# Bayesian analysis of pair-matched case-control studies subject to outcome misclassification 

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## Prodromal Multiple Sclerosis: the ProMS study

- Ongoing Canada-wide study (BC, NS, MA, SK) investigating the existence of a prodrome in multiple sclerosis (MS).
- Prevalence in Canada about .3\%, one of the highest in the world.
- No definite diagnostic test and highly heterogeneous symptoms lead to diagnostic delays.
- Focus lies on five years prior to the first recognized symptom of MS.
- Among others, presence of 14 morbidities in prodromal phase (e.g. hypertension, depression).
- Study data extracted from provincial administrative health databases.


## Health administrative databases of British Columbia

- Medical Services Plan (MSP) Database
- claim information of fee-for-service practitioners in BC
- since 1991, includes one to five ICD codes for reason of visit (e.g. 340 for MS)
- Canadian Discharge Abstract Database
- captures administrative records for all hospital discharges
- includes a maximum of 25 ICD codes per discharge
- PharmaNet
- prescription medication dispensed by pharmacies across BC
- includes information on drug type, quantity, directions for use

Databases are linkable, giving near-universal coverage of healthcare contacts for British Columbians.

## ProMS study design

- Matched case-control study
- MS cases identified from admin data using case definition of $\geq 3$ MS-specific records, i.e.
- ICD 340 in MSP or hospital discharge files
- MS-specific prescription drugs in PharmaNet
- Date of first MS-specific claim (index date) marks end of five-year prodromal phase.
- Matched controls selected from peers without MS-related records.
- Matching variables are sex, postal code and age at index date.
- Linkage with British Columbia Multiple Sclerosis (BC MS) database.


## Quality issues for administrative data

- ICD codes do not guarantee presence of a disease
- ICD coding errors
- Lack of specificity (e.g. ICD 780 - general symptoms)
- High misdiagnosis rate for multiple sclerosis (false positive rate of $35 \%$ reported by Poser [3])
- Possibility of misclassified disease status in ProMS, leading to
- apparent cases that are in fact controls
- apparent controls that are in fact MS cases
- Analysis must take potentially imperfect MS status of study subjects into account


## Preliminaries

- Suppose interest lies in the odds ratio $O R$ between a binary exposure $E$ and outcome $D$.
- $D$ is unobserved and only available via surrogate $D^{*}$ produced by a non-differential classifier.
- "Apparent" cases with $D^{*}=1$ are matched to "apparent" controls with $D^{*}=0$ on a set of confounders.
- Let $\left(E_{1 k}, E_{2 k}\right),\left(D_{1 k}, D_{2 k}\right)$ and $\left(D_{1 k}^{*}, D_{2 k}^{*}\right)$ denote the exposure, true and observed outcome of the apparent case and control in the $k$ th of $n$ pairs.
- Cell counts (probabilities):



## Analysis of matched case-control data under perfect outcome classification

- Consider the exposure risk model

$$
\operatorname{logit}\left(P\left(E_{i k}=1\right)\right)=\beta_{k}+\delta I(i=1), \quad i=1,2
$$

where $\beta_{k}$ is a pair-specific random effect.

- Assuming $E_{1 k}$ and $E_{2 k}$ are independent given $\beta_{k}$, Prescott et al. (2005) show that

$$
\begin{equation*}
O R=\exp (\delta)=\frac{P\left(E_{1}=1, E_{2}=0\right)}{P\left(E_{1}=0, E_{2}=1\right)}=\frac{\theta_{10}}{\theta_{01}} \tag{1}
\end{equation*}
$$

- This gives

$$
\begin{equation*}
\widehat{O R}=\frac{n_{10}}{n_{01}} \tag{2}
\end{equation*}
$$

- How do $\theta_{10} / \theta_{01}$ and $O R$ relate under outcome misclassification?


## Bias under outcome misclassification

- Denote

$$
\theta_{l m \mid j j}=P\left(E_{1}=I, E_{2}=m \mid D_{1}=i, D_{2}=j\right), \quad i, j, I, m=0,1
$$

- Under non-differential misclassification, the numerator of (1) is

$$
\begin{aligned}
\theta_{10} & =\sum_{i, j \in\{0,1\}} \theta_{10 \mid i j} P\left(D_{1}=i, D_{2}=j \mid D_{1}^{*}=1, D_{2}^{*}=0\right) \\
& =\sum_{i, j \in\{0,1\}} \theta_{10 \mid i j} P\left(D_{1}=i \mid D_{1}^{*}=1\right) P\left(D_{2}=j \mid D_{2}^{*}=0\right)
\end{aligned}
$$

where

$$
p p=P\left(D_{1}=1 \mid D_{1}^{*}=1\right) \quad \text { and } \quad n p=P\left(D_{2}=0 \mid D_{2}^{*}=0\right)
$$

- Similarly for the denominator,

$$
\theta_{01}=\sum_{i, j \in\{0,1\}} \theta_{01 \mid i j} P\left(D_{1}=i \mid D_{1}^{*}=1\right) P\left(D_{2}=j \mid D_{2}^{*}=0\right)
$$

## Bias under outcome misclassification (continued)

- Using

$$
\begin{array}{ll}
\theta_{01 \mid 10}=\theta_{10 \mid 01}, & \theta_{01 \mid 01}=O R \theta_{10 \mid 01}  \tag{3}\\
\theta_{01 \mid 00}=\theta_{10 \mid 00}, & \theta_{01 \mid 11}=\theta_{10 \mid 11},
\end{array}
$$

manipulations yield

$$
\frac{\theta_{10}}{\theta_{01}}=O R \frac{1+\left(\frac{(1-n p)}{n p} a+\frac{(1-p p)}{p p} c\right)+\frac{(1-p p)(1-n p)}{p p n p} b}{1+O R\left(\frac{(1-n p)}{n p} a+\frac{(1-p p)}{p p} c\right)+O R^{2} \frac{(1-p p)(1-n p)}{p p n p} b}
$$

where

$$
a=\frac{\theta_{10 \mid 11}}{\theta_{10 \mid 10}}, \quad b=\frac{\theta_{10 \mid 01}}{\theta_{10 \mid 10}}, \quad c=\frac{\theta_{10 \mid 00}}{\theta_{10 \mid 10}} .
$$

- Therefore,

$$
\frac{\theta_{10}}{\theta_{01}} \leq O R \quad \text { if } \quad O R \geq 1 \quad \text { and } \quad \frac{\theta_{10}}{\theta_{01}}>O R \quad \text { if } \quad O R<1
$$

## A Bayesian model for matched studies under outcome misclassification

- Assuming independence between pairs,

$$
\left(n_{11}+n_{00}, n_{10}, n_{01}\right) \sim \text { Multinomial }\left(n,\left(\theta_{11}+\theta_{00}, \theta_{10}, \theta_{01}\right)\right)
$$

where

$$
\begin{aligned}
& \theta_{10}=p p n p \theta_{01 \mid 10} O R+(1-p p)(1-n p) \theta_{01 \mid 10}+p p(1-n p) \theta_{10 \mid 00}+(1-p p) n p \theta_{10 \mid 11} \\
& \theta_{01}=p p n p \theta_{01 \mid 10}+(1-p p)(1-n p) \theta_{01 \mid 10} O R+p p(1-n p) \theta_{10 \mid 00}+(1-p p) n p \theta_{10 \mid 11}
\end{aligned}
$$

- Taking the difference between cell probabilities,

$$
\theta_{10}-\theta_{01}=\theta_{01 \mid 10}(O R-1)(p p n p-(1-p p)(1-n p))
$$

- Problem is non-identifiable when $p p, n p$ or $\theta_{01 \mid 10}$ are unknown.
- Needed: prior input to inform prior distributions of $p p, n p$ and $\theta_{01 \mid 10}$.


## Prior distributions

- Six model parameters: $\left(p p, n p, O R, \theta_{01 \mid 10}, \theta_{01 \mid 00}, \theta_{01 \mid 11}\right)^{\prime}$
- Choose informed, independent priors for $p p, n p$ and $\theta_{01 \mid 10}$

$$
\begin{aligned}
p p & \sim \operatorname{Beta}\left(\alpha_{1}, \alpha_{2}\right) \\
n p & \sim \operatorname{Beta}\left(\beta_{1}, \beta_{2}\right) \\
\theta_{01 \mid 10} & \sim \operatorname{Beta}\left(\gamma_{1}, \gamma_{2}\right)
\end{aligned}
$$

- Determine $\alpha_{j}$ and $\beta_{j, j=1,2}$ from previous estimates $\widehat{p p}, \widehat{n p}$ and $s e(\widehat{p p})$, se( $\widehat{n p})$.
- Determine $\gamma_{j}$ from validation data via

$$
\begin{aligned}
m_{01} \mid \theta_{01 \mid 10} & \sim \operatorname{Bin}\left(n_{v a l}, \theta_{01 \mid 10}\right) \\
\theta_{01 \mid 10} & \sim \operatorname{Unif}(0,1)
\end{aligned}
$$

where $m_{01}$ is the number of case-control pairs with $\left(E_{1}=0, E_{2}=1\right)$.

- Implies $\gamma_{1}=m_{01}+1$ and $\gamma_{2}=n_{\text {val }}-m_{01}+1$.


## Prior distributions (continued)

Choose uniform priors for $O R, \theta_{01 \mid 00}$ and $\theta_{01 \mid 11}$ as

$$
\begin{gathered}
O R \mid p p, n p, \theta_{01 \mid 10} \sim \operatorname{Unif}\left(0, t_{1}\right) \\
\theta_{01 \mid 00} \mid O R, p p, n p, \theta_{01 \mid 10} \sim \operatorname{Unif}\left(0, t_{2}\right) \\
\theta_{01 \mid 11} \mid O R, p p, n p, \theta_{01 \mid 10}, \theta_{01 \mid 00} \sim \operatorname{Unif}\left(0, t_{3}\right)
\end{gathered}
$$

where

$$
\begin{aligned}
& t_{1}=\min \left(\frac{1}{\theta_{01 \mid 10}}, \frac{1}{\theta_{01 \mid 10}(p p n p+(1-p p)(1-n p))}-1\right) \\
& t_{2}=\min \left(1, \frac{1-(O R+1) \theta_{01 \mid 10}(p p n p+(1-p p)(1-n p))}{2 p p(1-n p)}\right) \\
& t_{3}=\min \left(1, \frac{1-(O R+1) \theta_{01 \mid 10}(p p n p+(1-p p)(1-n p))-2 p p(1-n p) \theta_{01 \mid 00}}{2(1-p p) n p}\right)
\end{aligned}
$$

to ensure that $\theta_{10}+\theta_{01} \leq 1$ and $\theta_{i j \mid / m} \leq 1$.

## Simulation study

- Generate
- $n$ apparent case-control pairs
- $n_{\text {val }}$ true case-control pairs, matched on a binary confounder $U$ with $D-E$ association $O R$.
- Evaluate

1. posterior median of $O R$,
2. length and coverage of $95 \%$ posterior credible interval of $O R$,
3. empirical size and power of the hypothesis test $H_{0}: O R=1$ for naive and proposed analysis.

- Examine different settings of
- disease-exposure association $O R$,
- cohort sizes $n$ and $n_{\text {val }}$,
- misclassification (SN, SP),
- prior uncertainty about $p p$ and $n p$.
- deviations of $\widehat{p p}, \widehat{n p}$ from true $p p, n p$


## Results: Median, length and coverage

Median of posterior distribution of $O R$, coverage and length of $95 \%$ posterior credible interval, averaged over 1000 runs:

|  |  | naive |  |  |  |  | adjusted |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $S N$ | $S P$ | median | coverage | length |  | median | coverage | length |  |
| 0.7 | 0.7 | 1.29 | 0.00 | 0.47 |  | 2.00 | 0.96 | 1.72 |  |
|  | 0.9 | 1.53 | 0.15 | 0.56 |  | 1.97 | 0.95 | 1.11 |  |
|  | 1.0 | 1.84 | 0.83 | 0.68 |  | 2.05 | 0.96 | 0.92 |  |
| 0.9 | 0.7 | 1.44 | 0.06 | 0.53 |  | 2.00 | 0.96 | 1.26 |  |
|  | 0.9 | 1.70 | 0.53 | 0.63 |  | 2.03 | 0.97 | 1.01 |  |
|  | 1.0 | 1.94 | 0.92 | 0.73 |  | 2.04 | 0.97 | 0.82 |  |
| 1 | 0.7 | 1.53 | 0.16 | 0.57 |  | 1.99 | 0.96 | 1.10 |  |
|  | 0.9 | 1.78 | 0.71 | 0.67 |  | 2.01 | 0.96 | 0.93 |  |
|  | 1.0 | 2.00 | 0.95 | 0.75 |  | 2.00 | 0.95 | 0.70 |  |

## Application - Morbidities in MS prodrome

- Estimate odds ratio of MS and presence of 14 morbidities in the prodromal phase.
- Study cohort of 7250 apparent case-control pairs.
- Determine presence of morbidities via case definitions of Marrie et al. [1].
- E.g. hypertension is considered prevalent if $\geq 4$ disease-related records within 2 years.
- Assume $n p=1$ and use $\widehat{p p}=0.83, \operatorname{se}(\widehat{p p})=0.02$ based on Marrie et al. [2] for prior input on pp.
- Validation cohort defined as subset with $\geq 20 \mathrm{MS}$-specific ICD codes $\left(n_{\text {val }}=929\right)$.


## Results



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## References

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