EXOTIC REPRESENTATIONS

in non-abelian and abelian F-theory models

Nikhil Raghuram (MIT)

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BASED ON

For non-Abelian models

- arXiv:1706.08194 - D. Klevers, D. Morrison, NR, W. Taylor

For abelian models

- arXiv:1711.03210 - NR

BROAD QUESTIONS

Which charged matter representations can be obtained in F-theory?

- How do codim. 2 singularities $\rightarrow$ charged matter?
- How do you construct explicit Weierstrass models w/ certain matter spectra?

In F-theory, tough to get more than a few simple reps.

- Some reps. drop out easily
  - e.g. in Tate's algorithm constructions
- For reps beyond these, models are complicated
  - Greater algebraic complexity
  - Few systematic methods for obtaining models

EXOTIC VS. NON-EXOTIC REPS.

EXOTIC REPS: Reps difficult to obtain in F-theory constructions

	SU(N)	U(1)
NOT EXOTIC	Fundamentals 2-antisymmetrics Adjoints	Charge 1 and 2
EXOTIC	3-antisym. 4-antisym. Symmetric 3-sym.	Charge 3 and above

WHY STUDY EXOTICS?

We cannot characterize full F-theory landscape without understanding exotic representations

- Match between SUGRA and F-theory
  - Can all 6D SUGRAs be realized as F-theory compactifications?
  - Non-abelian: Models with some reps, spectra cannot
  - Abelian: Potentially infinite number of consistent SUGRA models
    - See upcoming work by [Taylor and Turner]
    - Which abelian models have F-theory constructions?
- Learn more about codim-2 singularities & physical interpretation
- Classification of EFCY manifolds

OUTLINE

PART I NON-ABELIAN MODELS

1. Higher Genus Representations
2. Non-Realizable Representations and Matter Spectra

PART II ABELIAN MODELS

1. Models with $q = 3$ and $q = 4$ Matter
2. Conjectures on Larger Charges

PART I NON-ABELIAN MODELS

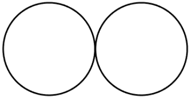
TYPICAL REPRESENTATIONS

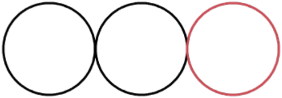
Typical charged matter: singularity type enhances on codim-two locus

- Resolution introduces exceptional curves forming Dynkin diagram

EXAMPLE *Fundamental of SU(n)*

Codim-one Singularity		Codim-Two Singularity
I_n	$\rightarrow$	I_{n+1}
A_{n-1}		A_n



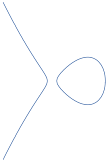


HIGHER GENUS REPRESENTATIONS

- Certain reps. involve 7-branes wrapped on higher genus divisors
- Exotic reps. can be localized at singular loci

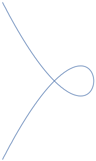
FOR SU(N)

Smooth Curve with Genus g



g Adjoints

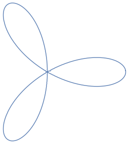
Double Point Singularity



Adjoint or

$\square\square + \begin{smallmatrix}\square\\\square\end{smallmatrix}$

Triple Point Singularity

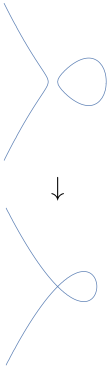


3 Adjoints or

$\square\square\square + 2 \times \square$

HIGHER GENUS DIFFICULTIES I

[Sadov '96] Double points give symmetrics



ISSUE

1. Start with smooth higher genus curve
   - Adjoints supported, no symmetrics
2. Tune a double point
3. Has the matter content changed?

[Morrison, Taylor '12]

- Double points can also give adjoints
- Just tuning double point doesn't give symmetrics

How do you distinguish adjoint vs. symmetric double points?
How do you construct models with symmetrics?

HIGHER GENUS DIFFICULTIES II

There are prior models with higher genus exotics:

SU(3) with symmetrics	[Cvetic, Klevers, Piragua, Taylor '15] [Anderson, Gray, NR, Taylor '15]
SU(2) with 3-sym.	[Klevers, Taylor '16]

But they

- Relied on previous constructions w/ different gauge groups
  - **How would we systematically construct models from scratch?**
- Realize a limited set of matter spectra
  - **Can we find more general models?**
- Have complicated "non-Tate" structure in Weierstrass models
  - **Can we explain this structure?**

AN EXAMPLE OF NON-TATE STRUCTURE

$$y^2 = x^3 + fx + g \qquad \Delta = 4f^3 + 27g^2 \qquad \text{SU(N)}: \Delta \propto \sigma^N$$

Expand f and g as

$$f = f_0 + f_1\sigma + f_2\sigma^2 + \ldots \qquad g = g_0 + g_1\sigma + g_2\sigma^2 + \ldots$$

For zeroth order cancellation: $4{f_0}^3 + 27g_0^2 \equiv 0 \bmod \sigma$

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OPTION 1 *Exact Cancellation (Tate's algorithm)*

$$f_0 = -3\phi^2 \qquad g_0 = 2\phi^3.$$

$$4{f_0}^3 + 27{g_0}^2 = 0$$

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OPTION 1 *Exact Cancellation (Tate's algorithm)*

$$f_0 = -3\phi^2 \qquad g_0 = 2\phi^3.$$

$$4{f_0}^3 + 27{g_0}^2 = 0$$

OPTION 2 *Suppose $\sigma = \xi^3 - b\eta^3$ w/ triple point at $\xi = \eta = 0$*

$$f_0 = -3b\xi\eta \qquad g_0 = 2b^2\eta^3$$

$$4f_0^3 + 27{g_0}^2 = -108b^3\eta^3\left(\xi^3 - b\eta^3\right)$$

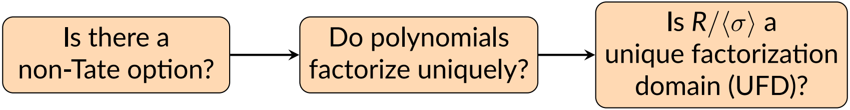
Models with exotics have structures similar to Option 2

NON-UFD STRUCTURE

Why is non-Tate structure allowed?

- Consider quotient ring $R/\langle\sigma\rangle$ ($x = x + a\sigma$)
- Cancellation condition becomes

$$4{f_0}^3 = -27g_0^2$$



- **When σ is singular, quotient ring is not a UFD.**
  - One can consider normalization of $\sigma = 0$
  - Add elements from field of fractions to $R/\langle\sigma\rangle$
  - Resulting ring is called the **normalized intrinsic ring (NIR)**
  - Find appropriate tunings by treating NIR as a UFD

NON-UFD TUNINGS

For $\sigma = \xi^3 - b\eta^3 = 0$.

1. Introduce new parameter $\tilde{B}$, with

$$\tilde{B}^3 = b \qquad \xi = \tilde{B}\eta$$

   Adding $\tilde{B}$ gives us the normalized intrinsic ring

2. Start with the UFD tunings

$$f_0 \sim -3\phi^2 \qquad g_0 \sim 2\phi^3$$

3. Let ϕ depend on $\tilde{B}$, but f_0, g_0 cannot directly depend on $\tilde{B}$

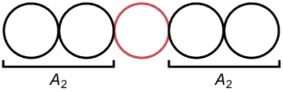
$$\phi = \tilde{B}^2\eta$$

$$f_0 \sim -3\tilde{B}^4\eta^2 \rightarrow -3b\xi\eta \qquad g_0 \sim 2\tilde{B}^6\eta^3 \rightarrow 2b^2\xi^3$$

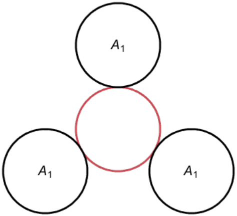
These are the non-Tate tunings from before.

DERIVED MODELS

SU(N) W/ SYMMETRICS
(DOUBLE PTS)



SU(2) W/ 3-SYM.
(TRIPLE PTS)



- Generalizes previous constructions
- Adjoint models & exotic models connected by matter transitions
  - At transition point: f, g vanish to orders (4,6) on codim-two locus
  - See [Anderson, Gray, NR, Taylor '15] or [Klevers, Morrison, NR, Taylor, '17] for more description

NON-REALIZABLE REPS

Reps must involve embedding in standard Dynkin diagram
Extended Dynkin not allowed

REASON

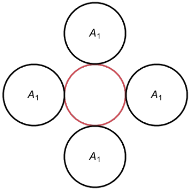
- Resolution introduces exceptional curves
- (Negative of) Cartan matrix gives intersection numbers
- Must contract all curves in diagram
- For extended diagram, intersection matrix not negative definite

IN PRACTICE

- Attempts lead to codim-2 (4,6) singularities

Hypothetical 4-sym. of SU(2)

$A_1^4 \to \hat{D}_4$



$$\begin{pmatrix} -2 & 0 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 & 0 \\ 1 & 1 & -2 & 1 & 1 \\ 0 & 0 & 1 & -2 & 0 \\ 0 & 0 & 1 & 0 & -2 \end{pmatrix}$$

Negative of $\hat{D}_4$ Cartan Matrix

NON-REALIZABLE REPS II

Examples of non-realizable reps include

- 3-sym. of SU(3) (**35**)
- 4-antisym. of SU(8) (**70**)
- 4-sym. of SU(2) (**5**)

even though they appear in seemingly consistent 6D SUGRAs

Further analysis suggests *Sp*, *SO*, exceptional gauge groups cannot support exotics in F-theory.

- Suggests F-theory can only realize “standard” reps plus a few exotics

NON-REALIZABLE SPECTRA

Some matter spectra seem non-realizable in F-theory

EXAMPLE *Quintic Curve on* $\mathbb{P}^2$

6D SUGRA suggests there should be a model with

- A $\mathbb{P}^2$ base
- An SU(2) tuned on a quintic curve
- Two triple points supporting 3-sym. (**4**) matter

SUGRA anomalies care only about whether genus is high enough

- Quintic has genus 6
- Each triple point eats up genus 3
- Should be enough genus

But you cannot have a quintic curve on $\mathbb{P}^2$ with two triple points

- Suggests this model cannot be realized in F-theory

NON-ABELIAN SUMMARY

- Exotic reps associated with singular divisors can be understood
  - Models can be systematically derived using normalized intrinsic ring
  - Non-UFD nature of models with singular divisors explains intricate Weierstrass structure
- Some models seem non-realizable in F-theory
  - Certain reps seem non-realizable
  - Certain combinations of reps non-realizable

PART II ABELIAN MODELS

ABELIAN WEIERSTRASS MODELS

$$\textit{Global Weierstrass Form:} \qquad y^2 = x^3 + f\,x\,z^4 + g\,z^2$$
$$[x : y : z] \equiv [\lambda^2 x : \lambda^3 y : \lambda z]$$

Interested in models w/ a U(1) gauge group, no non-abelian factors

- Generating section $\hat{s}$
- Section described by components $[\hat{x} : \hat{y} : \hat{z}]$
- $(\hat{x}, \hat{y}, \hat{z})$ depend on position in base

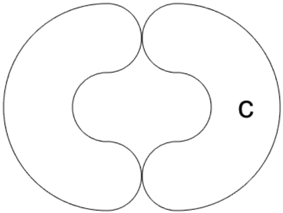
I_2 SINGULARITIES

Global Weierstrass Form: $y^2 = x^3 + f\,x\,z^4 + g\,z^2$

Codim-two I_2 singularities occur at

$$\hat{y} = 3\hat{x}^2 + f\hat{z}^4 = 0$$

- After resolution, fiber splits into two components
- "Extra" component denoted c
- All charged matter, regardless of charge, occurs at I_2 singularities



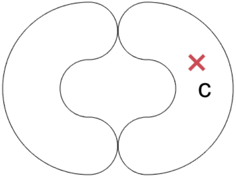
CHARGED MATTER

Shioda Map σ: Homomorphism from MW group to Neron-Severi

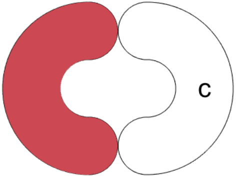
I_2 singularities occur at $\hat{y} = 3\hat{x}^2 + f\hat{z}^4 = 0$	Charge of matter $q = \sigma(\hat{s}) \cdot c$

Two ways matter can appear

1. Standard Intersection
   - Typically gives $q = 1$ matter



2. $\hat{x}$, $\hat{y}$, and $\hat{z}$ simultaneously vanish
   - Naively seems ill-defined
   - Must resolve section
   - Section wraps a component
   - Can give $q > 1$



MORRISON-PARK FORM

Well-understood model with Charge 1 & 2 matter [Morrison, Park '12]

$$f = c_1 c_3 - \frac{1}{3} c_2^2 - c_0 b^2 \quad g = c_0 c_3^2 - \frac{1}{3} c_1 c_2 c_3 + \frac{2}{27} c_2^3 - \frac{2}{3} c_0 c_2 b^2 + \frac{1}{4} c_1^2 b^2$$

$$\hat{z} = b \qquad \hat{x} = c_3^2 - \frac{2}{3} c_2 b^2 \qquad \hat{y} = -c_3^3 + c_2 c_3 b^2 - \frac{c_1}{2} b^4$$

Charge-2 matter occurs at $b = c_3 = 0$

- $(\hat{z}, \hat{x}, \hat{y})$ vanish to orders (1,2,3) on this locus

Are there constructions admitting charges greater than 2?

PRIOR MODELS WITH LARGE CHARGES

Not many models with charge greater than 2

- There is a class of charge-3 models
  - [Klevers, Mayorga-Pena, Oehlmann, Piragua, Reuter '14]
  - Found within set of constructions (toric hypersurface fibrations)
  - Weierstrass model has intricate structure, not in MP form
- Charge-4+ even more challenging
  - To my knowledge, no previously published models

QUESTIONS

- How would we construct charge-3 models from scratch?
- Can we explain intricate structure in charge-3 Weierstrass model?
- Can we get charge-4 or greater?

BASIC IDEA

Orders of vanishing of the $(\hat{z}, \hat{x}, \hat{y})$ section components tell us about the charge

Charge-2 Loci $(\hat{z}, \hat{x}, \hat{y})$ vanish to orders $(1, 2, 3)$ *(Morrison-Park form)*

Charge-3+ Loci $(\hat{z}, \hat{x}, \hat{y})$ vanish to higher orders

Evidence comes from

- Explicit models supporting charge-3 and charge-4 matter
- Non-generator sections in $q = 1$ model
- 6D anomaly relations (won't discuss here)

DERIVING U(1) MODELS

For a single U(1), need an additional rational section $[\hat{x} : \hat{y} : \hat{z}]$

$$\text{Global Weierstrass Form: } \hat{y}^2 - \hat{x}^3 = \hat{z}^4 \left(f\,\hat{x} + g\hat{z}^2 \right)$$

LHS has similar algebraic form to discriminant.

STRATEGY FOR CONSTRUCTION

1. Start with ansatz for $\hat{z}$. Assume $\hat{z}$, $\hat{x}$ and $\hat{y}$ are holomorphic.
2. Expand $\hat{x}$, $\hat{y}$ as series in $\hat{z}$.
3. Tune $\hat{x}$ and $\hat{y}$ so that $\hat{y}^2 - \hat{x}^3 \propto \hat{z}^4$
    - Similar to tuning an I_4 singularity
4. If necessary, further tune $\hat{x}$ and $\hat{y}$ so that $\hat{y}^2 - \hat{x}^3$ takes form above
5. Read off f and g

OBTAINING MORRISON-PARK FORM

Natural First Attempt: Assume $R/\langle\hat{z}\rangle$ is a UFD

1. Write $\hat{x}$ and $\hat{y}$ as
$$\hat{x} = x_0 + x_1\hat{z} + x_2\hat{z}^2 + \ldots \qquad \hat{y} = y_0 + y_1\hat{z} + y_2\hat{z}^2 \ldots$$
2. To have $\hat{y}^2 - \hat{x}^3 \propto \hat{z}^4$, use UFD I_4 tuning with altered coefficients:
$$\hat{x} = \phi^2 + x_2\,\hat{z}^2 \qquad \hat{y} = \phi^3 + \frac{3}{2}\phi\, x_2\,\hat{z}^2 + y_4\,\hat{z}^4$$
3. Without any further tuning,
$$\hat{y}^2 - \hat{x}^3 = \hat{z}^4\left[\underbrace{\left(2\phi y_4 - \frac{3}{4}x_2^2 + f_2\hat{z}^2\right)}_{f}\hat{x} + \underbrace{\left(x_2 y_4\phi - \frac{x_2^3}{4} + y_4\hat{z}^2 - f_2\hat{x}\right)\hat{z}^2}_{g}\right]$$
4. With the redefinitions
$$\hat{z} \to b \qquad x_2 \to -\frac{2}{3}c_2 \qquad \phi \to c_3 \qquad y_4 \to \frac{1}{2}c_1 \qquad f_2 \to -c_0$$
we recover Morrison-Park form!

OBTAINING CHARGE-3 MODEL

Using UFD tunings leads to Morrison-Park form

- $\hat{z} = b$ vanishes to order 1 at charge-2 loci $b = c_3 = 0$

Suppose $\hat{z}$ has singular structure

- $\hat{z}$ vanishes to orders higher than 1
- $R/\langle\hat{z}\rangle$ may not be a UFD
- Now can have non-UFD structure in the tunings
  - Introduces deviations from Morrison-Park form
- Use normalized intrinsic ring techniques to tune U(1)

DERIVING CHARGE 3 MODELS

1) Start with ansatz $\hat{z} = b_2\eta_a^2 + 2b_1\eta_a\eta_b + b_0\eta_b^2$

- Double point singularities at $\eta_a = \eta_b = 0$
- Identical $\hat{z}$ to that in the previous $q = 3$ models

2) Tuning steps lead to generalization of previous $q = 3$ construction

- Can derive $q = 3$ models essentially from scratch
- Entire structure motivated by singular nature of $\hat{z}$
- Can obtain new models with previously unrealized matter spectra

CHARGE 4 MODELS

NIR process is algebraically difficult, use alternative strategy

1. Start with U(1) $\times$ U(1) model admitting $(2,2)$ matter
   - [Cvetic, Klevers, Piragua, Taylor '15]
   - Two generating sections Q and R
   - A codim-2 I_2 locus for which $\sigma(Q) \cdot c = 2$,$\sigma(R) \cdot c = 2$
2. Deform model in a way that preserves $Q[+]R$ but not Q, R individually
   - $[+]$: elliptic curve addition law
   - Now only a single generator
3. Now have a single U(1) with charge-4 matter
   - Previous $(2,2)$ locus now supports charge-4, as

$$\sigma\left(Q[+]R\right) \cdot c = \sigma\left(Q\right) + \sigma\left(R\right) = 2 + 2 = 4$$

Charge-4 model has higher orders of vanishing and NIR structure

LEARNING ABOUT LARGER CHARGES

Based on [Morrison, Park '12]

Can we conjecture about charge-5+ matter without explicit models?

Consider a U(1) model and only charge-1 matter:

- Has a generating section $\hat{s}$.
- There are codim-two I_2 loci at which $\sigma(\hat{s}) \cdot c = 1$
- There are also sections $m\hat{s}$ for all integers m
  - Generated using elliptic curve addition
- At codimension-two loci, $\sigma(m\hat{s}) \cdot c = m$
  - Looks like charge m
  - Local behavior of $m\hat{s}$ likely mimics that of generator for an actual charge-m model

Punchline: Use $m\hat{s}$ sections to conjecture about higher charge models

ORDERS OF VANISHING I

EXAMPLE What is order of vanishing of $m\hat{s}$ section components at the codim-two loci?

- Should be related to orders of vanishing for charge-m models.
- Calculate sections one by one and read off orders of vanishing:

	$\hat{z}$	$\hat{x}$	$\hat{y}$
$m = 1$	0	0	1
$m = 2$	1	2	3
$m = 3$	2	4	7
$m = 4$	4	8	12
$m = 5$	6	12	19
$m = 6$	9	18	24
$\vdots$			

These match known behavior at charge-1 through charge-4 loci

Maybe these match as well?

ORDERS OF VANISHING II

	$\hat{z}$	$\hat{x}$	$\hat{y}$
$m = 1$	0	0	1
$m = 2$	1	2	3
$m = 3$	2	4	7
$m = 4$	4	8	12
$m = 5$	6	12	19
$m = 6$	9	18	24
	$\vdots$		

The orders seem to follow a pattern
For even m, the orders of vanishing are

$$\left(\frac{m^2}{4}, \frac{2m^2}{4}, \frac{3m^2}{4}\right)$$

For odd m, the orders of vanishing are

$$\left(\frac{m^2-1}{4}, \frac{2(m^2-1)}{4}, \frac{3(m^2-1)}{4} + 1\right)$$

- I've verified these patterns up to $m = 26$
- Would be interesting to verify/prove patterns for arbitrary m.

GENERAL CHARGE LOCI

CONJECTURE

At charge-q loci, the $(\hat{z}, \hat{x}, \hat{y})$ of the generator $\hat{s}$ vanish to orders

$$\text{For even } q\text{:} \quad \left(\frac{q^2}{4}, \frac{2q^2}{4}, \frac{3q^2}{4}\right)$$

$$\text{For odd } q\text{:} \quad \left(\frac{q^2-1}{4}, \frac{2(q^2-1)}{4}, \frac{3(q^2-1)}{4}+1\right)$$

- If true, could provide heuristic way of reading off charges from Weierstrass model

ABELIAN CONCLUSIONS

- Orders of vanishing of $(\hat{x}, \hat{y}, \hat{z})$ seem related to charges supported
- Can derive charge-3 models from scratch using normalized intrinsic ring
- Charge-4 models found, also display normalized intrinsic ring structure
- Conjectures on larger charge models

Thank you!

PART III BACK UP SLIDES

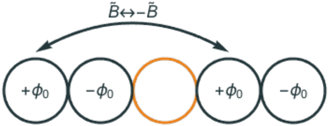
SYMMETRICS AND THE SPLIT CONDITION

To tune SU(N) on $\sigma = \xi^2 - b\eta^2$:

1. Introduce parameter $\tilde{B}$: $\tilde{B}^2 = b$, $\tilde{B}\eta = \xi$
2. Tunings: $f = -3\phi^2 + \dots$ $g = 2\phi^2 + \dots$
3. Must implement Split Condition: $\phi = \phi_0^2$
4. Near double point, curve looks like $(\xi + \tilde{B}\eta)(\xi - \tilde{B}\eta)$
   - The two "components" should be identified with each other

Adjoint

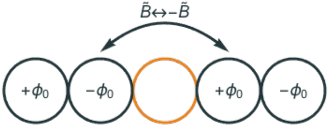
Generic ϕ_0



Dynkin index $[1, 1]$

Symmetric

$\phi_0 = \tilde{B}$



Dynkin index $[2, 0]$

INTERESTING DIRECTION

Direction for further understanding: 3-antisym of SU(9) (84)

- Argument suggests 3-antisym. of SU(9) (**84**) cannot be realized in F-theory
- But there are heterotic orbifolds with the **84** rep
  - Example: In 6D, heterotic on $T^4/\mathbb{Z}_3$ with SU(9)$\times E_8$ gauge group
  - When orbifold smoothed to K3, SU(9) Higgsed down to SU(8)
  - 3-antisym. of SU(8) is allowed in F-theory

CHARGE-4 DEFORMATION

Initial U(1)×U(1) Model: Describe via embedding in $\mathbb{P}^2$

$$u\left(s_1u^2+s_2uv+s_3v^2+s_5uw+s_6vw+s_8w^2\right) + (a_1v+b_1w)(a_2v+b_2w)(a_3v+b_3w)=0$$

Three Sections: $P=[0:-b_1:a_1]$ $Q=[0:-b_2:a_2]$ $R=[0:-b_3:a_3]$

- P taken as zero section
- Q, R interchanged under $a_2 \leftrightarrow a_3$, $b_2 \leftrightarrow b_3$

DEFORMATION Remove all instances of a_2, a_3, b_2, b_3 using

$$a_2a_3 \to d_0 \qquad a_2b_3+a_3b_2 \to d_1 \qquad b_2b_3 \to d_2$$

- Deformation involve expressions invariant under a_2, a_3, b_2, b_3
- Preserve $Q[+]R$, not Q or R

ANOMALIES AND ORDER OF VANISHING

6D anomalies hint at order of vanishing behavior:

1. Start with anomaly equations

$$-K_B \cdot h(\hat{s}) = \frac{1}{6} \sum_{\text{hypers}} q^2 \qquad h(\hat{s}) : \text{ Height of the section}$$

$$-h(\hat{s}) \cdot h(\hat{s}) = \frac{1}{3} \sum_{\text{hypers}} q^4 \qquad K_B : \text{ Canonical class of the base}$$

2. Sum to get new relation

$$(-2K_B + h(\hat{s})) \cdot h(\hat{s}) = \frac{1}{3} \sum_{\text{hypers}} q^2(q^2 - 1)$$

   which can often be rewritten as

$$(-K_B + [\hat{z}]) \cdot [\hat{z}] = \frac{1}{12} \sum_{\text{hypers}} q^2(q^2 - 1)$$

3. $\frac{1}{12} q^2(q^2 - 1)$ is always an integer, non-zero only for $q \geq 2$

ANOMALIES AND ORDER OF VANISHING II

$$\boxed{(-K_B + [\hat{z}]) \cdot [\hat{z}] = \frac{1}{12} \sum_{\text{hypers}} q^2(q^2 - 1)}$$

In all the examples considered

$$\hat{x} = t^2 + \mathcal{O}(\hat{z}) \qquad \hat{y} = t^2 + \mathcal{O}(\hat{z}) \qquad [t] = -K_B + [\hat{z}]$$

- Section components vanish wherever $t = \hat{z} = 0$
- Anomaly eqn. tells us about section components vanishing
- For Morrison-Park (only charges 1 and 2)

  $$\hat{z} = b \quad \hat{x} = c_3^2 + \mathcal{O}(b) \quad \hat{y} = c_3^3 + \mathcal{O}(b) \quad [c_3] = -K_B + [b]$$

  The anomaly equation suggests that, as expected

  $$[c_3] \cdot [b] = \text{No. of } q = 2 \text{ hypers}$$

- For $q = 3, 4$ models: $\frac{1}{12}q^2(q^2 - 1)$ numbers automatically appear in $\text{Res}(t, \hat{z})$!