

Positivity of Linear Series and Vector Bundles

February 2-7, 2014

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

***Please remember to scan your meal card at the host/hostess station in the dining room for each meal.**

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

Morning lectures on all days except Wednesday will be held in Room 202, afternoon lectures and Wednesday morning lectures will be held in Room 201.

SCHEDULE

5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon.

Sunday

16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30 Buffet Dinner, Sally Borden Building
20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and a small assortment of snacks are available on a cash honor system.

Monday	
7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:30	Greg Smith , <i>Toric vector bundles</i> , Room 202
9:30–9:45	Break
9:45–10:15	Kelly Jabbusch , <i>Parliaments of polytopes</i> , Room 202
10:15–10:45	Coffee Break
10:45–11:15	Sandra Di Rocco , <i>Positivity of toric vector bundles</i> , Room 202
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors).
14:15–14:45	Mihnea Popa , <i>On higher direct images of pluricanonical bundles</i> , Room 201
14:45–15:15	Coffee Break
15:15–15:45	Jack Huizenga , <i>Higher rank interpolation problems and the birational geometry of moduli spaces of sheaves</i> , Room 201
15:45–16:00	Break
16:00–16:30	Yusuf Mustopa , <i>Ulrich Sheaves and Higher-Rank Brill-Noether Theory</i> , Room 201
16:30–16:45	Break
16:45–17:15	Arend Bayer , <i>Nef divisors via Bridgeland stability conditions</i> , Room 201
17:30–19:30	Dinner

Tuesday	
7:00–9:00	Breakfast
9:00–9:30	Stefano Urbinati , <i>Positivity for Weil divisors (part 1)</i> , Room 202
9:30–9:45	Break
9:45–10:15	Alberto Chieccio , <i>Positivity for Weil divisors (part 2)</i> , Room 202
10:15–10:45	Coffee Break
10:45–11:15	Ernesto Mistretta , <i>Asymptotic Base Loci for Vector Bundles</i> , Room 202
11:30–13:30	Lunch
13:30–14:30	Klaus Altmann , <i>Okounkov bodies and versal deformations of toric singularities</i> , Room 201, [Joint session with parallel program]
14:30–14:45	Coffee Break
14:45–15:15	Giuseppe Pareschi , <i>Gaussian maps, generic vanishing, and Fourier-Mukai transforms</i> , Room 201
15:15–15:30	Coffee Break
15:30–16:00	Karol Palka , <i>On the log minimal model program for surfaces with fractional boundary and its applications</i> , Room 201
16:00–16:15	Break
16:15–16:45	Sándor Kovács , <i>Singularities of low degree complete intersections</i> , Room 201
17:30–19:30	Dinner

Wednesday

7:00–9:00 Breakfast

Joint session with parallel program

9:00–9:30 **Alex Küronya**, *Local positivity in convex geometric terms*, Room 201

9:30–9:45 Break

9:45–10:15 **Xin Zhou**, *Asymptotic Schur decomposition of Veronese syzygy functors*, Room 201

10:15–10:45 Coffee Break

10:45–11:15 **Tomasz Szemberg**, *Minkowski decomposition of Okounkov bodies on surfaces*, Room 201

11:30–13:30 Lunch

Free Afternoon

17:30–19:30 Dinner

Thursday

7:00–9:00 Breakfast

9:00–9:30 **David Schmitz**, *Minkowski decomposition of Newton-Okounkov bodies for toric varieties*, Room 202

9:30–9:45 Break

9:45–10:15 **Chih-Chi Chou**, *TBA*, Room 202

10:15–10:45 Coffee Break

10:30–11:30 **Michel Brion**, *On linearization of line bundles*, Room 201 [Joint session with parallel program]

11:30–13:30 Lunch

13:30–14:00 **Daniel Greb**, *Construction and Variation of moduli spaces of vector bundles on higher-dimensional base manifolds*, Room 201

14:00–14:15 Break

14:15–14:45 **Brian Harbourne**, *Ascenzi's Fundamental Bundle Splitting Result and Related Open Problems and Conjectures*, Room 201

14:45–15:15 Coffee Break

15:15–15:45 **Joaquim Roé**, *Very general monomial valuations of \mathbb{P}^2 and a Nagata type conjecture*, Room 201

15:45–16:00 Break

16:00–16:30 **Aaron Bertram**, $\chi = 0$ moduli, Room 201

17:30–19:30 Dinner

Friday

7:00–9:00 Breakfast

9:00–10:00 Discussion

10:00–10:30 Coffee Break

10:30–11:30 Discussion

11:30–13:30 Lunch

12:00 noon **Checkout by 12 noon**

Name of 2013 5-day Workshop
Date of 2013 5-day Workshop

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Bayer, Arend** (University of Edinburgh)

Title: *Nef divisors via Bridgeland stability conditions*

Abstract: I will explain a construction of a canonically defined nef divisor on any moduli space of Bridgeland-stable objects (based on joint work with Emanuele Macri). It has been used to study birational geometry of moduli space of sheaves on K3 surfaces in our own work, and by others for P^2 and rational surfaces.

Speaker: **Bertram, Aaron** (University of Utah)

Title: $\chi = 0$ *moduli*

Abstract:

Speaker: **Chiechio, Alberto** (University of Washington)

Title: *Positivity for Weil divisors (part 2)*

Abstract: With the notion of positivity defined in Part 1, we prove vanishing and non-vanishing theorems for cohomology, global generations statements, and a result related to log Fano.

Speaker: **Chou, Chih-Chi** (University of Illinois at Chicago)

Title:

Abstract:

Speaker: **Di Rocco, Sandra** (KTH Stockholm)

Title: *Positivity of toric vector bundles*

Abstract: In this talk, we will examine some more refined positivity properties of toric vector bundles such as generation of higher order jets. This will lead to some alternative characterizations of very ample toric vector bundles.

Speaker: **Greb, Daniel** (Ruhr-Universitaet Bochum)

Title: *Construction and Variation of moduli spaces of vector bundles on higher-dimensional base manifolds*

Abstract: While the variation of moduli spaces of H-slope/Gieseker-semistable sheaves on surfaces under change of the ample polarisation H is well-understood, research on the corresponding question in the case of higher-dimensional base manifolds revealed a number of pathologies. After presenting these, I will discuss recent joint work with Matei Toma (Nancy) and Julius Ross (Cambridge) which resolves some of these pathologies by looking at curves instead of divisors, and by embedding the moduli problem for sheaves into a moduli problem for quiver representations.

Speaker: **Harbourne, Brian** (University of Nebraska-Lincoln)

Title: *Ascenzi's Fundamental Bundle Splitting Result and Related Open Problems and Conjectures*

Abstract: Consider a closed curve $C \subseteq \mathbf{P}^2$ of degree $d = \deg(C)$ with a birational morphism $\phi : \mathbf{P}^1 \rightarrow C$ such that C has singularities only at general points $p_1, \dots, p_r \in \mathbf{P}^2$. Assume that C has multiplicity m_i at p_i , indexed so that $m_1 \geq \dots \geq m_r \geq 0$, and let $d = \deg(C)$. A fundamental result of M.G. Ascenzi determines the splitting of the rank 2 bundle $\phi^*(\Omega_{\mathbf{P}^2})$ when $2m_1 + 1 \geq d$. Ascenzi's result has been rediscovered a number of times (e.g., by Fitchett, 1997, related to resolutions of ideals of fat points, and by Song, Chen and Goldman, 2007, in work related to computer aided design) and reproved in these and

other works (such as by Cox, Kustin, Polini and Ulrich, 2013), all without being aware of Ascenzi’s original work in the mid-1980s.

When $2m_1 + 1 < d$, it is an open problem in general to determine the splitting of the bundle $\phi^*(\Omega_{\mathbf{P}^2})$, or even to determine when the splitting is unbalanced. Work on these problems is related to other open problems of interest in algebraic geometry, and has led to additional open problems and conjectures, which I will discuss.

Speaker: **Huizenga, Jack** (University of Illinois at Chicago)

Title: *Higher rank interpolation problems and the birational geometry of moduli spaces of sheaves*

Abstract: Questions like the Nagata conjecture seek to determine when certain zero-dimensional schemes impose independent conditions on sections of a line bundle on a surface. Understanding analogous questions for vector bundles instead amounts to studying the birational geometry of moduli spaces of sheaves on a surface. We explain how to use higher-rank interpolation problems to compute the cone of effective divisors on any moduli space of sheaves on the plane. This is joint work with Izzet Coskun

Speaker: **Jabbusch, Kelly** (University of Michigan - Dearborn)

Title: *Parliaments of polytopes*

Abstract: We will develop an explicit dictionary between toric vector bundles and certain collections of polytopes. Using this equivalence, we will compare and contrast various properties, such as ample, nef, and globally generated, for toric vector bundles over a smooth complete toric variety

Speaker: **Kovács, Sándor** (University of Washington)

Title: *Singularities of low degree complete intersections*

Abstract: I will show how a simple condition for Du Bois singularities may be used to prove that low degree hypersurfaces with small singular sets must have log canonical singularities.

Speaker: **Küronya, Alex** (Budapest University of Technology and Economics)

Title: *Local positivity in convex geometric terms*

Abstract: The close ties between Newton-Okounkov bodies and invariants of line bundles have long been known; the purpose of this talk is to concentrate on positivity questions. Following the principle that local positivity of a line bundle should be determined by the collection of Newton-Okounkov bodies centered at given points, we describe a new ampleness criterion in terms of convex geometry. This is an account of joint work with Victor Lozovanu.

Speaker: **Mistretta, Ernesto Carlo** (University of Padova)

Title: *Asymptotic Base Loci for Vector Bundles*

Abstract: We generalize some asymptotic base loci definitions and properties to vector bundles. In particular we define the stable, augmented, and restricted base loci, and we relate them to some positivity properties that have been defined for vector bundles such as bigness, strong bigness, Viehweg weak positivity. We construct a Itaka fibration as well, which is very much related to the Itaka fibration of the determinant of the vector bundle. A lot of this is still work in progress, joint with Bauer, Küronya, Kovacs, Szemberg, Urbinati.

Speaker: **Mustopa, Yusuf** (Northeastern University)

Title: *Ulrich Sheaves and Higher-Rank Brill-Noether Theory*

Abstract: Ulrich sheaves, which are in a sense the "nicest" ACM sheaves, occur naturally in a wide variety of topics, ranging from Brauer groups to Boij-Soderberg theory. The question of whether a given subvariety of projective space admits an Ulrich sheaf is quite hard in general, and in the case of surfaces an affirmative answer usually solves a Brill-Noether problem for linear series on a special curve. In this talk, I will report on a result obtained jointly with Rajesh Kulkarni and Ian Shipman which shows that the existence of an Ulrich sheaf on a variety of any dimension is equivalent to the solution of a Brill-Noether problem for vector bundles on special curves.

Speaker: **Palka, Karol** (Polish Academy of Sciences)

Title: *On the log Minimal Model Program for surfaces with fractional boundary and its applications*

Abstract: If we want to study a quasiprojective surface Y , it is natural to take its completion (X, D) , where X is projective, $X \setminus D = Y$ and D is a reduced snc-divisor, the curve 'at infinity', and to study the log minimal model program run for the pair (X, D) . This theory of 'open surfaces' has been developed by Miyanishi, Fujita and others for more than 40 years and turned out to be very successful. Now the program itself, although without an obvious geometric interpretation, works equally well when D is a \mathbb{Q} -divisor. Recently we were able to adapt it to obtain progress in the classical subject of classification of planar rational cuspidal curves. The positivity of the log-canonical bundle plays a major role in bounding the length of the minimalization process.

Speaker: **Pareschi, Giuseppe** (Universita di Roma "Tor Vergata")

Title: *Gaussian maps, generic vanishing, and Fourier-Mukai transforms*

Abstract: I will describe the relationship between gaussian (Wahl) maps and vanishing of higher cohomology, together with some applications to generic vanishing.

Speaker: **Popa, Mihnea** (University of Illinois at Chicago)

Title: *On higher direct images of pluricanonical bundles*

Abstract: I will make a few remarks on regularity results for higher direct images of powers of canonical bundles via projective morphisms between smooth varieties. By focusing methods of Viehweg and Kollár on regularity rather than weak positivity, these lead to some more precise versions of well-known positivity statements of Viehweg. In the special case when the base is an abelian variety, I will explain how these results imply that direct images of pluricanonical bundles satisfy generic vanishing (and why this is somewhat surprising).

Speaker: **Roé, Joaquim** (Universitat Autònoma de Barcelona)

Title: *Very general monomial valuations of \mathbb{P}^2 and a Nagata type conjecture*

Abstract: It is well known that multi-point Seshadri constants for a small number s of points in the projective plane are submaximal. It is predicted by the Nagata conjecture that their values are maximal for $s \geq 9$ points. Tackling the problem in the language of valuations one can make sense of s points for any positive real $s \geq 1$. We show somewhat surprisingly that a Nagata-type conjecture should be valid for $s \geq 8 + \frac{1}{36}$ points and we compute explicitly all Seshadri constants for $s \leq 7 + \frac{1}{9}$.

Speaker: **Schmitz, David** (Phillips-Universität-Marburg)

Title: *Minkowski decomposition of Newton-Okounkov bodies for toric varieties*

Abstract: I will report on how to generalize the idea of decomposing Newton-Okounkov bodies for surfaces presented by T. Szemberg to higher dimensions. It turns out that finding a Minkowski base for a toric variety with respect to a torus-invariant flag recovers the structure of the secondary fan. I will describe an algorithm that can be used to obtain such a base. This is joint work with Piotr Pokora and Stefano Urbinati.

Speaker: **Smith, Gregory** (Queen's University)

Title: *Toric vector bundles*

Abstract: We will motivate and define the category of toric vector bundles over smooth complete toric varieties. After highlighting some applications and potential applications, we will concentrate on extending various properties of line bundles on smooth toric varieties to higher-rank toric vector bundles.

Speaker: **Szemberg, Tomasz** (Pedagogical University of Cracow)

Title: *Minkowski decomposition of Okounkov bodies on surfaces*

Abstract: I will report on decomposing Okounkov bodies on surfaces with rational polyhedral effective cone into Minkowski sums of some elementary "building bricks". This report builds upon recent work of Patrycja Łuszcz-Świdecka and David Schmitz (arXiv:1304.4246).

Speaker: **Urbinati, Stefano** (Università degli Studi di Padova)

Title: *Positivity for Weil divisors (part 1)*

Abstract: We define a notion of positivity for Weil divisors inspired by the behaviour on toric varieties. We compare this definition with another notion coming from b-divisor and we explain the relations between the two.