Phase transitions in noncentrosymmetric superconductors: Lifshitz invariants and nonuniform states

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Outline

- New features in the Ginzburg-Landau theory
- Noncentrosymmetric superconductors in a nutshell:
  - Examples in 3D, 2D, and 1D
  - Electron-lattice spin-orbit coupling and electron band splitting
  - Rashba model
- Cooper pairing in nondegenerate bands:
  - Intraband vs interband pairing
  - Two-band model
- Unusual nonuniform superconducting states:
  - Helical states
  - Interband phase solitons
  - Zero-field instabilities

Ginzburg-Landau free energy

Bardeen-Cooper-Schrieffer theory: superconductivity is due to the coherent motion of the pairs of electrons with $\boldsymbol{k}$ and $-\boldsymbol{k}$ near the Fermi surface (Cooper pairs)

Order parameter in superconductors = wave function of the pairs

Single component: $\eta(\boldsymbol{r})$ (e.g., classic BCS or high-T_c SCs)

Many components: $\eta_1(\boldsymbol{r}), ..., \eta_N(\boldsymbol{r})$ (e.g., $N = 2$ in Sr_2RuO_4, $N = 9$ in superfluid ^{3}He)

Simplest model of noncentrosymmetric SCs:
$N = 2$, order parameters $\eta_+(\boldsymbol{r}), \eta_-(\boldsymbol{r})$

Ginzburg-Landau free energy

Free energy density: $F = F_{uniform} + F_{gradient} + F_{magnetic}$

Single component $\to$ standard GL:

$$F = a(T - T_c)|\eta|^2 + \frac{\beta}{2}|\eta|^4 + K|\boldsymbol{D}\eta|^2 + \frac{(\boldsymbol{B} - \boldsymbol{H})^2}{8\pi}, \; \boldsymbol{D} = -i\boldsymbol{\nabla} + \frac{2e}{\hbar c}\boldsymbol{A}$$

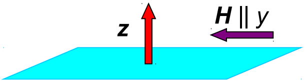
Noncentrosymmetric superconductors:

$$\begin{aligned} F_{uniform} &= A_1(T)|\eta_+|^2 + A_2(T)|\eta_-|^2 + A_3(\eta_+^*\eta_- + \text{c.c.}) \\ &+ B_1|\eta_+|^4 + B_2|\eta_-|^4 + B_3|\eta_+|^2|\eta_-|^2 \\ &+ B_4(\eta_+^{*,2}\eta_-^2 + \text{c.c.}) + (B_5|\eta_+|^2 + B_6|\eta_-|^2)(\eta_+^*\eta_- + \text{c.c.}) \end{aligned}$$

Sensible approximation: $B_3 = B_4 = B_5 = B_6 = 0$

Ginzburg-Landau free energy

2D SC
in a parallel field



no orbital effect:
$\boldsymbol{A}(z=0)=0$
$\boldsymbol{B}(\boldsymbol{r})=\boldsymbol{H}$

GL free energy density: $F = F_+ + F_- + \gamma_m(\eta_+^*\eta_- + \eta_-^*\eta_+)$

Intraband contributions:

$$F_\lambda = \alpha_\lambda|\eta_\lambda|^2 + \beta_\lambda|\eta_\lambda|^4 + K_\lambda|\boldsymbol{\nabla}\eta_\lambda|^2 + \underbrace{\tilde{K}_\lambda \,\mathrm{Im}\,[\eta_\lambda^*(\boldsymbol{H}\times\boldsymbol{\nabla})_z\eta_\lambda]}_{\text{Lifshitz invariant}} + \underbrace{L_\lambda H^2|\eta_\lambda|^2}_{\text{"diamagnetic" term}}$$

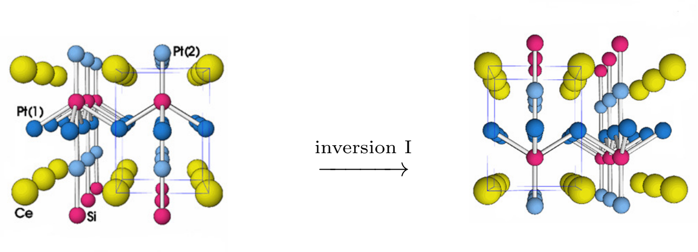
Superconducting current by the pairs:

$$\boldsymbol{j} = -4e\sum_\lambda K_\lambda\,\mathrm{Im}(\eta_\lambda^*\boldsymbol{\nabla}\eta_\lambda) + 2e\sum_\lambda \tilde{K}_\lambda(\boldsymbol{H}\times\hat{\boldsymbol{z}})|\eta_\lambda|^2$$

Lifshitz invariants (Mineev & KS ’94; Agterberg ’03; KS ’04) $\Rightarrow$
unusual nonuniform SC states, etc

3D noncentrosymmetric superconductors

O	Li_2Pt_3B (2K), Li_2Pd_3B (8K), Mo_3Al_2C (10K)
$\mathbf{T}_d$	Ti_5Re_{24} (6.6K), Y_2C_3 (17K), TLa_3S_4 (8K)
T	LaRhSi (4K), LaIrSi (2K)
$\mathbf{C}_{4v}$	$CePt_3Si$ (0.5K), $CeRhSi_3$ (1K), $CeIrSi_3$ (1.5K)
$\mathbf{C}_4$	$La_5B_2C_6$ (7K)
$\mathbf{C}_{6v}$	MoN (15K), GaN (6K)
$\mathbf{D}_{3h}$	MoC (9K), NbSe (6K), ZrPuP (13K)
$\mathbf{C}_{3v}$	MoS_2 (1K)
$\mathbf{C}_2$	UIr (0.1K)



(from E. Bauer *et al*, PRL **92**, 027003 (2004))

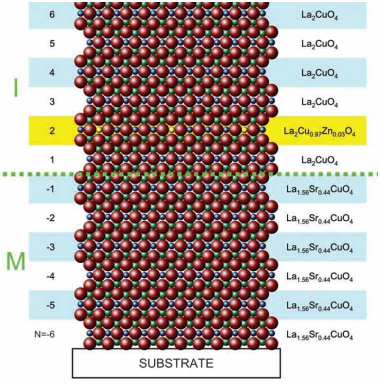
2D noncentrosymmetric superconductors

Insulator/insulator interface:
$LaAlO_3/SrTiO_3$ (LAO/STO)
$LaTiO_3/SrTiO_3$ (LTO/STO)

Metal/insulator interface:
LSCO/LCO

Doped insulator surface:
STO, WO_3

Typically: $T_c <$1K
FeSe single layers on doped STO substrate: T_c=109K

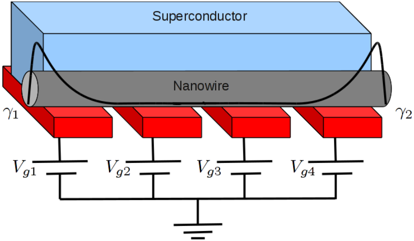


(from J. Pereiro *et al*, Phys. Express **1**, 208 (2011))

1D noncentrosymmetric superconductors

Proximity-induced superconductivity
in semiconducting wires

(experiment: InSb nanowire on NbTiN SC substrate, $H = 100$mT)



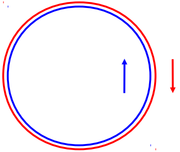
(from M. Leijnse and K. Flensberg, Semicond. Sci. Technol. **27**, 124003 (2012))

γ_1 and γ_2 – topologically-protected zero-energy bound states
a.k.a. Majorana quasiparticles

Electron bands

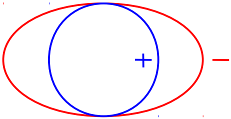
No inversion + spin-orbit coupling $\rightarrow$ nondegenerate Bloch bands

Time-reversal K ($K = i\hat{\sigma}_2 K_0$) and inversion I:
$|\boldsymbol{k}\rangle, KI|\boldsymbol{k}\rangle$ belong to $\boldsymbol{k}$
$K|\boldsymbol{k}\rangle, I|\boldsymbol{k}\rangle$ belong to $-\boldsymbol{k}$



bands twofold degenerate at all $\boldsymbol{k}$

Time-reversal K, no inversion:
$|\boldsymbol{k}\rangle$ belongs to $\boldsymbol{k}$
$K|\boldsymbol{k}\rangle$ belongs to $-\boldsymbol{k}$



bands nondegenerate at (almost) all $\boldsymbol{k}$

Spin-orbit coupling

Electron-lattice SO coupling: $H = \frac{\hat{\boldsymbol{p}}^2}{2m} + U(\boldsymbol{r}) + \frac{\hbar}{4m^2c^2}\hat{\boldsymbol{\sigma}}[\boldsymbol{\nabla} U(\boldsymbol{r}) \times \hat{\boldsymbol{p}}]$

Noninteracting electrons:

$$\hat{H}_0 = \sum_{\boldsymbol{k},\mu\nu} \sum_{\alpha,\beta=\uparrow,\downarrow} [\underbrace{\epsilon_\mu(\boldsymbol{k})\delta_{\mu\nu}\delta_{\alpha\beta}}_{I-\text{symmetric}} + \underbrace{iA_{\mu\nu}(\boldsymbol{k})\delta_{\alpha\beta} + \boldsymbol{B}_{\mu\nu}(\boldsymbol{k})\boldsymbol{\sigma}_{\alpha\beta}}_{I-\text{asymmetric}}]\hat{a}^\dagger_{\boldsymbol{k}\mu\alpha}\hat{a}_{\boldsymbol{k}\nu\beta}$$

$$A_{\mu\nu}(\boldsymbol{k}) = -A_{\nu\mu}(\boldsymbol{k}) = -A_{\mu\nu}(-\boldsymbol{k})$$

$$\boldsymbol{B}_{\mu\nu}(-\boldsymbol{k}) = \boldsymbol{B}_{\nu\mu}(\boldsymbol{k}) = -\boldsymbol{B}_{\mu\nu}(\boldsymbol{k})$$

+ additional constraints due to point-group symmetry

Band degeneracy is lifted if $\boldsymbol{B}_{\mu\nu}(\boldsymbol{k}) \neq \boldsymbol{0}$

$\Downarrow$

Nondegenerate Bloch bands $\xi_n(\boldsymbol{k}) = \xi_n(-\boldsymbol{k})$ labelled by n

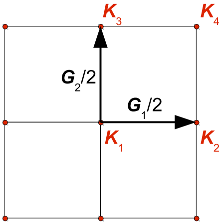
Spin-orbit coupling

TR invariant points: $-\boldsymbol{K} = \boldsymbol{K} + \boldsymbol{G}$

2D square lattice (spacing = d)

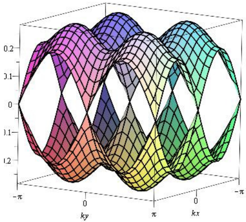
$$\{\boldsymbol{K}_i\} = \left\{ \boldsymbol{0},\ \frac{\boldsymbol{G}_1}{2},\ \frac{\boldsymbol{G}_2}{2},\ \frac{\boldsymbol{G}_1 + \boldsymbol{G}_2}{2} \right\}$$

$$\boldsymbol{G}_1 = \frac{2\pi}{d}\hat{x}, \quad \boldsymbol{G}_2 = \frac{2\pi}{d}\hat{y}$$



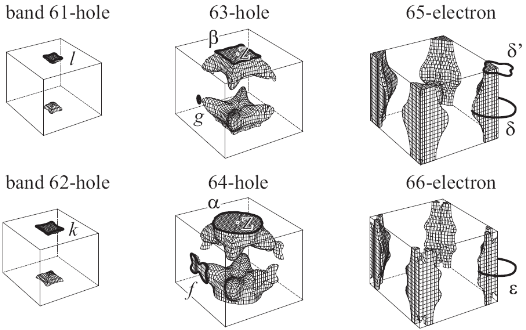
$$\boldsymbol{B}_{\mu\nu}(\boldsymbol{K}) = 0, \quad \boldsymbol{A}_{\mu\nu}(\boldsymbol{K}) = 0$$

Bloch bands $\xi_n(\boldsymbol{k})$ remain pairwise degenerate at the TRI points



Electron band structure

Band structure of $CePt_3Si$:



SO band splitting:

$CePt_3Si$: $E_{SO} \simeq 200$ meV
Li_2Pd_3B: $E_{SO} \simeq 30$ meV
Li_2Pt_3B: $E_{SO} \simeq 200$ meV
LAO/STO: $E_{SO} \simeq 1..10$ meV

$E_{SO} \gg$ SC energy scales

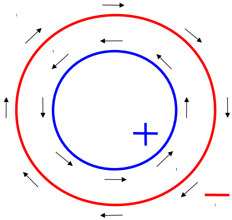
Minimal model of SO coupling

Generalized Rashba model: $\hat{H}_0 = \sum_{\boldsymbol{k},\alpha\beta=\uparrow,\downarrow} [\epsilon_0(\boldsymbol{k})\delta_{\alpha\beta} + \boldsymbol{\gamma}(\boldsymbol{k})\boldsymbol{\sigma}_{\alpha\beta}]\hat{a}^\dagger_{\boldsymbol{k}\alpha}\hat{a}_{\boldsymbol{k}\beta}$

antisymmetric SO coupling, $\boldsymbol{B}_{00}(\boldsymbol{k}) \equiv \boldsymbol{\gamma}(\boldsymbol{k}) = -\boldsymbol{\gamma}(-\boldsymbol{k})$

Two Bloch bands: $\xi_\lambda(\boldsymbol{k}) = \epsilon(\boldsymbol{k}) + \lambda|\boldsymbol{\gamma}(\boldsymbol{k})|$
(band index $n = \lambda = \pm$ – helicity)

The original Rashba model:
$\boldsymbol{\gamma}(\boldsymbol{k}) = a(k_y\hat{x} - k_x\hat{y})$
$\xi_\lambda(\boldsymbol{k}) = \epsilon(\boldsymbol{k}) + |a|\sqrt{k_x^2 + k_y^2}$



Symmetry of the SO coupling

Point-group symmetry: $g\boldsymbol{\gamma}(g^{-1}\boldsymbol{k}) = \boldsymbol{\gamma}(\boldsymbol{k})$ (g - lattice rotation or reflection)

<u>21 PGs in 3D</u>	<u>$\boldsymbol{\gamma}_{3D}(\boldsymbol{k})$</u>
O	$a(k_x\hat{x} + k_y\hat{y} + k_z\hat{z})$
$\mathbf{C}_{4v}$	$a_1(k_y\hat{x} - k_x\hat{y}) + ia_2(k_+^4 - k_-^4)k_z\hat{z}$
$\mathbf{T}_d$	$a[k_x(k_y^2 - k_z^2)\hat{x} + k_y(k_z^2 - k_x^2)\hat{y} + k_z(k_x^2 - k_y^2)\hat{z}]$
...	...

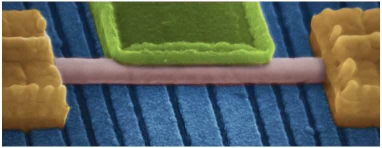
<u>10 PGs in 2D</u>	<u>$\boldsymbol{\gamma}_{2D}(\boldsymbol{k})$</u>
$\mathbf{C}_1$	$(a_1k_x + a_2k_y)\hat{x} + (a_3k_x + a_4k_y)\hat{y} + (a_5k_x + a_6k_y)\hat{z}$
$\mathbf{C}_2$	$(a_1k_x + a_2k_y)\hat{x} + (a_3k_x + a_4k_y)\hat{y}$
...	
$\mathbf{D}_4$	$a(k_y\hat{x} - k_x\hat{y})$
$\mathbf{D}_6$	$a(k_y\hat{x} - k_x\hat{y})$

Symmetry of the SO coupling

5 point groups in 1D $\quad$ $\boldsymbol{\gamma}_{1D}(k_x) = \boldsymbol{a} k_x$

$\mathbf{C}_1 = \{E\}$	$\boldsymbol{a} = a_1\hat{x} + a_2\hat{y} + a_3\hat{z}$
$\mathbf{D}_x = \{E, \sigma_x\}$	$\boldsymbol{a} = a_2\hat{y} + a_3\hat{z}$
$\mathbf{D}_y = \{E, \sigma_y\}$	$\boldsymbol{a} = a_2\hat{y}$
$\mathbf{C}_2 = \{E, \sigma_x\sigma_y\}$	$\boldsymbol{a} = a_1\hat{x} + a_2\hat{y}$
$\mathbf{V} = \{E, \sigma_x, \sigma_y, \sigma_x\sigma_y\}$	$\boldsymbol{a} = a_2\hat{y}$

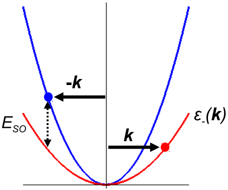
SC in quantum wires:
no reflection symmetry $z \to -z$
due to substrate

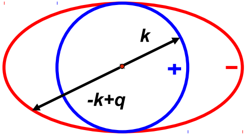


Superconducting pairing in nondegenerate bands

In real noncentrosymmetric SCs: $T_c \ll \varepsilon_c \ll E_{\mathrm{SO}}, \epsilon_F$

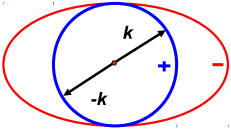
interband pairing is suppressed



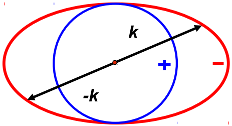


Fulde-Ferrell Larkin-Ovchinnikov state

only intraband pairing survives



$\Delta_+(\boldsymbol{k})$



$\Delta_-(\boldsymbol{k})$

Superconducting pairing in nondegenerate bands

Cooper pairing of the time-reversed states in the same band:

$$\hat{H}_{int} = \frac{1}{2\mathcal{V}} \sum_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}} \sum_{nn'} V_{nn'}(\boldsymbol{k},\boldsymbol{k}') \hat{c}^{\dagger}_{\boldsymbol{k}+\boldsymbol{q},n} \hat{\tilde{c}}^{\dagger}_{\boldsymbol{k},n} \hat{\tilde{c}}_{\boldsymbol{k}',n'} \hat{c}_{\boldsymbol{k}'+\boldsymbol{q},n'}$$

$$\hat{\tilde{c}}^{\dagger}_{\boldsymbol{k},n} = K \hat{c}^{\dagger}_{\boldsymbol{k},n} K^{-1} = t_n(\boldsymbol{k})\ \hat{c}^{\dagger}_{-\boldsymbol{k},n},$$

phase factor, $t_n(\boldsymbol{k}) = -t_n(-\boldsymbol{k})$

Mean field (M = # of nondegenerate bands crossing the Fermi level):

$$\hat{H}_{MF} = \frac{1}{2} \sum_{\boldsymbol{k}\in \mathrm{BZ}} \sum_{n=1}^{M} \left[\Delta_n(\boldsymbol{k}) \hat{c}^{\dagger}_{\boldsymbol{k},n} \hat{\tilde{c}}^{\dagger}_{\boldsymbol{k},n} + \Delta^*_n(\boldsymbol{k}) \hat{\tilde{c}}_{\boldsymbol{k},n} \hat{c}_{\boldsymbol{k},n} \right]$$

Gap functions are even: $\Delta_n(\boldsymbol{k}) = \Delta_n(-\boldsymbol{k})$

Superconducting pairing in nondegenerate bands

Symmetry properties: $K: \ \Delta_n(\boldsymbol{k}) \to \Delta_n^*(\boldsymbol{k})$

$g \in \mathbb{G}: \ \Delta_n(\boldsymbol{k}) \to \Delta_n(g^{-1}\boldsymbol{k})$

Basis-function expansion (in IREP Γ): $\Delta_n(\boldsymbol{k}) = \sum_{a=1}^{d_\Gamma} \eta_{n,a} \ \phi_a(\boldsymbol{k})$

$M d_\Gamma$ order parameter components → $\eta_{n,a}$

Example: $\mathbb{G}_{2D} = \mathbf{D}_4$ (e.g. oxide interfaces)

Γ	d_Γ	$\phi_\Gamma(\boldsymbol{k}) = \phi_\Gamma(-\boldsymbol{k})$
A_1	1	1
A_2	1	$k_x k_y (k_x^2 - k_y^2)$
B_1	1	$k_x^2 - k_y^2$
B_2	1	$k_x k_y$
E	2	$-$

Superconducting pairing in two-band model

Band representation: $\Delta_+(\boldsymbol{k}) = \Delta_+(-\boldsymbol{k}), \quad \Delta_-(\boldsymbol{k}) = \Delta_-(-\boldsymbol{k})$

Spin representation:

$$\hat{\Delta}_{\alpha\beta} = \Delta_s(\boldsymbol{k})(i\hat{\sigma}_2)_{\alpha\beta} + \underbrace{\Delta_t(\boldsymbol{k})\hat{\gamma}(\boldsymbol{k})}_{\boldsymbol{d}(\boldsymbol{k})=-\boldsymbol{d}(-\boldsymbol{k})}(i\hat{\boldsymbol{\sigma}}\hat{\sigma}_2)_{\alpha\beta}$$

singlet-triplet mixing

$$\Delta_s(\boldsymbol{k}) = \frac{\Delta_+(\boldsymbol{k}) + \Delta_-(\boldsymbol{k})}{2}, \quad \Delta_t(\boldsymbol{k}) = \frac{\Delta_+(\boldsymbol{k}) - \Delta_-(\boldsymbol{k})}{2}$$

Novel features in superconducting state

This talk:

- Unusual nonuniform states: helical, phase solitons, zero-field instabilities

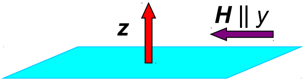
Not today:

- Topology in normal state: Berry flux, Z_2 invariants
- Topology in SC state: bulk-boundary correspondence, Majorana quasiparticles
- Magnetoelectric effect
- Unusual effects of disorder
- ...

Ginzburg-Landau free energy

Simplest case: unit IREP, two bands: $n = \lambda = \pm \quad \Rightarrow \quad \eta_+(\boldsymbol{r}),\, \eta_-(\boldsymbol{r})$

2D SC
in a parallel field



GL free energy density: $F = F_+ + F_- + F_m$

Intraband:

$$F_\lambda = (\text{uniform terms}) + K_\lambda |\boldsymbol{\nabla}\eta_\lambda|^2 + \underbrace{\tilde{K}_\lambda \,\mathrm{Im}\left[\eta_\lambda^*(\boldsymbol{H}\times\boldsymbol{\nabla})_z\eta_\lambda\right]}_{\text{Lifshitz invariant}} + L_\lambda H^2|\eta_\lambda|^2$$

$$K_\lambda \sim \frac{N_{F,\lambda}}{T_{c0}^2}v_{F,\lambda}^2, \quad |\tilde{K}_\lambda| \sim \frac{N_{F,\lambda}}{T_{c0}^2}\mu_B v_{F,\lambda}, \quad L_\lambda \sim \frac{N_{F,\lambda}}{T_{c0}^2}\mu_B^2$$

Interband pair tunneling: $F_m = \gamma_m(\eta_+^*\eta_- + \eta_-^*\eta_+)$

Lifshitz invariants $\Rightarrow$ nonuniform instability

Low fields: helical state

Helical state – stable at low field

$$\eta_\lambda(\boldsymbol{r}) = \eta_\lambda e^{iqx}$$

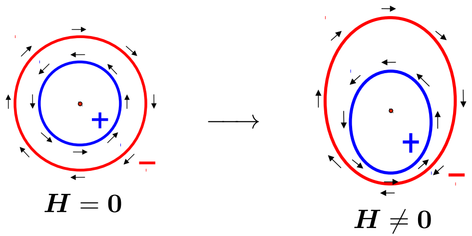
$$q = C_1 H, \quad T_c(H) = T_{c0} - C_2 H^2$$

$$C_{1,2} = C_{1,2}(\alpha_\pm, K_\pm, \tilde{K}_\pm, L_\pm)$$

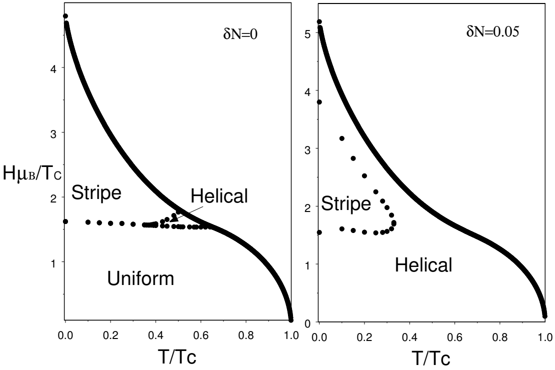
No supercurrent in the helical state: $j_x = -\frac{c}{\mathcal{V}}\frac{\partial \mathcal{F}}{\partial A_x} = \frac{2e}{\mathcal{V}}\frac{\partial \mathcal{F}}{\partial q} = 0$

Origin of the modulation: band displacement and deformation by $\boldsymbol{H}$

$$\xi_\lambda(\boldsymbol{k}) \to \Xi_\lambda(\boldsymbol{k}) = \xi_\lambda(\boldsymbol{k}) - \lambda\mu_B\hat{\gamma}(\boldsymbol{k})\boldsymbol{H}, \quad \Xi_\lambda(\boldsymbol{k}) \neq \Xi_\lambda(-\boldsymbol{k})$$



(Possible) phase diagram in 2D



(from D. Agterberg and R. Kaur, PRB **75**, 064511 (2007))

helical (single $\boldsymbol{q}$):
$\eta_\pm(\boldsymbol{r}) = \Delta_0 e^{i\boldsymbol{q}\boldsymbol{r}}$

stripe (multiple $\boldsymbol{q}$):
$\eta_\pm(\boldsymbol{r}) = \sum_{\boldsymbol{q}} \Delta_{\boldsymbol{q}} e^{i\boldsymbol{q}\boldsymbol{r}}$

High fields: phase soliton lattice

High fields: competing phases?

London approximation: $\eta_\lambda(\boldsymbol{r}) = |\eta_\lambda| e^{i\varphi_\lambda(x)}$

Supercurrent: $j_x = -4e \sum_\lambda K_\lambda |\eta_\lambda|^2 \nabla_x \varphi_\lambda + 2eH \sum_\lambda \tilde{K}_\lambda |\eta_\lambda|^2 = 0$

current conservation + boundary conditions

$$\nabla_x \varphi_+ = \frac{1}{1+\rho} \nabla_x \theta + q, \quad \nabla_x \varphi_- = -\frac{\rho}{1+\rho} \nabla_x \theta + q$$

$$\theta = \varphi_+ - \varphi_-, \quad \rho = \frac{K_+ |\eta_+|^2}{K_- |\eta_-|^2}, \quad q = \frac{H}{2} \frac{\sum_\lambda \tilde{K}_\lambda |\eta_\lambda|^2}{\sum_\lambda K_\lambda |\eta_\lambda|^2}$$

φ_+ and φ_- are locked ($\theta = 0$ or π) $\Rightarrow$ helical state

$\nabla_x \theta \neq 0 \Rightarrow$ phase soliton state

High fields: phase soliton lattice

London free energy density:

$$f = (...) + \frac{1}{2}(\nabla_x\theta)^2 + V_0(1-\cos\theta) - \underbrace{h(\nabla_x\theta)}_{\text{bias}}$$

$$h = \frac{H}{2}\left(\frac{\tilde{K}_+}{K_+} - \frac{\tilde{K}_-}{K_-}\right), \quad V_0 \propto |\gamma_m|$$

Sine-Gordon equation

$$\nabla_x^2\theta - V_0\sin\theta = 0$$

$\longrightarrow$ single soliton ($\gamma_m < 0$):

$\theta(x) = \pi + 2\arcsin\tanh(x/\xi_s)$

$\xi_s = 1/\sqrt{V_0}$, energy = ϵ_1

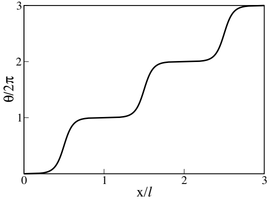
At low soliton density n_s: $F_{\text{solitons}} - F_{\text{no solitons}} = (\epsilon_1 - 2\pi h)n_s + ...$

Soliton lattice

at $h > h_s = \epsilon_1/2\pi$

lattice spacing

$\simeq 2\xi_s \ln H_s/(H - H_s)$



Zero-field nonuniform superconducting states

Lifshitz gradient terms are possible even at $\boldsymbol{H} = \mathbf{0}$!

GL energy for a tetragonal SC, point group $\mathbf{C}_{4v}$:
$F = F_+ + F_- + F_m + F_L$

Additional Lifshitz invariant: $F_L = K_L \,\mathrm{Re}(\eta_+^* \nabla_z \eta_- - \eta_-^* \nabla_z \eta_+)$

Zero-field nonuniform instability: $\eta_\lambda = \eta_{\lambda,0} e^{iqz}$ if $K_L > K_{L,c}$

Attempt at microscopic derivation:

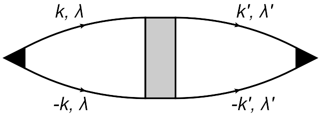
$$\hat{H}_{int} = \frac{1}{2\mathcal{V}} \sum_{\boldsymbol{k}\boldsymbol{k}'\boldsymbol{q}} \sum_{\lambda\lambda'} V_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q}) \hat{c}^\dagger_{\boldsymbol{k}+\boldsymbol{q},\lambda} \hat{\tilde{c}}^\dagger_{\boldsymbol{k},\lambda} \hat{\tilde{c}}_{\boldsymbol{k}',\lambda'} \hat{c}_{\boldsymbol{k}'+\boldsymbol{q},\lambda'}$$

$\boldsymbol{q}$-expansion: $V_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}';\boldsymbol{q}) = v_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}') + i\boldsymbol{b}_{\lambda\lambda'}(\boldsymbol{k},\boldsymbol{k}')\boldsymbol{q} + \mathcal{O}(q^2)$

treat as a perturbation

Zero-field nonuniform superconducting states

Correction to the free energy =



Magnitude of the Lifshitz term: $K_L = \frac{1}{2} N_+ N_- \ln^2\left(\frac{2e^{\mathbb{C}}\epsilon_c}{\pi T_c}\right)|\boldsymbol{\beta}|$

$\boldsymbol{\beta} = \langle \boldsymbol{b}_{+-}(\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}')\rangle_{\hat{\boldsymbol{k}}, \hat{\boldsymbol{k}}'}$ – invariant polar vector (for $\mathbf{C}_{4v}$: $\boldsymbol{\beta} \parallel \hat{z}$)

$\boldsymbol{\beta} \neq 0$ in pyroelectric crystals:
$\mathbb{G} = \mathbf{C}_1, \mathbf{C}_s, \mathbf{C}_2, \mathbf{C}_{2v}, \mathbf{C}_4, \mathbf{C}_{4v}, \mathbf{C}_3, \mathbf{C}_{3v}, \mathbf{C}_6, \mathbf{C}_{6v}$

Zero-field nonuniform superconducting states

Other types of zero-field Lifshitz invariants:

weak SO coupling + spin-triplet pairing: $\hat{\Delta}(\boldsymbol{k},\boldsymbol{r}) = \boldsymbol{d}(\boldsymbol{k},\boldsymbol{r})(i\hat{\boldsymbol{\sigma}}\hat{\sigma}_2)$

in IREP Γ: $\boldsymbol{d}(\boldsymbol{k},\boldsymbol{r}) = \sum_{a=1}^{d_\Gamma} \eta_a(\boldsymbol{r})\boldsymbol{\varphi}_a(\boldsymbol{k}), \quad \boldsymbol{\varphi}_a(\boldsymbol{k}) = -\boldsymbol{\varphi}_a(-\boldsymbol{k})$

e.g., 3-component order parameter $\boldsymbol{\eta} = (\eta_1, \eta_2, \eta_3)$ in a cubic crystal

$$\mathbb{G} = \mathbf{O}, \quad \Gamma = F_1, \quad \boldsymbol{\gamma}(\boldsymbol{k}) = \gamma_0 \boldsymbol{k}, \quad \varphi_{a,i}(\boldsymbol{k}) \propto e_{aij}\hat{k}_j$$

Lifshitz invariant: $F_L = K_L(\eta_1^*\nabla_y\eta_3 + \eta_2^*\nabla_z\eta_1 + \eta_3^*\nabla_x\eta_2 + c.c.)$

LI magnitude depends on the SO band splitting: $K_L \propto |\gamma_0|$

Open questions

In the presence of the Lifshitz gradient terms:

- Stable SC states, $H-T$ phase diagram, in 2D, 3D?
- Single vortex structure? Abrikosov vortex lattice structure?
- Nonequilibrium properties (TDGL)?

Same questions – in other SC systems with "built-in" periodic instabilities
e.g. for the Fulde-Ferrell-Larkin-Ovchinnikov state (high-field paramagnetically-limited BCS):

$$F_{gradient} = -K_2|\boldsymbol{D}\eta|^2 + K_4|\boldsymbol{D}^2\eta|^2$$

Conclusions

- Absence of inversion symmetry + electron-lattice spin-orbit coupling = nondegenerate electron bands
- Momentum-space symmetry of the SC states in noncentrosymmetric crystals differs from the standard centrosymmetric case
- The SC order parameter in noncentrosymmetric SCs has at least two components, one per each helicity band
- Linear gradient terms in the GL energy (Lifshitz invariants) are responsible for a variety of new effects, e.g. nonuniform SC states, even at zero applied field

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