



# Banff International Research Station

for Mathematical Innovation and Discovery

## Cohomological Realizations of Motives December 7 – December 12, 2014

### MEALS

\*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

\*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

\*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

**\*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

### MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

### SCHEDULE

#### Sunday

**16:00** Check-in begins (Front Desk - Professional Development Centre - open 24 hours)

**17:30–19:30** Buffet Dinner, Sally Borden Building

**20:00** Informal gathering in 2nd floor lounge, Corbett Hall

Beverages and a small assortment of snacks are available on a cash honor system.

#### Monday

**7:00–8:45** Breakfast

**8:45–9:00** Introduction and Welcome by BIRS Station Manager, TCPL

**9:00–10:00** **Bruno Kahn:** *On the generalised Hodge and Tate conjectures for products of elliptic curves*

**10:00–10:30** Coffee Break, TCPL

**10:30–11:30** **Susama Agarwala:** *Graphical Motives*

**11:30–13:00** Lunch

**13:00–14:00** Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall

**14:00–15:00** **Masanori Asakura:** *Period and regulator for fibration with CM structure and hypergeometric functions*

**15:00–15:30** Coffee Break, TCPL

**15:30–16:30** **Pablo Pelaez:** *Filtering the Chow groups via motivic homotopy theory*

**17:30–19:30** Dinner

## Tuesday

- 7:00–9:00 Breakfast  
9:00–10:00 **Philippe Gille:** *New homogeneous spaces with a 0-cycle of degree one and without rational points*  
10:00–10:30 Coffee Break, TCPL  
10:30–11:30 **Gonçalo Tabuada:** *Non-commutative motives of separable algebras*  
11:30–11:45 Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).  
11:45–13:30 Lunch  
14:00–15:00 **Mao Sheng:** *Higgs-de Rham flow in positive and mixed characteristic*  
15:00–15:30 Coffee Break, TCPL  
15:30–16:30 **Charles Doran:** *From elliptic surfaces to Calabi-Yau moduli via Euler transform*  
17:30–19:30 Dinner

## Wednesday

- 7:00–9:00 Breakfast  
9:00–10:00 **Ravindra Girivaru:** *An infinitesimal Weak Lefschetz theorem for Chow groups*  
10:00–10:30 Coffee Break, TCPL  
10:30–11:30 **Matilde Lalin:** *Mahler measure and elliptic curve  $L$ -functions at  $s = 3$*   
11:30–13:30 Lunch  
Free Afternoon  
17:30–19:30 Dinner

## Thursday

- 7:00–9:00 Breakfast  
9:00–10:00 **Greg Pearlstein:** *Principal VHS arising from mirror symmetry and middle convolution*  
10:00–10:30 Coffee Break, TCPL  
10:30–11:30 **Roy Joshua:** *Generalized  $t$ -structures and étale realizations of motives*  
11:30–13:30 Lunch  
14:00–15:00 **Jaya Iyer:** *Degeneration of Gross-Schoen algebraic cycle*  
15:00–15:30 Coffee Break, TCPL  
15:30–16:30 **Matt Kerr:** *The simplicial Abel-Jacobi map*  
17:30–19:30 Dinner

## Friday

- 7:00–9:00 Breakfast  
9:00–10:00 **Nikita Karpenko:** *Incompressibility of products of projective homogeneous varieties*  
10:00–10:15 Coffee Break, TCPL  
10:15–11:15 **Stefan Gille:** *Milnor-Witt groups over local rings*  
11:30–13:30 Lunch  
Checkout by  
12 noon.

\*\* 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. \*\*



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### ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: **Agarwala, Susama** (University of Oxford)

Title: *Graphical Motives*

Abstract: In this talk I introduce a graphical representation of a subalgebra of algebraic cycles that themselves can be represented graphically. These correspond to Mixed Tate Motives. The graphical point of view gives great insight into calculating admissibility and decomposability of these cycles. It also gives insight into the cohomology of the bar construction. Time permitting, I will make some comments about the periods associated to a class of these cycles.

Speaker: **Asakura, Masanori** (Hokkaido University)

Title: *Period and regulator for fibration with CM structure and hypergeometric functions*

Abstract: We discuss the period and regulator for a fibration  $X$  onto  $\mathbb{P}^1$  with certain CM structure on the generic fiber. We will first show that the period of  $H^2(X)$  is a product of Gamma values. This gives a special case of the period conjecture of Deligne-Gross. Secondly, we show that the regulator of certain motivic elements are written in terms of special values of the hypergeometric function  ${}_3F_2$  and the digamma function. This is joint work with N. Otsubo.

Speaker: **Doran, Charles** (University of Alberta)

Title: *From elliptic surfaces to Calabi-Yau moduli via Euler transform*

Abstract: Starting from extremal elliptic surfaces, we construct a large class of one-parameter families of K3-surface fibered Calabi-Yau threefolds together with an explicit description of their periods. By a quadratic twist we construct models for moduli spaces of lattice polarized K3 surfaces of high Picard-rank such that their multi-parameter K3 periods can be computed explicitly. Restricting to sub-loci and carrying out another quadratic twist one obtains the desired one-parameter families of Calabi-Yau threefolds. The period computation makes essential use of a generalization of the classical Euler transform for the hypergeometric function. All symplectically rigid rank four differential operators of Calabi-Yau type are realized in this way. This is joint work with Andreas Malmendier.

Speaker: **Gille, Philippe** (IMAR Bucharest and CNRS)

Title: *New homogeneous spaces with a 0-cycle of degree one and without rational points*

Abstract: This is a report on joint work with C. Beli (Bucharest) and T.-Y. Lee (Lausanne). Given a connected affine algebraic  $G$  defined over a field  $k$ , B. Totaro raised the question whether a homogeneous space  $X$  under  $G$  having a 0-cycle of degree one has a rational point, that is  $X(k) \neq \emptyset$ . The question is widely open for principal homogeneous spaces and there are counterexamples by Florence and Parimala respectively with finite stabilizers and parabolic stabilizers. The goal of this talk is to present a new class of homogeneous spaces without rational points but with quadratic and cubic points. Those homogeneous spaces are geometrically isomorphic to  $G/T$ , that is the quotient of a group  $G$  of type  $G_2$  by a maximal torus.

Speaker: **Gille, Stefan** (University of Alberta)

Title: *Milnor-Witt groups over local rings*

Abstract: The Milnor-Witt group of a field arises in  $\mathbb{A}^1$ -homotopy theory and has a nice presentation found by Morel and Hopkins. In this talk I will discuss a possible definition of Milnor-Witt groups and the analog of the Morel-Hopkins theorem for “nice” local rings.

Speaker: **Girivaru, Ravindra** (University of Missouri–St. Louis)

Title: *An infinitesimal Weak Lefschetz theorem for Chow groups*

Abstract: Given a smooth, projective variety  $X$ , and a smooth, ample hyperplane section  $Y$  in  $X$ , the conjectures of Bloch and Beilinson imply that the natural restriction map of rational Chow groups  $CH^p(X)_{\mathbb{Q}} \rightarrow CH^p(Y)_{\mathbb{Q}}$  is an isomorphism for  $p < \dim(Y)/2$ . This conjecture has been settled only in the case  $p = 1$ , in which case this is true even integrally and goes by the name of the Grothendieck-Lefschetz theorem. Following Grothendieck’s proof of this theorem, we can break up the above conjecture into an “infinitesimal” part and an “algebraisation” part. We will sketch a proof of the infinitesimal part. This is joint work with Deepam Patel.

Speaker: **Iyer, Jaya** (Institute of Mathematical Sciences)

Title: *Degeneration of Gross-Schoen algebraic cycle*

Abstract: In this talk, we will concretely describe the degeneration of the Gross-Schoen cycle when the curve is nodal, as a higher Chow cycle.

Speaker: **Joshua, Roy** (Ohio State University)

Title: *Generalized  $t$ -structures and étale realizations of motives*

Abstract:  $t$ -structures were originally introduced by Beilinson, Bernstein and Deligne to define and study perverse sheaves and soon afterwards by Ekedahl to study crystalline cohomology problems. Since then, they have appeared in other contexts, for example in the construction of suitable categories of mixed Tate-motives by Deligne, Bloch and Kriz. They also appear prominently in certain conjectures on algebraic cycles where a formalism similar to  $\ell$ -adic derived categories for algebraic cycles is formulated.

In this talk we utilize the techniques of pre-aisles and aisles introduced by Keller and Vossieck to provide a painless way to define and study (generalized)  $t$ -structures for many of the above contexts. In particular, we show how to define  $t$ -structures on motivic derived categories that are compatible with étale realization.

Speaker: **Kahn, Bruno** (CNRS)

Title: *On the generalised Hodge and Tate conjectures for products of elliptic curves*

Abstract: The Hodge conjecture is true for products of elliptic curves over  $\mathbb{C}$ , and the Tate conjecture is true for products of elliptic curves over finite fields. How far can one go when considering the generalised Hodge or Tate conjecture for such products? I will explain that it holds for products of elliptic curves “in general position” (to be defined). The first case of not general position is that of 4 non-isogenous CM (resp. ordinary) elliptic curves whose fields of endomorphisms are not linearly disjoint over  $\mathbb{Q}$ . Then a new, 4-dimensional simple Abelian variety enters the picture.

Speaker: **Karpenko, Nikita** (University of Alberta)

Title: *Incompressibility of products of projective homogeneous varieties*

Abstract: For a given prime number  $p$ , we provide a list of projective homogeneous varieties  $X$  such that the product  $X \times Y$  by an arbitrary projective homogeneous variety  $Y$  is  $p$ -incompressible if and only if the varieties  $X_{F(Y)}$  and  $Y_{F(X)}$  are so. The proofs are based on properties of their Chow motives with coefficients in  $\mathbb{Z}/p\mathbb{Z}$ . Some of the results are applied to computation of the essential dimension of representations of finite groups.

Speaker: **Kerr, Matt** (Washington University in St. Louis)

Title: *The simplicial Abel-Jacobi map*

Abstract: We describe an explicit morphism of complexes that induces the cycle-class maps from (simplicially described) higher Chow groups to Deligne cohomology, and give a couple of applications of the construction. This is joint work with James Lewis and Patrick Lopatto.

Speaker: **Lalin, Matilde** (Université de Montréal)

Title: *Mahler measure and elliptic curve  $L$ -functions at  $s = 3$*

Abstract: The Mahler measure of a Laurent polynomial  $P$  is defined as the integral of  $\log|P|$  over the unit torus with respect to the Haar measure. For multivariate polynomials, it often yields special values of  $L$ -functions. In this talk I will discuss some of these relationships and current developments, and present some results involving  $L(E, 3)$  for  $E$  an elliptic curve.

Speaker: **Pearlstein, Greg** (Texas A&M University)

Title: *Principal VHS arising from mirror symmetry and middle convolution*

Abstract: We present evidence in support of a conjecture of Griffiths, Green and Kerr on the arithmetic of extension classes of limiting mixed Hodge structures arising from semistable degenerations over a number field. A crucial role is played by the Mumford-Tate group (of type  $G_2$ ) of the family of 6-folds, and the theory of boundary components of Mumford-Tate domains.

Speaker: **Pelaez, Pablo** (UNAM)

Title: *Filtering the Chow groups via motivic homotopy theory*

Abstract: The goal of this talk is to introduce a finite filtration on the Chow groups and on motivic cohomology using techniques of motivic homotopy theory.

Speaker: **Sheng, Mao** (University of Science and Technology of China)

Title: *Higgs-de Rham flow in positive and mixed characteristic*

Abstract: In this talk, I would like to advertise the notion “Higgs-de Rham flow” both in characteristic  $p$  and in mixed characteristic, introduced in the paper “Semistable Higgs bundles, periodic Higgs bundles and representations of algebraic fundamental groups” by Guitang Lan, Kang Zuo and myself. There are two significant applications of this notion so far. One is the construction of representations of étale fundamental groups from semistable Higgs bundles of trivial Chern classes over  $p$ -adic varieties, obtained in the paper. The other is a characteristic  $p$  proof of Bogomolov-Gieseker’s inequality for semistable Higgs bundles and Miyaoka-Yau’s Chern number inequalities for algebraic surfaces, obtained by Adrian Langer.

Speaker: **Tabuada, Gonçalo** (MIT)

Title: *Non-commutative motives of separable algebras*

Abstract: I will describe a simple and explicit model of the category of non-commutative (=NC) motives of separable algebras  $\text{Sep}(k)$  over a field  $k$ . Making use of it, I will obtain a complete dictionary between direct sums of NC motives of central simple algebras (=CSA) and sequences of elements in the Brauer group of  $k$ . Among other applications, I will establish two families of motivic relations between CSA which hold for every additive invariant (e.g., algebraic  $K$ -theory, cyclic homology, and topological Hochschild homology) and compute the additive invariants of twisted flag varieties using solely the Brauer classes of the corresponding CSA. This is joint work with Michel van den Bergh.