Exercises.

- If $F(n)$ is the nth Fibonacci number, then $F(2^{1000})$ is an integer with 10^{300} decimal digits. Determine the 20 least significant decimal digits of $F(2^{1000})$.
- Fix a random matrix $A \in \mathbb{Z}^{100\times 101}$ and a set of primes $p_1, \ldots, p_{100}$ with $p_i \approx 2^i$. For each i check how long your computer needs to find a basis of $\ker A$ mod p_i.
- Use Chinese remaindering and rational reconstruction to find a basis vector of $\ker A$ in $\mathbb{Q}^{101}$. How can we tell in advance how many primes are needed?

Exercises.

- How long does it take on your computer to compute a Gröbner basis for 3 random polynomials in 4 variables of total degree 5?
- Let $I, J \subseteq \mathbb{Q}[x, y, z]$ be ideals. Show that $I \cap J$ is also an ideal, and that $\dim I = \dim J = 0 \iff \dim(I \cap J) = 0$. What does this mean geometrically?
- Given the minimal polynomials of two algebraic functions $f(x), g(x)$, how can we find the minimal polynomial of their composition $h(x) := f(g(x))$?

Exercises.

- Find a linear recurrence equation for $\binom{2n}{n} + 2^n - \sum_{k=1}^{n} \frac{1}{1+k^2}$, and a differential equation for its generating function.
- How do we need to define σ and δ in order to obtain an Ore algebra where ∂ acts like $\partial \cdot f(x) = f(x+1) - f(x)$?
- Show that when $f(x)$ is differentially algebraic, then so are $1/f(x)$, $\sqrt{f(x)}$, $\exp(f(x))$, and $\log(f(x))$.