Vojta's Conjectures
September 28 to October 3, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

Sunday

16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30	Buffet Dinner, Sally Borden Building

Monday

7:00–8:45	Breakfast
9:15–9:30	**Introduction and Welcome by BIRS Station Manager, TCPL**
9:30–10:20	**Introductory lecture by Paul Vojta**
10:30–11:00	Coffee Break, TCPL
11:00–11:50	**Introductory lecture by Frédéric Campana**
12:00–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	**Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).**
14:30–15:00	Coffee Break, TCPL
15:00–15:50	**Introductory lecture by Aaron Levin**
16:00–17:00	**Open Problem Session**
17:30–19:30	Dinner

Tuesday

7:00–9:30	Breakfast
9:30–10:20	**Hector Pasten**
10:30–11:00	Coffee Break, TCPL
11:00–11:50	**Gordon Heier**
12:00–13:30	Lunch
13:30–14:20	**Amos Turchet**
14:30–15:00	Coffee Break, TCPL
15:00–15:50	**Khoa Nguyen**
16:00–16:50	**Jan-Hendrik Evertse**
17:30–19:30	Dinner

Wednesday

7:00–9:30	Breakfast
9:30–10:20	**Paul Vojta**
10:30–11:00	Coffee Break, TCPL
11:00–11:50	**William Cherry**
12:00–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:30	Breakfast
9:30–10:20	**Ekaterina Amerik**
10:30–11:00	Coffee Break, TCPL
11:00-11:50	**Min Ru**
12:00–13:30	Lunch
13:30–14:20	**Kálmán Győry**
14:30–15:00	Coffee Break, TCPL
15:00–15:50	**Yann Bugeaud**
16:00–16:50	**Felipe Voloch**
17:30–19:30	Dinner

Friday

7:00–9:30	Breakfast
9:30–10:20	**Pietro Corvaja**
10:30–11:00	Coffee Break, TCPL
11:00–11:50	**Julie Tzu-Yueh Wang**
12:00–13:30	Lunch

Checkout by 12 noon.

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Ekaterina Amerik** (Higher School of Economics)
Title: *Characteristic foliation on smooth divisors in holomorphically symplectic manifolds*
Abstract: This is a joint work with Frédéric Campana.

Let D be a smooth divisor in a holomorphically symplectic manifold X. Then D carries a rank-one foliation obtained as a kernel of the restriction of the symplectic form to D. It is called the characteristic foliation. Hwang and Viehweg have proved that when D is of general type, the foliation cannot be algebraic (unless in the trivial case when X is a surface). On the other hand, it is easy to see that the foliation is algebraic when D is uniruled. We explain what happens in the case when D is not uniruled. In particular, if X is an irreducible holomorphic symplectic manifold (that is, simply connected and such that the holomorphic symplectic form is unique up to a constant), we prove that the characteristic foliation is never algebraic in this case.

The main new ingredient for our results is the observation that the canonical bundle of the orbifold base of the family of leaves must be torsion. This implies, in particular, the isotriviality of the family of leaves.

Speaker: **Yann Bugeaud** (Université de Strasbourg)
Title: *Extensions of the Cugiani-Mahler theorem*
Abstract: In 1955, Roth established that if ξ is an irrational number such that there are a positive real number ϵ and infinitely many rational numbers p/q with $q \geq 1$ and $|\xi - p/q| < q^{-2-\epsilon}$, then ξ is transcendental. A few years later, Cugiani obtained the same conclusion with ϵ replaced by a function $q \mapsto \epsilon(q)$ that decreases very slowly to zero, provided that the sequence of rational solutions to $|\xi - p/q| < q^{-2-\epsilon(q)}$ is sufficiently dense, in a suitable sense. We give an alternative, and much simpler, proof of Cugiani's Theorem and extend it to simultaneous approximation.

Speaker: **William Cherry** (University of North Texas)
Title: *Value Distribution Theory of non-Archimedean Meromorphic Functions*
Abstract: I will survey the non-Archimedean analog of Nevanlinna's theory of value distribution initiated by Ha Huy Khoai, focussing on similarities to and differences from the complex case. I will also discuss the work of An, Levin, and Wang which introduces a Vojta like analogy between non-Archimedean analytic curves and integral points over the rationals and imaginary quadratic number fields. Finally, I will survey criteria for degeneracy of non-Archimedean analytic maps from the affine line into algebraic varieties and highlight an open problem whose solution might shed some light on the Green-Griffiths conjecture in complex geometry.

Speaker: **Pietro Corvaja** (University of Udine)
Title: *Integral points in orbits for a two-dimensional dynamical system*
Abstract: Let K be a number field, let $f : \mathbb{P}_1 \dashrightarrow \mathbb{P}_1$ be a nonconstant rational map of degree greater than 1, let S be a finite set of places of K, and suppose that $u, w \in \mathbb{P}_1(K)$ are not preperiodic under f. We prove that the set of $(m, n) \in \mathbb{N}^2$ such that $f^{\circ m}(u)$ is S-integral relative to $f^{\circ n}(w)$ is finite and effectively computable. This may be thought of as a two-parameter analog of a result of Silverman on integral points in orbits of rational maps.

This issue can be translated in terms of integral points on an open subset of $\mathbb{P}^2_1$; then one can apply a modern version of the method of Runge, after increasing the number of components at infinity by iterating the rational map. Alternatively, an ineffective result comes from a well-known theorem of Vojta.

This is a joint work with Thomas Tucker, Vijay Sookdeo and Umberto Zannier.

Speaker: **Jan-Hendrik Evertse** (Universiteit Leiden)
Title: *Results and open problems related to Schmidt's Subspace Theorem*
Abstract: Let $L_i = \sum_{i=1}^n \alpha_{ij} X_j$ $(i = 1 \dots n)$ be n linearly independent linear forms with algebraic coefficients (from $\mathbb{C}$), and let $c_1 \dots c_n$ be reals. Denote by $\|\mathbf{x}\|$ the maximum norm of $\mathbf{x} \in \mathbb{Z}^n$. Consider the system of inequalities

$$|L_i(\mathbf{x})| \le \|\mathbf{x}\|^{c_i} \ (i = 1 \dots n) \quad \text{in } \mathbf{x} \in \mathbb{Z}^n. \tag{1}$$

The following result is equivalent to Schmidt's Subspace Theorem from 1972:

Assume that $c_1 + \cdots + c_n < 0$. Then the set of solutions of (1) *is contained in finitely many proper linear subspaces of $\mathbb{Q}^n$.*

The proofs of the Subspace Theorem given so far are ineffective, in that they do not allow to compute the subspaces containing the solutions. But work of Vojta (1989), Schmidt (1993) and Faltings and Wüstholz (1994) implies that Schmidt's Subspace Theorem can be refined as follows:

Assumptions being as above, there is an effectively computable, proper linear subspace T^{exc} of $\mathbb{Q}^n$ such that (1) *has only finitely many solutions outside T^{exc}. Moreover, T^{exc} can be chosen from a finite collection independent of $c_1 \dots c_n$.*

Here, the method of proof does not allow to compute the solutions outside T^{exc}. But one can prove the following 'semi-effective' result. Assume that the coefficients of $L_1 \dots L_n$ have heights at most H, and that they generate a number field K of degree D. Further, let $c_1 + \cdots + c_n \le -\delta$ with $0 < \delta < 1$, and $\max(c_1 \dots c_n) = 1$. Then one can show that for the solutions $\mathbf{x} \in \mathbb{Z}^n$ of (1) outside T^{exc} one has

$$\|\mathbf{x}\| \le \max\left(B^{\text{ineff}}(n, K, \delta),\ H^{c^{\text{eff}}(n,D,\delta)}\right),$$

where c^{eff} is effectively computable, B^{ineff} not effectively computable from the method of proof, and both constants depend only on the parameters between the parentheses.

We would like to pose as an open problem to replace the above bound by one in which B^{ineff} depends on D instead of K. This would have various interesting consequences, of which we would like to mention a few during the talk.

Speaker: **Kálmán Győry** (University of Debrecen)
Title: *Effective results for Diophantine equations over finitely generated domains*
Abstract: This will be a survey of some recent results, partly obtained jointly with A.Berczes and J.H.Evertse on various classes of Diophantine equations, including unit equations and some of their generalizations, Thue equations, superelliptic equations with unknown exponents, discriminant equations and their applications.The results make effective several ineffective results of Lang and others, and generalize many earlier results established over number fields.

Speaker: **Gordon Heier** (University of Houston)
Title: *A generalization of the Schmidt subspace theorem*
Abstract: We will discuss a generalization of the Schmidt subspace theorem (and Cartan's Second Main Theorem) in the setting of not numerically equivalent divisors. This is joint work with Aaron Levin.

Speaker: **Khoa Nguyen** (UBC and PIMS)
Title: *Primitive and doubly primitive divisors in dynamical sequences*
Abstract: Let K be a number field or a function field of characteristic 0, let $\varphi(z) \in K(z)$ having degree at least 2 and let $\alpha \in K$ such that the orbit $\{\varphi^n(\alpha)\}_{n \ge 0}$ is infinite. Consider the question: (A) except trivial counter-examples, is it true that for all sufficiently large n, the element $\varphi^n(\alpha)$ has a prime divisor $\mathfrak{p}$ that is not a divisor of $\varphi^k(\alpha)$ for every $k < n$. Ingram and Silverman are the first to consider this question in

such generality. They even go further and ask: (B) except trivial counter-examples, is it true that for all $m \geq 0$ and $n > 0$ such that $m+n$ is sufficiently large, the element $\varphi^{m+n}(\alpha) - \varphi^m(\alpha)$ has a prime divisor $\mathfrak{p}$ that is not a divisor of any $\varphi^{M+N}(\alpha) - \varphi^M(\alpha)$ for $M < n$ or $N < n$. Later on, Faber and Granville modify question (B) somewhat and provide certain evidence towards it.

In this talk, we explain how the *ABC* Conjecture implies that both questions have an affirmative answer. In the function field case our result is unconditional; when using a deep result of Yamanoi (previously conjectured by Vojta), we can show that $\mathfrak{p}$ appears with multiplicity 1. This is joint work with Chad Gratton and Tom Tucker for Question (A), and with Dragos Ghioca and Tom Tucker for Question (B).

Speaker: **Hector Pasten** (Harvard University)
Title: *On the radical of polynomials values*
Abstract: It is known that the abc conjecture gives a good lower bound for the radical of $F(n)$ in terms of n, for any fixed polynomial F without repeated factors. In this talk I will explain related results where one is interested in lower bounds in terms of F rather than n. This leads to new applications of the abc conjecture, such as counting square free values of polynomials at prime arguments (which also uses results of Green, Tao and Ziegler about arithmetic progressions of primes), and some consequences in undecidability questions (related to Hilbert's tenth problem).

Speaker: **Min Ru** (University of Houston)
Title: *Quantitative Geometric and Arithmetic Results for Complement of Divisors*
Abstract: In this talk, I will introduce, for an effective divisor D on a smooth projective variety X, the notion of Nevanlinna constant. We then prove a quantitative result, in terms of the Nevanlinna constant of D, which extends Schmidt's subspace thereom to D in X. The counter-part in Nevanlinna theory is also obtained. The result recovers the recent important results in this direction, including the results of Corvaja-Zannier, Evertse-Ferretti, Levin, Ru etc., as well as derives new results. The notion of "Nevanlinna constant of D" gives a unified description of the quantitative geometric and arithmetic properties of (X, D).

Speaker: **Amos Turchet** (Chalmers University of Technology and University of Gothenburg)
Title: *Geometric Lang-Vojta conjecture in the projective plane*
Abstract: We present a proof of the function field version of Lang-Vojta Conjecture on algebraic hyperbolicity for complements of very generic quartics in the projective plane with at most normal crossing singularities. The proof relies on a deformation argument applied to the known case for three components divisors (proved by Corvaja and Zannier) and uses a reformulation of the problem via moduli spaces of logarithmic stable maps as introduced by Q. Chen and Abramovich.

Speaker: **Paul Vojta** (University of California, Berkeley)
Title: *Toric geometry and Dyson's lemma*
Abstract: In 1989, I proved a Dyson lemma for products of two smooth projective curves of arbitrary genus. In 1995, M. Nakamaye extended this to a result for a product of an arbitrary number of smooth projective curves of arbitrary genus, in a formulation involving an additional "perturbation divisor." In 1998, he also found an example in which a hoped-for Dyson lemma is false without such a perturbation divisor. This talk will present work in progress on eliminating the perturbation divisor by using a different definition of "volume" at the points under consideration. The proof involves toric and toroidal geometry, and this is reflected in the statement as well.

Speaker: **Felipe Voloch** (University of Texas at Austin)
Title: *Unboundedness of the number of rational points on curves*
Abstract: We will discuss some results and speculations about the number of rational points on curves of genus bigger than one over global fields. In particular, we will describe the result obtained jointly with R. Concei and D. Ulmer, where we show that the number of rational points on (non-isotrivial) curves of fixed genus over a fixed function field can be arbitrarily large.

Speaker: **Julie Tzu-Yueh Wang** (Academia Sinica)
Title: *On the exponential local-global principle for meromorphic functions and algebraic functions*
Abstract: We prove the rank one case of Skolems Conjecture on the exponential local-global principle for algebraic functions and discuss its analog for meromorphic functions.