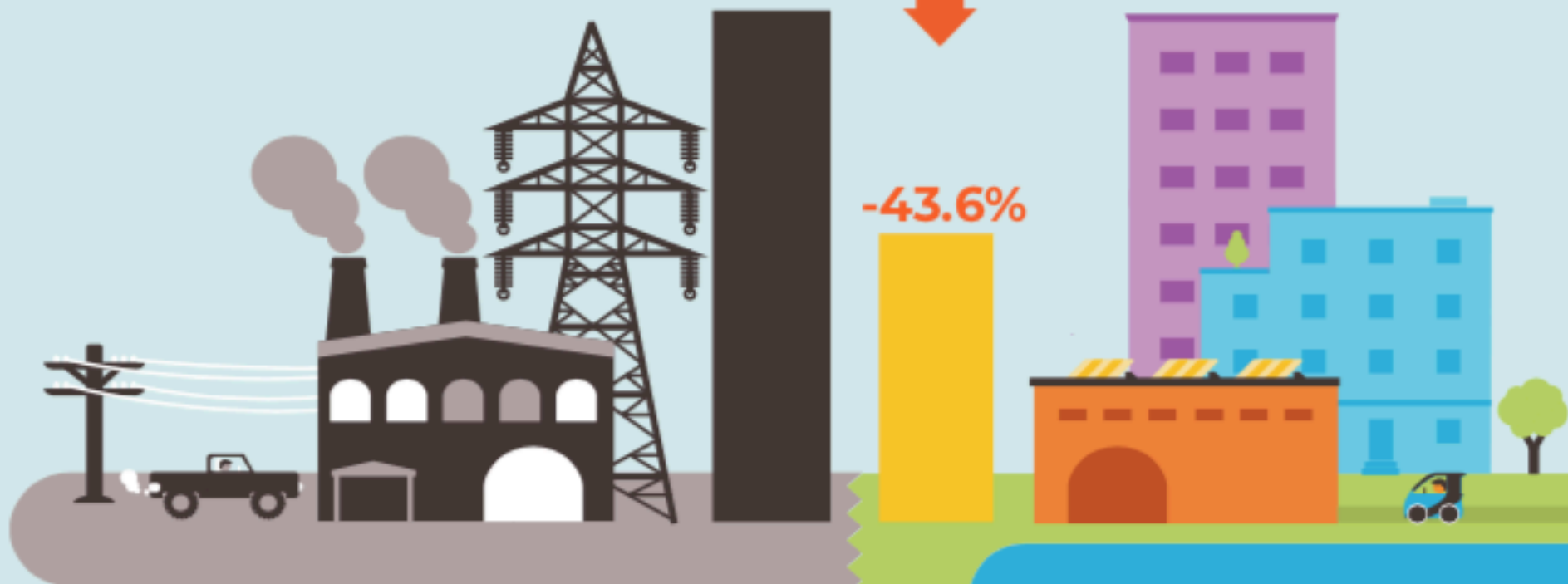
Operation jobs: **450**

## Reducing Energy Demand

Improving energy efficiency and powering the grid with electricity from the wind water and sun positively reduces the overall energy demand.

Current demand

Wind, water,



## Health Cost Savings

Avoided health costs per year:



**0.13%** of city  
GDP

**Lives lost to air pollution that we could  
save each year:**

**52**

The transition pays for itself in as little as  
**1.1 years** from air pollution and climate  
cost savings alone



## Average Energy Costs in 2050



- Fossil Fuels & Nuclear Energy
- Wind, Water & Solar

\*Health and climate externality costs of fossil fuels are another 5.7¢/kWh

## Money in your Pocket



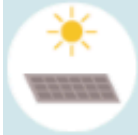
# **A Vision to Transition to 100% Wind, Water & Solar Energy in Calgary**

# 100% CALGARY

A vision for the transition to 100% wind, water & solar energy



Residential rooftop solar  
4%



Solar plants  
19%



Concentrating solar plants  
10%



Onshore wind  
35%



Offshore wind  
0%

## 2050

PROJECTED  
ENERGY MIX



Commercial &  
government rooftop solar  
4%



Wave devices  
0%



Geothermal  
9%



Hydroelectric  
19%



Tidal Turbines  
0%



## 40-Year Jobs Created

Number of jobs where a person is employed for 40 consecutive years



Construction jobs: **8,308**



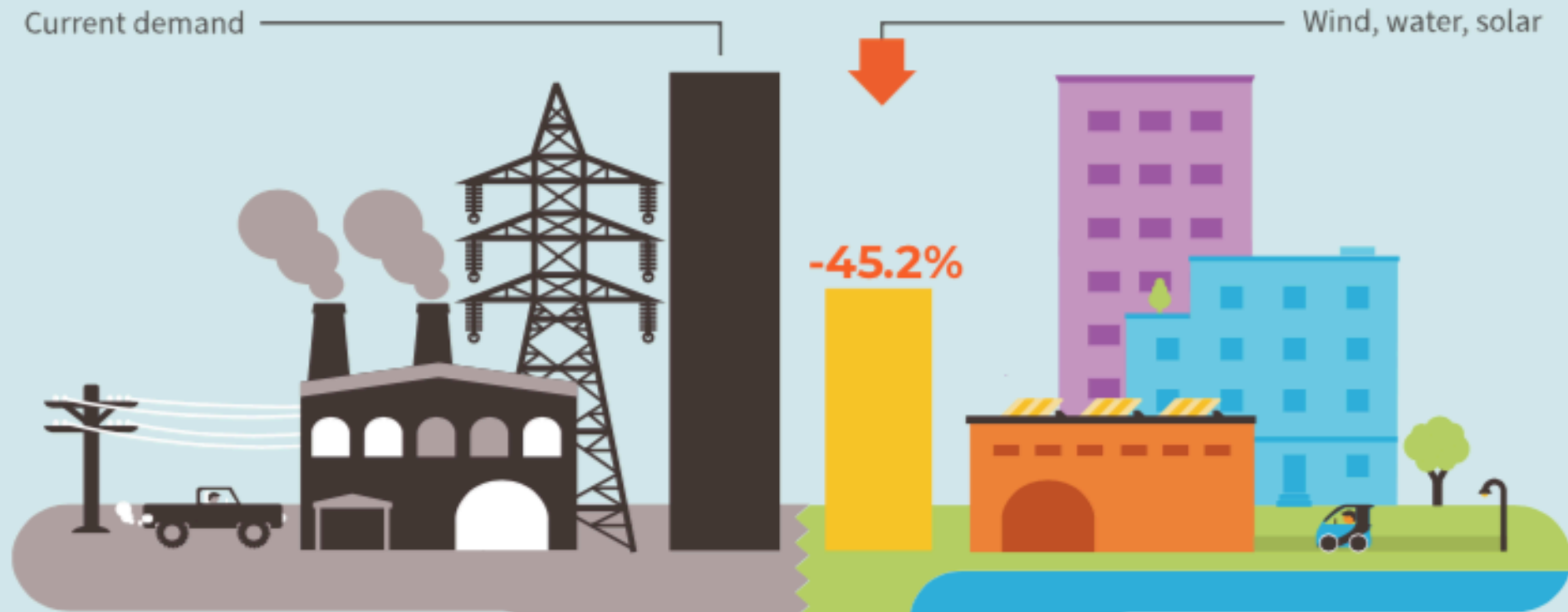
Operation jobs: **2,821**

## Reducing Energy Demand

Improving energy efficiency and powering the grid with electricity from the wind water and sun positively reduces the overall energy demand.

Current demand

Wind, water, solar



## Health Cost Savings

Avoided health costs per year:



1.72% of city  
GDP

Lives lost to air pollution that we could  
save each year:

**337**

The transition pays for itself in as little as  
**1.8 years** from air pollution and climate  
cost savings alone

## Average Energy Costs in 2050



- Fossil Fuels & Nuclear Energy
- Wind, Water & Solar

\*Health and climate externality costs of fossil fuels are another 5.7¢/kWh

## Money in your Pocket



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[www.thesolutionsproject.org](http://www.thesolutionsproject.org)



## Conclusion

In this talk we overviewed my recent results in energy market modelling, including:

- option pricing formula for a mean-reversion asset,
- variance and volatility swaps in energy markets,
- applications of weather derivatives in energy markets,
- pricing crude oil options using Lévy processes,
- energy contracts modelling with delayed and jumped volatilities, and some latest results on
- energy-switching and carbon pricing.

I also talked about the clean renewable energy prospective, and a vision to transition to 100% wind, water & solar energy in Canada, and, in particular, in Vancouver and Calgary.

**The End**

*Thank You!*

*Q&A time!*



**UNIVERSITY OF  
CALGARY**