Highest weights for certain algebras constructed from Yangians

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Introduction

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Joint with J. Kamnitzer, P. Tingley, B. Webster, and O. Yacobi

Notation

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- For an affine algebraic variety X over $\mathbb{C}$, denote the coordinate ring by $\mathbb{C}[X]$

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- Yangian case:

$$\mathfrak{k} = \mathfrak{g}[t], \quad \mathfrak{k}^* = t^{-1}\mathfrak{g}[[t^{-1}]], \quad K^* = G_1[[t^{-1}]]$$

Yangians

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- The *Yangian* $Y = Y(\mathfrak{g})$ is the associative $\mathbb{C}$-algebra with generators

$$E_i^{(r)}, H_i^{(r)}, F_i^{(r)} \quad \text{for } i \in I,\ \ r \geq 1$$

and relations $[E_i^{(r)}, F_j^{(s)}] = \delta_{i,j} H_i^{(r+s-1)}$, etc.

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- Y is filtered by $\deg_{NC} X^{(r)} = r - 1$, and

$$\mathrm{gr}_{NC}\, Y \cong U(\mathfrak{g}[t])$$

where $X^{(r)}$ corresponds to Xt^{r-1}.

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1. *$G_1[[t^{-1}]]$ is a Poisson-Lie group (via Yang's Manin triple)*
2. *There is an (explicit!) isomorphism of graded Poisson algebras*

$$\operatorname{gr} Y \cong \mathbb{C}\left[G_1[[t^{-1}]]\right]$$

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- Consider $H(u) = 1 + \sum_{r>0} H^{(r)} u^{-r}$. There exist unique $A^{(s)} \in Y$ such that

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$$A(u), \quad B(u) := A(u)E(u), \quad C(u) := F(u)A(u),$$

$$A(u)D(u-1) - B(u)C(u-1) = 1$$

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- Y_μ is a left coideal subalgebra

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Definition (KWWY)

The *truncated shifted Yangian* is the quotient

$$Y_\mu^\lambda(R) := Y_\mu / \langle A^{(s)} : s > m \rangle$$

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Theorem (KWWY)

There is a map of graded Poisson algebras

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which is an isomorphism modulo the nilradical of the LHS.

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Conjecture

The map is an isomorphism, and $Y_\mu^\lambda(\mathbf{R})$ provides the universal deformation quantization of Gr_μ^λ.

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2. Y_μ^λ is a "parabolic" W-algebra of type A (Webster-W-Yacobi)

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Theorem (BK, KTWWY)

There is a bijection

$$\left\{ \begin{array}{c} \textit{highest weights} \\ \textit{for } Y_\mu^\lambda(R) \end{array} \right\} \longleftrightarrow \left\{ \begin{array}{c} \textit{monic } S(u) \in \mathbb{C}[u], \\ \deg S(u) = m, \\ S(u) \textit{ divides } R(u) \end{array} \right\}$$

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- Write $R(u) = (u - r_1)^{\ell_1} \cdots (u - r_n)^{\ell_n}$
- Both sets above in bijection with basis for $\mathfrak{sl}_2$ weight space

$$\Big(V(\ell_1) \otimes \cdots V(\ell_n) \Big)_\mu$$

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$$M \in A-\mathrm{Mod} \quad\Longrightarrow\quad M^{\mathrm{high}} \in B(A)-\mathrm{Mod}$$

$$N \in B(A)-\mathrm{Mod} \quad\Longrightarrow\quad A \otimes_{A_{\geq 0}} N \in A-\mathrm{Mod}$$

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Conjecture

With data as above, there is an isomorphism

$$B(Y_\mu^\lambda(\mathbf{R})) \cong H^* \left(\mathcal{M}(\lambda, \mu)^{\mathbb{C}^\times} \right)$$

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3. Categorical $\mathfrak{g}^\vee$-action on $\bigoplus_\mu \mathcal{O}(Y_\mu^\lambda(\mathbf{R}))$

Thank you for listening!

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I refuse to answer that question on the grounds that I don't know the answer.

- Douglas Adams