HAMILTON CYCLES IN REGULAR MAPS

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The problem

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Does the underlying graph of every (orientably) regular map have a Hamilton cycle (simple cycle through every vertex)?

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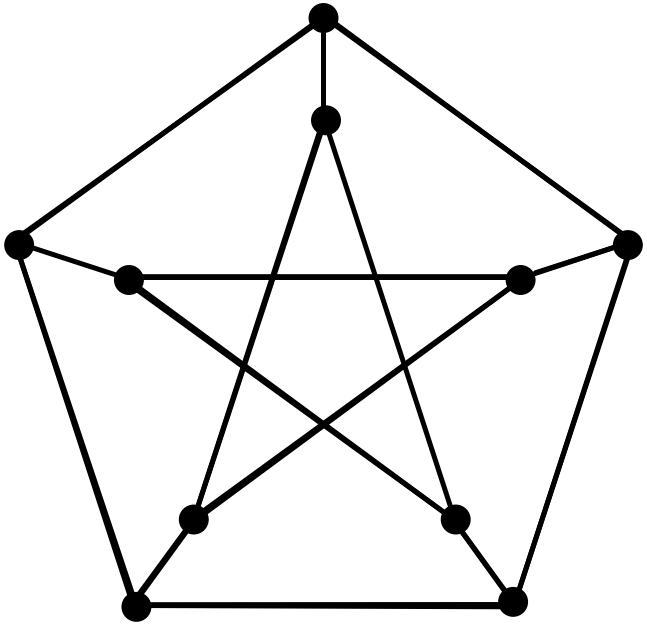
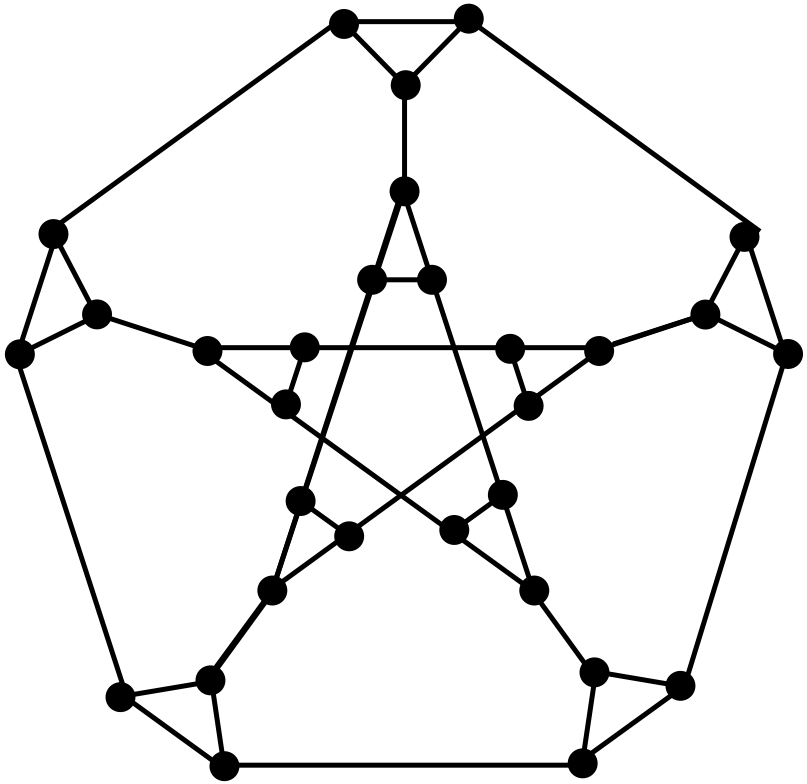
- The underlying graph of every regular map is arc-transitive and hence vertex-transitive.
- Only four vertex-transitive non-hamiltonian graphs are known (apart from K_2), two of them arc-transitive.

Non-hamiltonian vertex-transitive graphs





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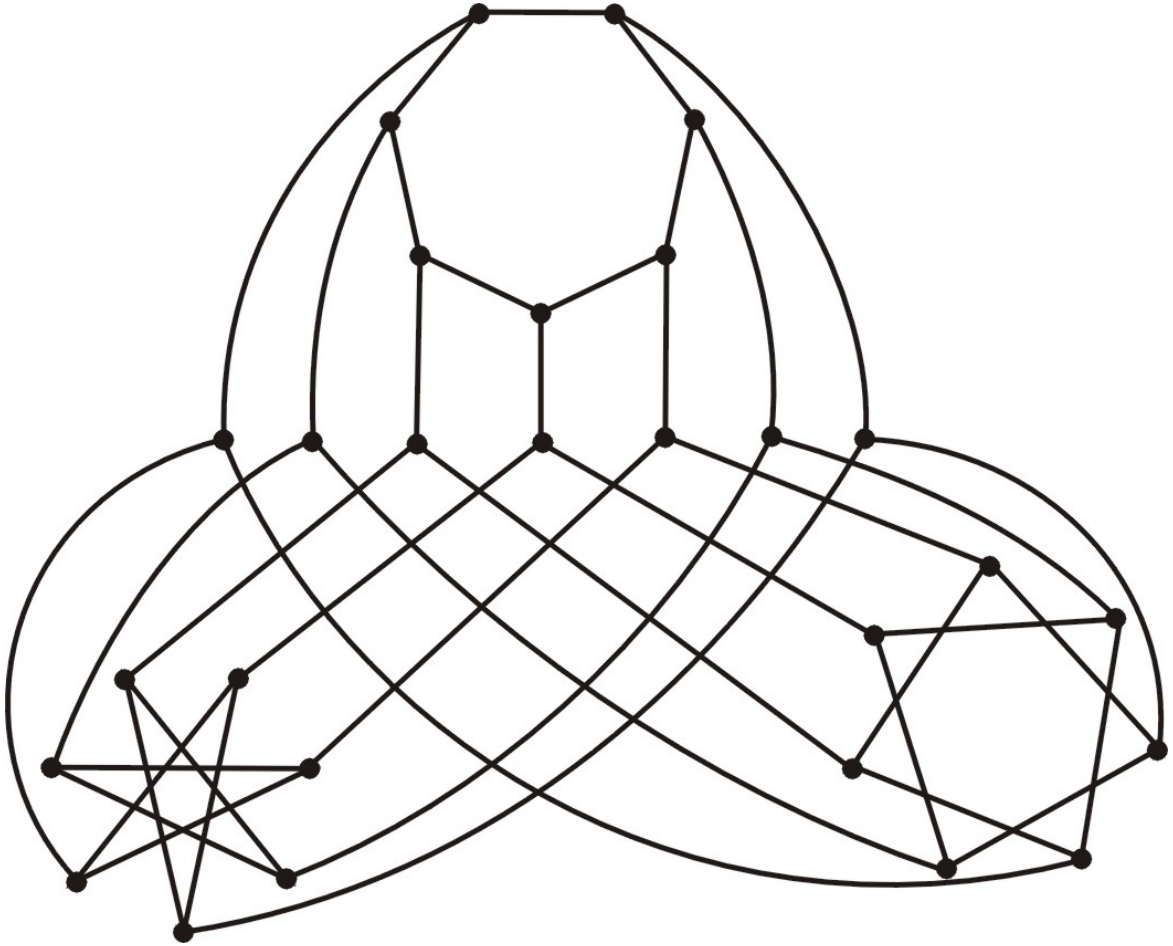

Hamilton cycles in regular maps

- The Petersen graph has a regular embedding on the projective plane (the half-dodecahadron).
- The Coxeter graph does not have a regular embedding on any closed compact surface, but if we double each edge, it does embed as a Moebius regular map.

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Thank you!