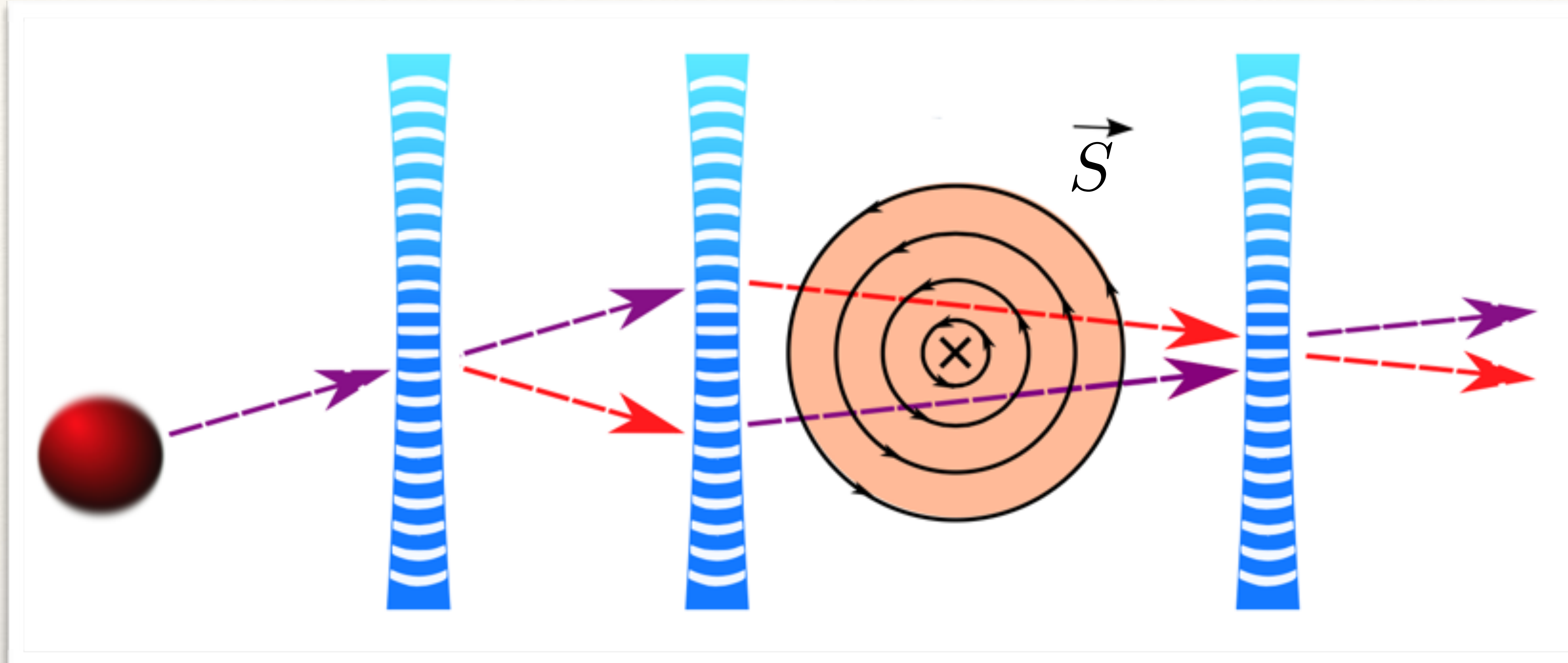


The Abraham-Minkowski controversy: an ultracold atom perspective

Duncan O'Dell,
McMaster University

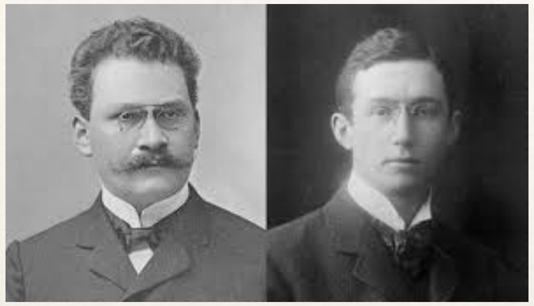
Quantum field framework for structured light interactions, Banff *April, 24 2017*

Light in a medium: Abraham vs Minkowski

Minkowski

Momentum density:

$$\mathbf{g}_M = \mathbf{D} \times \mathbf{B}$$



Abraham

Momentum density:

$$\mathbf{g}_A = \mathbf{E} \times \mathbf{H}/c^2$$

$$\rightarrow p_M = \frac{n_p^2}{n_g}\hbar k_0 \approx n\hbar k_0$$

$$\rightarrow p_A = \frac{1}{n_g}\hbar k_0 \approx \frac{\hbar k_0}{n}$$

"wave picture":

de Broglie: $p = h/\lambda$

In medium: $\lambda \rightarrow \lambda/n$

"particle picture": $p = mv = m\frac{c}{n}$

Einstein: $E = mc^2$

$$\rightarrow p = \frac{E}{c^2}\frac{c}{n} = \frac{E}{nc}$$

Planck: $E = \hbar ck$

H. Minkowski, *Nachr. Ges. Wiss. Göttn. Math.-Phys. Kl.* **53**, (1908).
M. Abraham, Rend. Circ. Matem. Palermo **28**, 1 (1909).

Origin of controversy is the difficulty of separating electromagnetic field from matter.

Large literature

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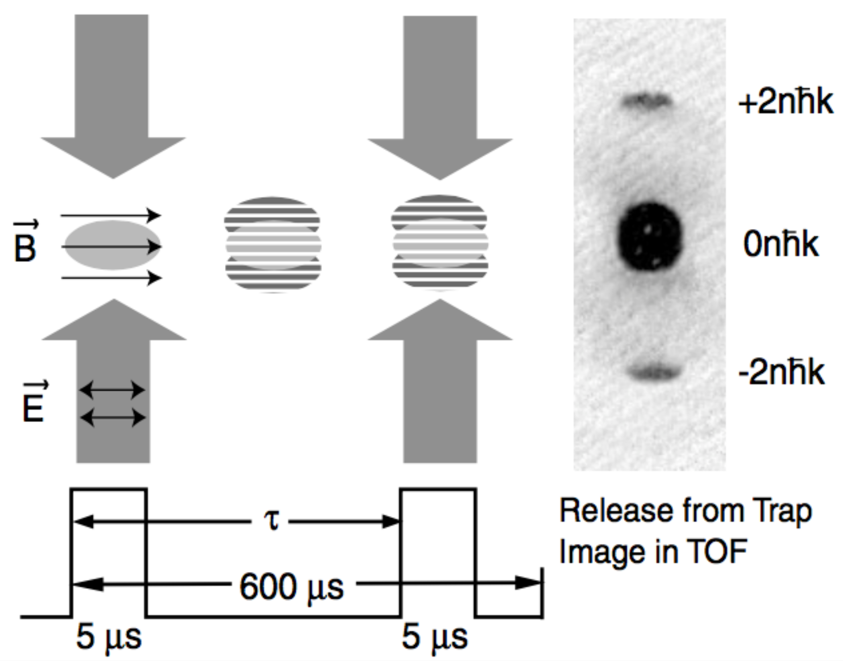
BEC experiment

PRL 94, 170403 (2005) PHYSICAL REVIEW LETTERS week ending 6 MAY 2005

Photon Recoil Momentum in Dispersive Media

Gretchen K. Campbell, Aaron E. Leanhardt,* Jongchul Mun, Micah Boyd, Erik W. Streed,
Wolfgang Ketterle, and David E. Pritchard†



Kapitza-Dirac interferometer (pulsed standing wave of laser light) gives momentum kick to atoms in BEC. **Result seems to support Minkowski.**

Force on an atom due to a light pulse

PRL 102, 050403 (2009) PHYSICAL REVIEW LETTERS week ending 6 FEBRUARY 2009

Momentum Exchange between Light and a Single Atom: Abraham or Minkowski?

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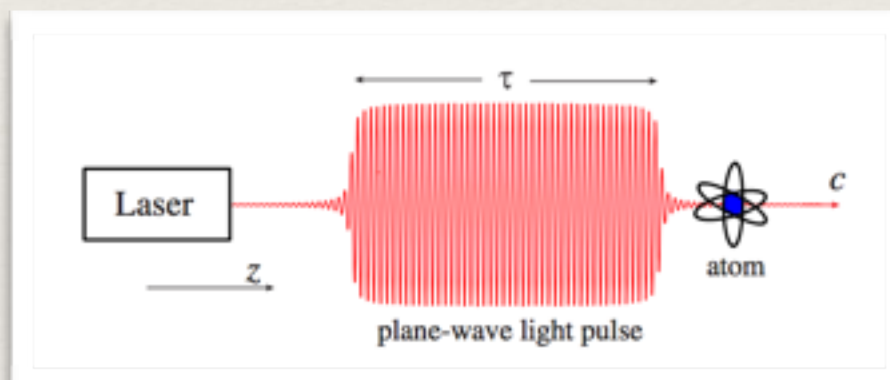
(Received 17 November 2008; published 3 February 2009)

Dielectric medium is a single atom



$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\text{atom}})$$

Lorentz force on induced dipole

$$F_i = \mathbf{d} \cdot \frac{\partial}{\partial x_i}\mathbf{E} + \frac{\partial}{\partial t}(\mathbf{d} \times \mathbf{B})_i$$

J.P. Gordon, Phys. Rev. A 8, 14 (1973)

Which way does atom recoil?

Extra force can be interpreted as the Lorentz force acting on the internal current.

Canonical vs kinetic momentum

Momentum density: $\mathbf{g}_M = \mathbf{D} \times \mathbf{B}$ $\mathbf{g}_A = \mathbf{E} \times \mathbf{H}/c^2$

For a dielectric "medium" consisting of a single atom:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{d}\delta(\mathbf{r} - \mathbf{r}_{\mathrm{atom}}) \qquad \mathbf{B} = \mu_0 \mathbf{H}$$

$$\Rightarrow \int \mathbf{g}_M \mathrm{d}^3 r = \int \mathbf{g}_A \mathrm{d}^3 r + \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$$

Thus [Lembessis et al, Phys. Rev. A. **48**, 1594 (1993)] :

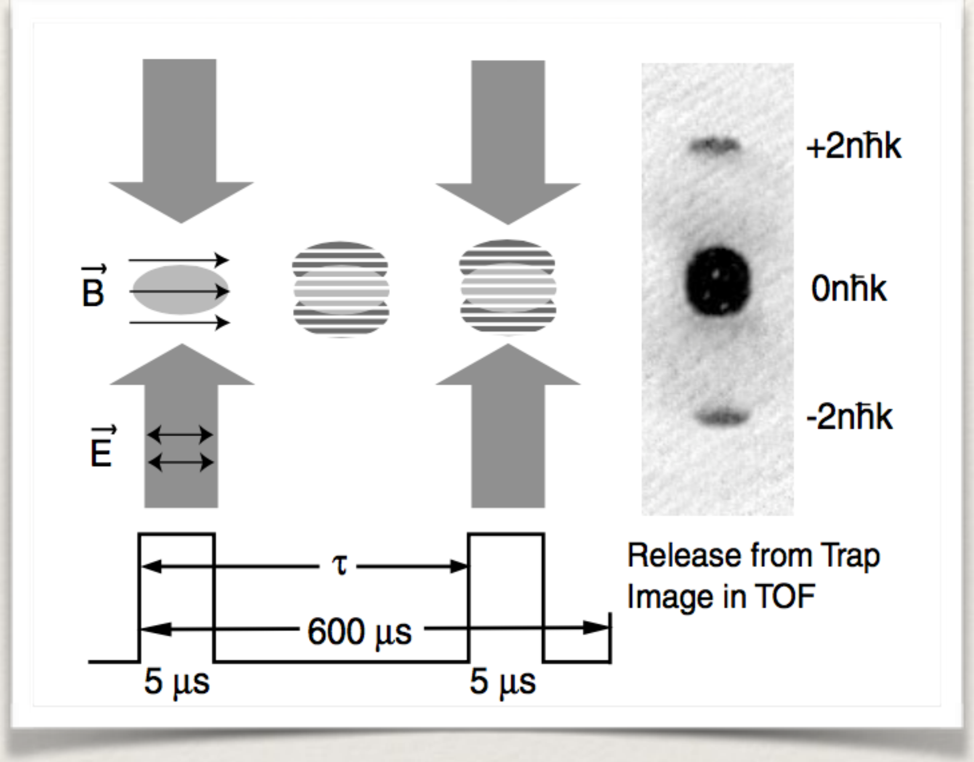
$$\mathbf{p}_{\mathrm{atom}} + \int \mathbf{g}_M \mathrm{d}^3 r = M\dot{\mathbf{r}}_{\mathrm{atom}} + \int \mathbf{g}_A \mathrm{d}^3 r$$

where: $\boxed{\mathbf{p}_{\mathrm{atom}}} = \boxed{M\dot{\mathbf{r}}_{\mathrm{atom}}} - \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$

canonical kinetic

Implications for Campbell et al.

$$\int \mathbf{g}_M \mathrm{d}^3 r = \int \mathbf{g}_A \mathrm{d}^3 r + \mathbf{d} \times \mathbf{B}(\mathbf{r}_{\mathrm{atom}})$$



Assume linear response: $\mathbf{d} = \alpha \mathbf{E}$

$$\mathbf{d} \times \mathbf{B} = \alpha \mathbf{E} \times \mathbf{B}$$
$$= \alpha \mu_0 \mathbf{S}$$

where $\mathbf{S} = (\mathbf{E} \times \mathbf{B})/\mu_0$

Poynting vector

In the experiment by Campbell et al. $\mathbf{S} = 0$ and so $\mathbf{g}_M = \mathbf{g}_A$

Abraham and Minkowski momenta are the same!

Electric dipole moving in a B-field: Röntgen interaction

$$L_{\text{dipole}} = \frac{1}{2}mv^2 + \mathbf{d}\cdot(\mathbf{E} + \mathbf{v}\times\mathbf{B})$$

neutral electric dipole

Wilkens, Phys. Rev. Lett. **72**, 5 (1994); Wei, Han, & Wei, Phys. Rev. Lett. **75**, 2071 (1995).

Compare with Lagrangian for a **charged** particle :

$$L_{\text{charge}} = \frac{1}{2}mv^2 + q\mathbf{v}\cdot\mathbf{A} - q\phi$$

$$\mathbf{B}\times\mathbf{d} \leftrightarrow (q\mathbf{A})_{\text{eff}} \qquad \mathbf{d}\cdot\mathbf{E} \leftrightarrow -(q\phi)_{\text{eff}}$$

Define:

$$\mathbf{E}_{\text{eff}} \equiv -\nabla\phi_{\text{eff}} - \frac{\partial\mathbf{A}_{\text{eff}}}{\partial t} = \frac{1}{q}\left[\nabla(\mathbf{d}\cdot\mathbf{E}) - \frac{\partial}{\partial t}(\mathbf{B}\times\mathbf{d})\right]$$

$$\mathbf{B}_{\text{eff}} \equiv \nabla\times\mathbf{A}_{\text{eff}} = \frac{1}{q}\nabla\times(\mathbf{B}\times\mathbf{d})$$

Force on atom in a plane wave

Assume plane wave laser beam: $\nabla \times (\mathbf{B} \times \mathbf{d}) = 0 \longrightarrow \mathbf{B}_{\text{eff}} = 0$

$$\mathbf{E}_{\text{eff}} \equiv -\nabla\phi_{\text{eff}} - \frac{\partial \mathbf{A}_{\text{eff}}}{\partial t} = \frac{1}{q}\left[\nabla(\mathbf{d}\cdot\mathbf{E}) - \frac{\partial}{\partial t}(\mathbf{B}\times\mathbf{d})\right]$$

$$\longrightarrow \mathbf{F}_{\text{atom}} = q\mathbf{E}_{\text{eff}}$$

Assume linear response: $\mathbf{d} = \alpha(\mathbf{E} + \mathbf{v} \times \mathbf{B})$

$$\longrightarrow \mathbf{F} = q\mathbf{E}_{\text{eff}} = \nabla\left(\frac{\alpha}{2}E^2\right) + \alpha\frac{\partial}{\partial t}(\mathbf{E}\times\mathbf{B})$$

[J.P. Gordon, Phys. Rev. A **8**, 14 (1973)]

Note that for structured light: $\nabla \times (\mathbf{B} \times \mathbf{d}) \neq 0$

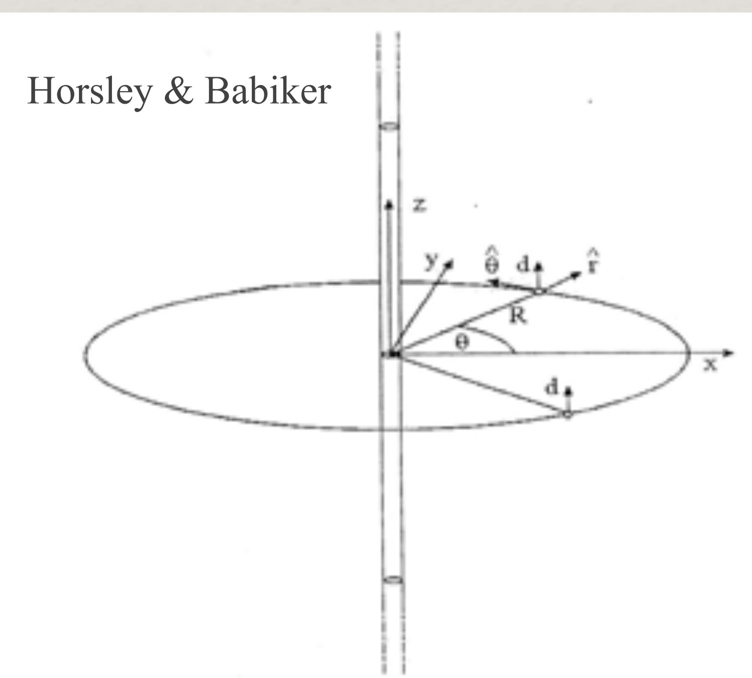
He-McKellar-Wilkens phase: electric dipole moving in B-field

Aharanov-Bohm phase: $$\phi_{\mathrm{AB}} = (q/\hbar)\oint \mathbf{A}(\mathbf{r})\cdot d\mathbf{r}$$

HMW phase: $$\phi_{\mathrm{HMW}} = \hbar^{-1}\oint [\mathbf{B}(\mathbf{r})\times\mathbf{d}]\cdot d\mathbf{r}$$ (neutral electric dipole moving in a STATIC magnetic field)

He & McKellar, Phys. Rev. A **47**, 3424 (1993), Wilkens, Phys. Rev. Lett. **72**, 5 (1994), Wei, Han, & Wei, Phys. Rev. Lett. 75, 2071 (1995), Horsley & Babiker Phys.Rev. Lett. **95**, 010495 (2005)…

Seems to require a straight line of magnetic monopoles…



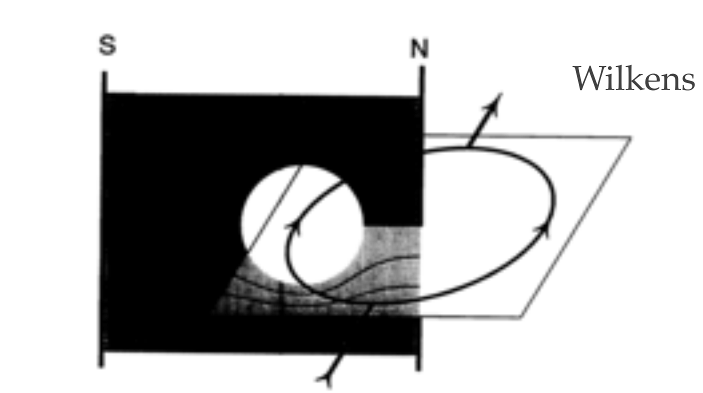


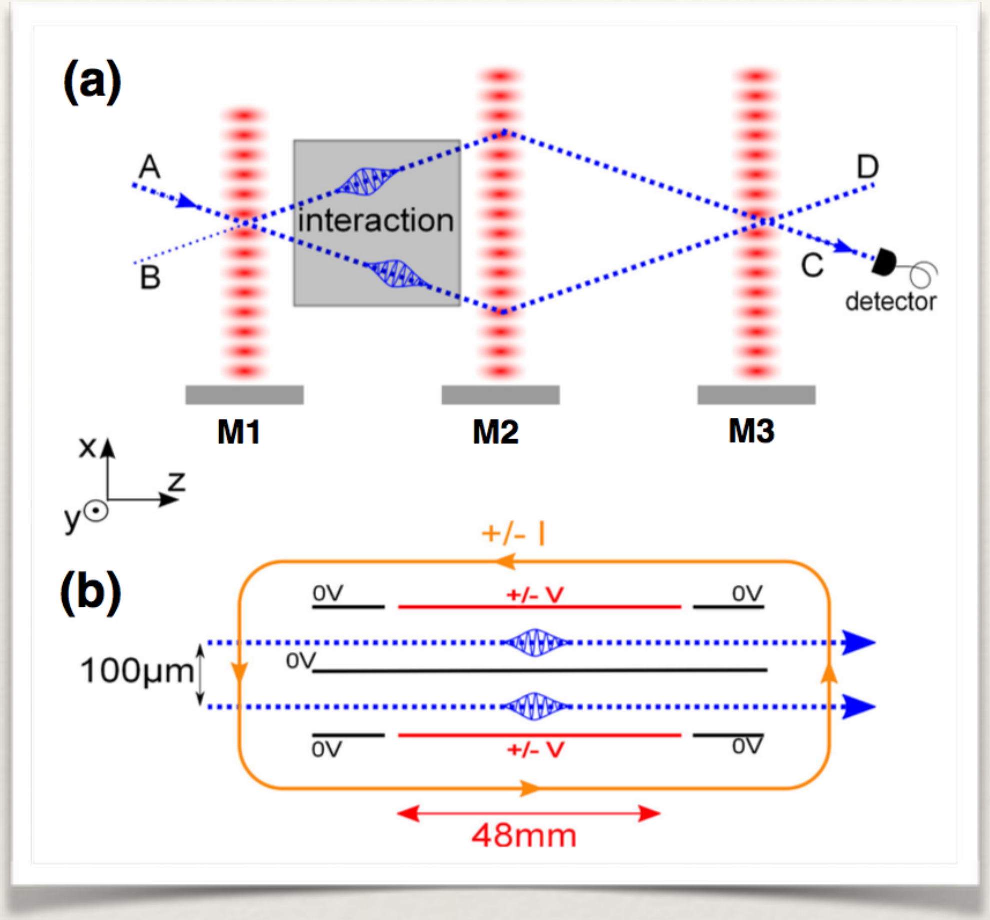
FIG. 1. Geometry of the interferometric experiment. Shown is the "Dirac sheet" with the two vertical edges effectively realizing two oppositely charged lines of magnetic monopoles. The "Dirac strings" (flux lines) which connect the two edges leave a hole for the interferometric path. The interferometric plane is orientated perpendicular to the magnet poles and the electric dipole moment of the interfering particles.

HMW is dual to the **Aharanov-Casher** phase:

$$\phi_{\mathrm{AC}} = (\hbar c^2)^{-1}\oint [\mathbf{E}(\mathbf{r})\times\mu]\cdot d\mathbf{r}$$

Phys. Rev. Lett. **53**, 319 (1984).

Observation of the He-McKellar-Wilkens phase



Observed for static electric and magnetic fields by S. Lepoutre, A. Gauguet, G. Trénec, M. Buchner & J. Vigué, Phys. Rev. Lett. **109**, 120401 (2012).

Geometric phase: depends only on path taken, not on the speed

Optical HMW phase?

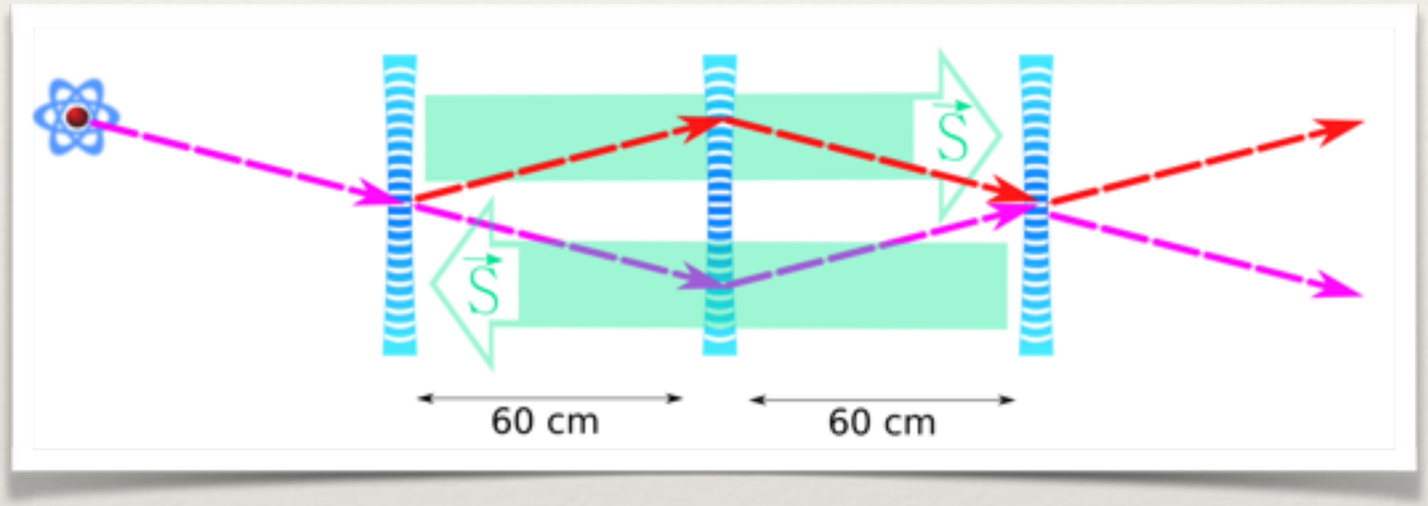
$$\psi \propto e^{(i/\hbar)\int L dt} = e^{i[\phi_{\rm dyn}(t_{\rm int}) + \phi_{\rm HMW}^{\rm optical}]}$$

$$\phi_{\rm dyn}(t_{\rm int}) = \frac{1}{\hbar}\int_0^{t_{\rm int}} \left(\frac{mv^2}{2} + \mathbf{d}\cdot\mathbf{E}\right) dt$$

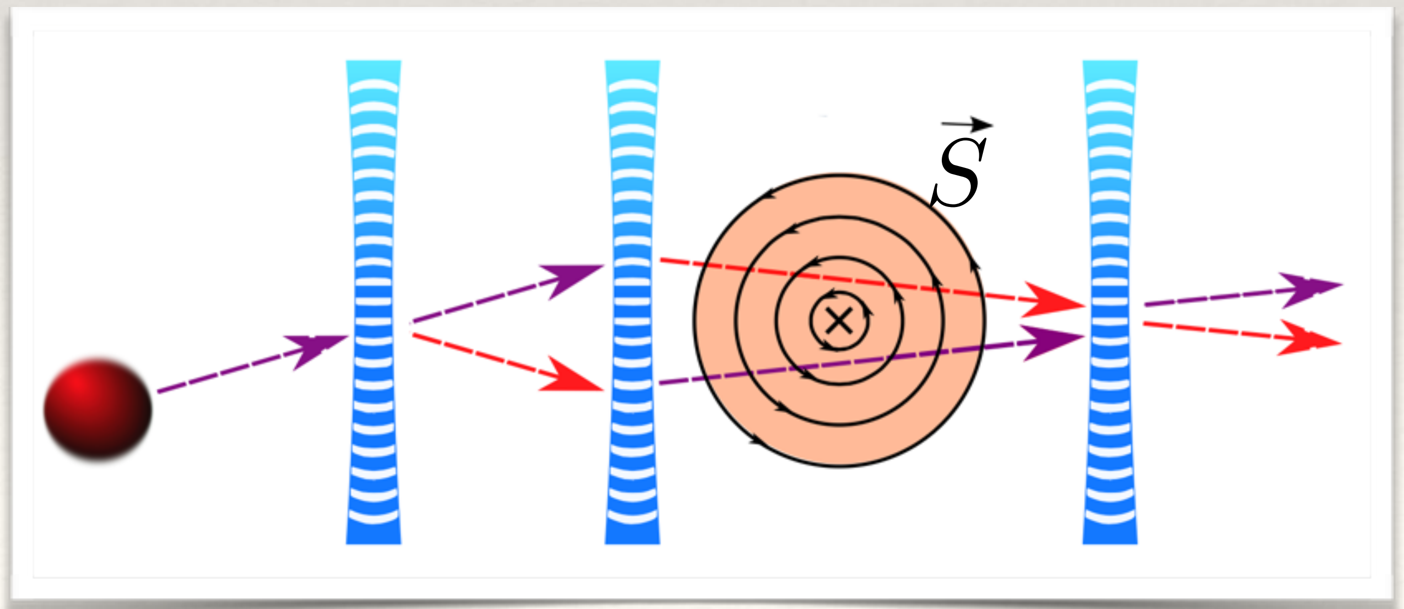
$$\phi_{\rm HMW}^{\rm optical} = \frac{\alpha\mu_0}{\hbar}\oint \mathbf{S}(\mathbf{r})\cdot d\mathbf{r}$$

Atom interferometer

Two gaussian beams



One Laguerre-Gauss beam



Note: this is different to the NIST experiments where an LG-beam + G-beam generates a rotation in a BEC: Andersen et al, PRL 97, 170406 (2006).

Summary

- Abraham-Minkowski physics is the classical part of the Röntgen interaction. The quantum part of the Röntgen interaction is a geometric phase which is the optical version of the He-McKellar-Wilkens phase.
- Structured light provides a way to realize this effect.
- Structured light also permits new ways to generate artificial magnetic fields for neutral atoms.

Laser configurations