Chance Constraints for Improving Security of AC Optimal Power Flow

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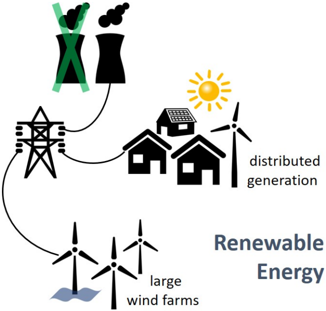
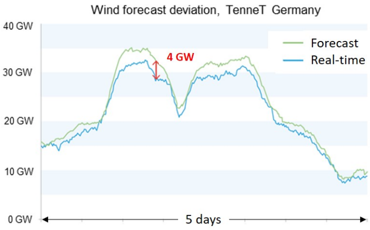
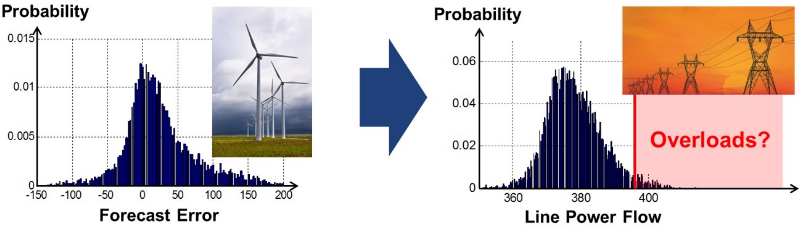
Uncertainty in Power Systems Operation





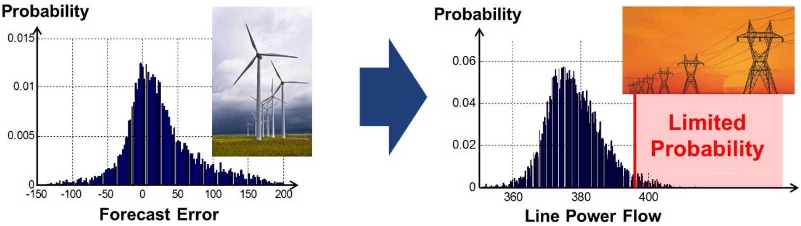
Renewable energy increases *variability* and *uncertainty* in power systems operation.

Uncertainty in Power Systems Operation



How to limit **adverse impacts** of uncertainty?

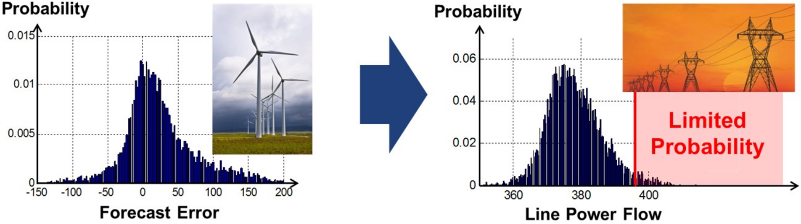
Uncertainty in Power Systems Operation



Chance-Constrained Optimal Power Flow

Limit **probability** of constraint violations

Uncertainty in Power Systems Operation



Chance-Constrained **AC** Optimal Power Flow

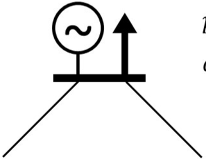
Non-linear dependence on uncertain variables!

Outline

1. AC Power Flow and System Response to Uncertainty
2. Chance-Constrained AC Optimal Power Flow
3. Handling Quadratic Chance Constraints
4. Numerical Results
5. Summary and Conclusion

AC Power Flow

Two **equations** per bus:
Nodal power balance equations
for *active* and *reactive* power



$$p_G - p_L = \sum p_{ij}$$
$$q_G - q_L = \sum q_{ij}$$

AC Power Flow

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Nodal power balance equations
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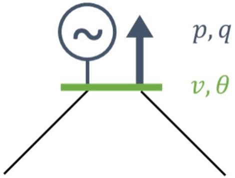

Four **variables** per bus:
p - active power
q - reactive power
v - voltage magnitude
θ - Voltage angle

AC Power Flow

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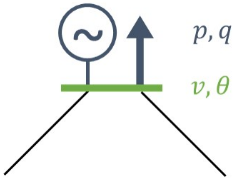

Four **variables** per bus:
p - active power
q - reactive power
v - voltage magnitude
θ - Voltage angle

Choose two variables per bus:
pq bus - load buses
pv bus - local voltage control
θv bus - reference bus/slack bus

AC OPTIMAL Power Flow

$$\min \sum_{i \in \mathcal{G}} c_{G,i}(p_{G,i})$$ Minimize generation cost

$$\text{s.t.} \quad F(f^p, f^q, \theta, v, p, q) = 0$$ Nodal power balance

$$p_G^{min} \leq p_G \leq p_G^{max}$$ Active generation

$$q_G^{min} \leq q_G \leq q_G^{max}$$ Reactive generation

$$v^{min} \leq v \leq v^{max}$$ Voltage magnitudes

$$\sqrt{(f^p)^2 + (f^q)^2} \leq s^{max}$$ Power flows

$$\theta_{\theta V} = 0$$

Not really necessary to differ between pq, pv and θv buses.

But - influences system response under uncertainty!

Modeling Uncertainty

- Active power $= p + \omega$

  p - forecasted active power
  ω - random fluctuation

- Reactive power $= q + \gamma\omega$

  q - forecasted reactive power
  $\gamma\omega$ - **constant power factor**

Power System Balancing

Consumed and produced power must be balanced at all times!

- **Balanced** for $\omega = 0$
- **Active power** - Automatic Generation Control (AGC)
  [Borkowska 1974], [Vrakopoulou 2013]
  $p_G^{new} = p_G - \alpha \left(\sum \omega\right)$
- **Reactive power** - Local voltage control at PV buses
  $v_i = const$, adjust reactive power q_G^{new} to achieve this!

Chance Constraints

Generator active power

$$\mathbb{P}(p_{G,i} \underbrace{-\alpha_{\mathbf{i}} \left(\sum \omega\right)}_{\text{RESERVE}} \leq p_{G,i}^{\max}) \geq 1-\epsilon$$

Insufficient reserves

Generator reactive power

$$\mathbb{P}\left(q_{G,i}(\omega) \leq q_{G,i}^{\max}\right) \geq 1-\epsilon$$

Voltages change

Voltage magnitudes

$$\mathbb{P}\left(v_i(\omega) \leq v_i^{\max}\right) \geq 1-\epsilon$$

Voltages out of bound

Power flows

$$\mathbb{P}\left(\sqrt{(f_{ij}^{p}(\omega))^2 + (f_{ij}^{q}(\omega))^2} \leq s^{max}\right) \geq 1-\epsilon$$

Temp. overload/redispatch

Chance Constraints Challenge

Non-linear uncertainty quantification + optimization friendliness

Some approaches:

- Linear DC power flow approximation
  [Vrakopoulou et al 2012], [Roald et al 2013], [Bienstock, Chertkov and Harnett 2014], [Lubin, Dvorkin, Backhaus 2016]
- SDP relaxation, sample-based reformulation
  [Vrakopoulou et al 2013]
- Full AC equations for $\omega = 0$, linearized uncertainty
  [Qu, Roald, Andersson 2015], [Schmidli et al 2016]
- Linearized AC power flow with voltage constraints
  [Baker, Summers, Dall'Anese 2016]

Goal: Include **power flow constraints** and **optimized response**!

Gaussian Chance Constraints

Power injections are **not** Gaussian.

Power flows are **typically close to** Gaussian.

$$\mathbb{P}\left(\sqrt{(f_{ij}^{p}(\omega))^2 + (f_{ij}^{q}(\omega))^2} \leq s^{max}\right) \geq 1 - \epsilon$$

Depends on high-dimensional random vector

$\rightarrow$ "Central limit theorem"!

A solution approach

We don't know how to impose chance constraints directly on ACOPF. Known tractable methods require a linear relationship between injections, voltages, and flows.

So we propose to...

1. Solve deterministic AC OPF
2. Linearize around solution point.
3. Solve convex optimization problem with chance constraints.

Linearization

Let $x := (f^p, f^q, \theta, v, p, q)$, so that the AC equations can be expressed as

$$F(x) = 0.$$

Given a feasible operating point $\tilde{x} := (\tilde{f}^p, \tilde{f}^q, \tilde{\theta}, \tilde{v}, \tilde{p}, \tilde{q})$, we instead enforce the linear equations

$$\nabla F(\tilde{x})^T (x - \tilde{x}) + F(\tilde{x}) = 0.$$

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Can be inverted to obtain flows (f^p, f^q) as affine function of voltages and injections (θ, v, p, q).

Recall line flow constraints

$$\sqrt{(f^p)^2 + (f^q)^2} \leq s^{max}$$

Using (approximate) linear relationship, to impose line flow constraints we end up with chance constraints of the form

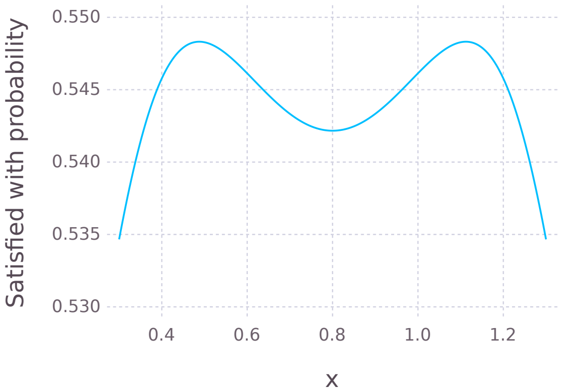
$$\mathbb{P}_\xi \left((a^T\xi + b)^2 + (c^T\xi + d)^2 \leq k \right) \geq 1 - \epsilon$$

where a, b, c, d are decision variables

Is this a convex constraint?

Not convex for $\epsilon = 0.445$

$$P((x\xi_1)^2 + (y\xi_2)^2 \leq 1) \geq 1 - \epsilon$$



Not convex for $\epsilon = 0.445$

$$P((x\xi_1)^2 + (y\xi_2)^2 \leq 1) \geq 1 - \epsilon$$

Counterexample does not apply for smaller ϵ, but anyway let's look for approximations

Start off by trying to understand the simpler (previously unstudied) constraint

$$\mathbb{P}(a \leq x^T \xi \leq b) \geq 1 - \epsilon$$

where $a \in \mathbb{R}, b \in \mathbb{R}$, and $x \in \mathbb{R}^n$ are decision variables, and ξ is jointly Gaussian with known mean and covariance

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Result: The "two-sided" chance constraint above defines a convex set in (a, b, x) when $\epsilon \leq \frac{1}{2}$ (L., Bienstock. Vielma, 2016)

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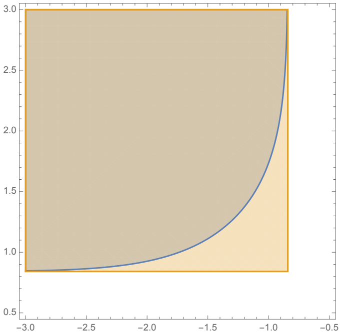
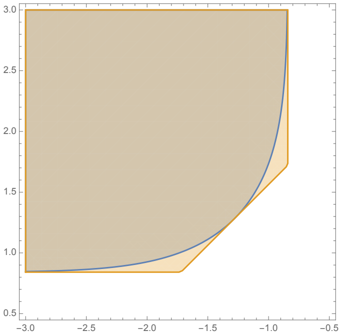
But representation requires perspective transformation, so nonsmooth. Not representable using standard cones.

2ϵ outer approximation



- With two linear constraints plus second-order cone, we guarantee that chance constraint holds with 2ϵ. (Can be made conservative)
- **Proof**: split into two linear chance constraints $\mathbb{P}(a \leq x^T\xi) \geq 1 - \epsilon$, $\mathbb{P}(x^T\xi \leq b) \geq 1 - \epsilon$

1.25ϵ **outer approximation**



- With **three** linear constraints plus second-order cone, we guarantee that chance constraint holds with 1.25ϵ
- **Proof**: L., Bienstock, Vielma (2016)

Approximating quadratic chance constraints

Fix $\epsilon < \frac{1}{2}$ and $\beta \in (0, 1)$. If $\exists$ f_1, f_2 such that

$$\mathbb{P}(|a^T\xi + b| \leq f_1) \geq 1 - \beta\epsilon$$
$$\mathbb{P}(|c^T\xi + d| \leq f_2) \geq 1 - (1 - \beta)\epsilon$$
$$f_1^2 + f_2^2 \leq k$$

then

$$\mathbb{P}\left((a^T\xi + b)^2 + (c^T\xi + d)^2 \leq k\right) \geq 1 - \epsilon$$

Proof: Union bound

So this gives us a convex, tractable (via SOCP) approximation for

$$\mathbb{P}\left((a^T\xi + b)^2 + (c^T\xi + d)^2 \leq k\right) \geq 1 - \epsilon$$

What about other approaches?

So this gives us a convex, tractable (via SOCP) approximation for

$$\mathbb{P}\left((a^T\xi + b)^2 + (c^T\xi + d)^2 \leq k\right) \geq 1 - \epsilon$$

What about other approaches?

- Robust optimization ($\rightarrow$ SDP)
- CVaR (sampling)

Experimental setup

1. Solve deterministic ACOPF (using local nonlinear solver)
2. Linearize around that solution, add chance constraints
3. Solve convex problem (SOCP) to obtain new production levels p, q and response parameters α, etc
4. Given realization of uncertainty (in sample), solve feasibility problem with injections fixed to check if computed solution induces feasible flows and voltages

Table 1: Comparison of Feasibility (%) Against In-Sample Uncertainty Realizations (1000 samples)

ϵ	10^{-1}	10^{-2}	10^{-3}
ACOPF ($\alpha_i = 1/N_g$)	0.073	0.076	0.076
ACOPF (cheating)	53.0	53.4	53.4
CCACOPF ($\alpha_i = 1/N_g$)	84.1	97.4	99.8
CCACOPF (opt)	85.4	98.6	99.8

Test system: IEEE RTS96 three area

JuMPChance

There's modeling software too!

`github.com/kersulis/IJulia-WPS`

`github.com/mlubin/JuMPChance.jl`

`jumpchance.readthedocs.io/en/latest/twoside.html`

Conclusions

- Preliminary results show that deterministic ACOPF + linearization + chance constraints can give more "robust" solutions than determistic ACOPF.
- Very little additional computational overhead compared with solving determistic ACOPF alone
- Can be used to study reactive power generation response policies.
- Lots of room to experiment and expand on the methodology.

Thanks!