Geometric Aspects of Semilinear Elliptic and Parabolic Equations: Recent Advances and Future Perspectives
Organizers: Manuel del Pino, Peter Polacik, Juncheng Wei

Final Schedule of Invited Talks

May 26th, 2014

8:45-9:00
Welcome Talk by BIRS

9:00-9:40
Prof. R. Mazzeo (Stanford Univ.)

9:45-10:25
Prof. N. Kapouleas (Brown Univ.)

10:25-10:45
Coffee Break

10:45-11:25
Prof. R. McCann (Univ. Toronto)

11:30-12:10
Prof. S. Angenent (Univ. Wisconsin)

12:10-2:00
Lunch

1:00-2:00pm
Tour of Banff Centre

2:00-2:10pm
Group Photo Session

2.10-2:50
Prof. Yanyan Li (Rutgers Univ.)

2:55-3:35
Prof. P. Daskalopoulos (Columbia Univ.)

3:35-4:00
Coffee Break

4:00-4:40
Prof. R. Jerrard (Univ. Toronto)

4:45-5:25
Prof. F. Hamel (Univ. Marseille)

5:30-6:10
Prof. J. King (Univ. Nottingham)

Dinner

May 27th, 2014

8:50-9:30
Prof. Chang-Shou Lin (National Taiwan Univ.)

9:35-10:15
Prof. X. Cabre (Univ. Catalunya)

10:15-10:45
Coffee Break

10:45-11:25
Prof. E. Valdinoci (Weierstrass Institute)

11:30-12:10
Prof. M. Fila (Comenius Univ.)

12:10-2:00
Lunch

2:00-2:40
Prof. P. Quittner (Comenius Univ.)

2:45-3:25
Prof. S. Terracini (Univ. Torino)

3:25-3:50
Coffee Break

3:50-4:30
Prof. Yong Liu (North China Electric Power Univ.)

4:35-5:15
Prof. Z-Q Wang (Utah State Univ.)

5:20-6:00
Prof. E. Yanagida (Tokyo Inst. Technology)

Dinner

May 28th, 2014

8:50-9:30
Prof. N. Sesum (Rutgers Univ.)

9:35-10:15
Prof. C. Gui (Univ. Connect.)

10:15-10:45
Coffee Break

10:45-11:25
Prof. P. Souplet (Univ. Paris 13)

11:30-12:10
Prof. H. Ninomiya (Meiji Univ.)

Lunch
Free Afternoon

August 29th, 2014

8:50-9:30
Prof. N. Ghoussoub (Univ. British Columbia)

9:35-10:15
Prof. F. Robert (Univ. Lorraine)

10:15-10:45
Coffee Break

10:45-11:25
Prof. Dancer (Univ. Sydney)

11:30-12:10
Prof. M. Winkler (Univ. Paderborn)

12:10-2:00
Lunch

2:00-2:40
Prof. Y. Sire (Univ. Marseille)

2:45-3:25
Prof. A. Pistoia (Univ. Roma)

3:25-3:50
Coffee Break

3:50-4:30
Prof. M. Taniguchi (Okayama Univ.)

4:35-5:15
Prof. S. Yan (Univ. New England)

5:20-6:00
Prof. X. Nguyen (Iowa State Univ.)

Dinner

May 30th 2014

8:30-9:10
Prof. K. Wang (Wuhan Inst. Phys. Math.)

9:15-9:55
Prof. L. Hauswirth (Univ. Marne La Valle)

9:55-10:25
Coffee Break

Lunch

END of Program. Thank You!

Have a Nice Trip!

Abstracts of Talks

1. Constant mean curvature in higher codimension, and a Ginzburg-Landau analogue
Rafe Mazzeo

Abstract: From joint work with Pacard and Zolotareva, I propose a new definition of constant mean curvature for submanifolds of codimension greater than one. We prove existence of small sphere-like solutions of this problem which are tied to critical points of a certain curvature functional. I will also some ongoing work with Marzuola concerning a Ginzburg-Landau type relaxation of this problem in three dimensions.

2. Some recent gluing constructions in Differential Geometry
Nick Kapouleas

3. Title: The spectrum of a family of fourth-order flows near the global attractor
Robert McCann

Abstract: The thin-film and quantum drift-diffusion equations belong to a fourth-order family of evolution equations proposed by Denzler and myself as analogous to the (second-order) porous medium family. They are 2-Wasserstein gradient flows of the generalized Fisher information (just as Otto showed the porous medium to be the 2-Wasserstein gradient flow of the Reyni entropy). In this talk we describe the linearization of the fourth-order dynamics around the self-similar solution. We diagonalize this linearization by relating it to analogous problem for the porous medium equation. This yields information about the leading-and higher-order asymptotics of the fourth-order flows on R^n which are outside of special cases were inaccessible previously. These results were obtained jointly with Christian Seis.

4. Title: The Soliton Zoo for Curve Shortening in $\mathbb{R}^n$.
Sigurd Angenent

Abstract: Compact self similar solutions for Curve Shortening are all planar and have been classified Abresch-Langer. In this talk I will describe the non compact self similar solutions to Curve Shortening of Space curves.

5. Title: Multi-bump solutions of $-\Delta u = K(x)u^{(n+2)/(n-2)}$ on lattices in R^n
Yanyan Li

Abstract: We consider the semilinear elliptic equation with critical exponent and with positive periodic function $K(x)$ in dimension n larger than 2. Under some natural condition on K near a critical point, we prove the existence of multi-bump solutions where the centers of bumps can be placed in some lattices in R^k, including infinite lattices, if $1 \leq k < (n-2)/2$. We also show that for $k \geq (n-2)/2$, no such solutions exist. This is a joint work with Juncheng Wei and Haoyuan Xu.

6. Title: Ancient Solutions to Geometric Flows
P. Daskalopoulos

Abstract: We will discuss the classification of ancient solutions to geometric flows such as the Ricci flow on surfaces and the Yamabe flow.

7. Title: saddle points and Gamma-convergence.
R. Jerrard
Abstract: The framework of Gamma-convergence can provide a way associating a "limiting functional" to a sequence of functionals E_ϵ, such that sequences of minimizers of E_ϵ converge to a minimizer of the limiting functional. It is also a classical fact that, given a local minimizer of the limiting functional, there exists a nearby local minimizers of E_ϵ, for small enough epsilon. We will first review a result from a few years ago which shows that, given a saddle point of the limiting functional, one can find associated critical points of E_ϵ for every small enough epsilon, and then discuss some recent related developments.

8. Title: Comparison results for semilinear elliptic equations in equimeasurable domains
Francois Hamel
Abstract: I will talk about pointwise comparison results between the solutions of some second-order semilinear elliptic equations in a domain of R^n and the solutions of some radially symmetric equations in the equimeasurable ball. The coefficients of the symmetrized equations in the ball satisfy similar constraints as the original ones in the original domain. I will consider both the case of equations with linear growth in the gradient and the case of equations with at most quadratic growth in the gradient. I will also discuss some improved quantified comparisons when the original domain is not a ball. The method is based on a symmetrization of the second-order terms and the talk will be based on a joint work with Emmanuel Russ.

9. Title: Asymptotic analysis of power-law nonlinear diffusion
John King
Abstract: The intermediate-asymptotic behaviour of a number of initial-value problems for the power-law nonlinear diffusion equation will be characterised, with emphasis on the role of critical exponents. Related issues for the higher-order nonlinear diffusion will also be touched upon.

10. Title:Mean-field equations, Hyperelliptic curves and Modular forms
Chang-Shou Lin
Abstract Let $E = E_\tau = C = (Z + \tau Z)$ be a two-dimensional at torus, where $\tau = a + ib$ with b ¿ 0. Consider the Liouville equation:

$$\Delta u + e^u = \rho\delta_0 \ \text{ in } E; (0.1)$$

where $\rho > 0$ and δ_0 is the Dirac measure at the lattice point 0. In a series of recent papers joint with C. L. Chai and C. L. Wang, we have been developing a theory to connect the Liouville equation, hyper-elliptic curves and modular forms. In particular, we construct a pre-modular form of degree $\frac{1}{2}n(n+1)$, and prove for E_τ that the Liouville equation (at $\rho = 8\pi n$) has a solution iff τ is a zero of this pre-modular form. Thus, nonlinear PDE and modular forms might help each other. In this talk, I will survey this development.

11. Title: On traveling waves under the presence of fractional diffusion
Xavier Cabre

Abstract: I will present three recent works concerning front propagation, and in particular the existence of traveling fronts, in the presence of fractional diffusion. With J.-M. Roquejoffre and A.-C. Coulon we have established the exponential in time propagation of fronts for the Fisher-KPP equation with fractional diffusion, both in homogeneous and in periodic media. Thus, no traveling waves exist in this case. Instead, with N. Consul and J.V. Mand, we find traveling fronts and their qualitative properties for the following fractional-diffusion problem: the classical homogeneous heat equation in a half-plane with a boundary Neumann condition of bistable or combustion type.

12. TITLE: Nonlocal minimal surfaces and free boundary problems.
E. Valdinoci
ABSTRACT: We present some recent results on the qualitative properties and the regularity theory for the minimizers of a nonlocal perimeter functional. Also, we discuss some related free boundary problems and we discuss the rigidity properties of their minimizers, in relation with density estimates and homogeneous solutions.

13. Title: Rate of Convergence to Separable Solutions of the Fast Diffusion Equation
M. Fila
Abstract: We study the asymptotic behaviour near extinction of positive solutions of the Cauchy problem for the fast diffusion equation with a subcritical exponent. We show that separable solutions are stable in some suitable sense by finding a class of functions which belong to their domain of attraction. For solutions in this class we establish optimal rates of convergence to separable solutions. This is a joint work with Michael Winkler.

14. Title:A priori estimates, existence and Liouville theorems for semilinear elliptic systems with power nonlinearities
Pavol Quittner
Abstract: We prove a priori estimates and existence of positive solutions of semilinear elliptic systems with power nonlinearities and homogeneous Dirichlet boundary conditions. In general, our systems are nonvariational and noncooperative, and our estimates are optimal in the class of very weak solutions. We also provide new Liouville-type theorems.

15. Title: Liouville theorems and qualitative properties of solutions to competitive systems with several components
Susanna Terracini
Abstract: In this talk we consider solutions of the competitive elliptic system

$$\begin{array}{l} \Delta u_i = \sum_{j\neq i} u_j^2 u_i, i = 1, ..., k, \text{ in } R^N, \\ u_i > 0 \text{ in } R^N, \end{array} \tag{1}$$

which appears in the analysis of phase separation phenomena for Bose-Einstein condensates with multiple states. We are concerned with the classification of entire solutions, according with their (algebraic) growth rate. The prototype of our main results is the following: for every $d > 0$ there exists $h = h(d, N) \in \mathbb{N}$ such that if $(u_1, ..., u_k)$ is a solution of (1) and

$$u_1(x) + ... + uk(x) \leq C(1 + |x|^d) \text{ for every } \quad x \in R^N$$

then $k \leq h(d, N)$. This means that a bound on the growth of a positive solution imposes a bound on the number of components k of the solution itself. If $N = 2$, the expression of h(d;N) is explicit and optimal, while in higher dimension it can be characterized in terms of an optimal partition problem. We discuss the sharpness of our results and, as a further step, for every $N \geq 2$ we can prove the 1-dimensional symmetry of the solutions of (1) satisfying suitable assumptions, extending known results which are available for $k = 2$. The proofs rest upon a blow-down analysis and on some monotonicity formulae. This is a joint work with Nicola Soave.

16. Title:Axially symmetric solutions to the Allen-Cahn equation in dimension 3
Yong Liu
Abstract: We study the Allen-Cahn equation in dimension 3:

$$-\Delta u = u - u^3, \text{ in } \mathbb{R}^3.$$

Recent advances in this direction indicates that those solutions which are bounded and axially symmetric will play a special role in the whole theory. Here we will construct new family of solutions whose behavior at infinity is well controlled: the nodal sets of these solutions outside of a large ball are of catenoid type. This is closely related to the minimal surface theory in dimension 3.

17. Title: Nonlinear Elliptic systems with mixed couplings
Wang Zhi-Qiang
Abstract: We report recent work on existence and multiplicity of positive vector solutions for nonlinear Schrodinger systems with mixed couplings. In symmetric domains we show symmetric breaking for least energy solutions whose asymptotic behaviors are also studied. Coexistence of synchronization and segregation is explored.

18. Title:Removable and non-removable singularities in some parabolic equations
Eiji Yanagida
Abstract: For parabolic partial differential equations, there may exist solutions with time-dependent singularities. In this talk we give a sufficient condition for the removability of such singularities. We also show the existence of solutions with non-removable singularities. This is a joint work with Jin Takahashi.

19. Title: Ricci flow neckpinches
Natasa Sesum
Abstract: We study "warped Berger" solutions $\left(S^1 \times S^3, G(t)\right)$ of Ricci flow: generalized warped products with the metric induced on each fiber $\{s\} \times \mathrm{SU}(2)$ a left-invariant Berger metric. We prove that this structure is preserved, that these solutions develop finite-time neckpinch singularities, and that they asymptotically approach round product metrics in space-time neighborhoods of their singular sets, in precise senses. This is the first concrete example of Ricci flow solutions without rotational symmetry that develop locally finite-time neck pinch singularities. This is a joint work with James Isenberg and Dan Knopf.

20.Title: Even Symmetry of Axially Symmetric Solutions of the Allen-Cahn Equation
C. Gui
Abstract: In this talk, we will discuss the even symmetry in the direction of the axis for axially symmetric solutions of Allen-Cahn equation. The dimension of the space will play an important role in the proof, which is based on the asymptotic behavior of nodal sets of such solutions and the moving plane method.

21.Title: No touchdown at zero points of the permittivity profile for the MEMS problem
P. Souplet
Abstract: (Joint work with Jong-Shenq Guo) We study the quenching behavior for a semilinear heat equation arising in models of micro-electro mechanical systems. The problem involves a source term with a spatially dependent potential, given by the dielectric permittivity profile, and quenching corresponds to a touchdown phenomenon. It is well known that quenching does occur. We prove that **touchdown cannot occur at zero points of the permittivity profile.** In particular, we remove the assumption of compactness of the touchdown set, made in all previous work on the subject and whose validity is unknown in most typical cases. This answers affirmatively a conjecture made in [Y. Guo, Z. Pan and M.J. Ward, SIAM J.Appl. Math 66 (2005), 309–338] on the basis of numerical evidence. The result crucially depends on a new type I estimate of the quenching rate, that we establish. In addition we obtain some sufficient conditions for compactness of the touchdown set, without convexity assumption on the domain. These results may be of some qualitative importance in applications to MEMS optimal design, especially for devices such as micro-valves.

22.Title: Traveling spots and obstacle-induced spirals in an excitable medium
Hirokazu Ninomiya
Abstract: In this talk, traveling spots observed in two-dimensional excitable media are explored. First, we introduce the singular limit problem of the FitzHugh-Nagumo equations. Then we show the existence of the traveling spot including the front and the back. Using this traveling spots, I will explain some mathematical understanding of the formation of spirals which is induced by obstacles. The formation of the spirals depends on the shapes of the obstacle. This spiral formation is deeply related to the ventricular brillation. This work is partially based on a joint work with Y.-Y. Chen and Y. Kohsaka.

23. Title:Decoupling DeGiorgi-type systems via multi-marginal optimal transport
Nassif Ghoussoub
Abstract: We use Monge-Kantorovich optimal transport theory to show that the components $u = (u_1, u_2, ..., u_m) : \mathbb{R}^N \to \mathbb{R}^m$ of certain solutions of elliptic partial differential systems of the form $\Delta u = \nabla H(u)$, where $H : \mathbb{R}^m \to \mathbb{R}$ is a given C^2 non-linearity, are also solutions of a –much more tractable– decoupled system of ODEs

$$\Delta u_i = V_i'(u_i(x)) \quad \text{for } i = 1, ..., m, \tag{2}$$

where $V_1, ..., V_m$ are appropriate potentials. This is a joint work with Brendan Pass.

24. Title: On the Hardy-Schrödinger operator with a singularity on the boundary (joint work with N.Ghoussoub)

F. Robert

Abstract: We consider borderline elliptic partial differential equations involving the Hardy-Schrödinger $L_\gamma := -\Delta - \gamma\frac{1}{|x|^2}$ operator on a domain $\Omega \subset \mathbf{R}^n$, when the singularity zero is on the boundary of the domain. This operator arises naturally when dealing with the Caffarelli-Kohn-Nirenberg inequalities and their associated Euler-Lagrange equations.

Now, it is well known that the operator L_γ is non-negative when 0 is in the interior of a domain as long as $\gamma \leq \frac{(n-2)^2}{4}$. The situation is much more interesting when $0 \in \partial\Omega$. For one, the operator is then non-negative for all $\gamma \leq \frac{n^2}{4}$. The problem of whether the Dirichlet boundary problem

$$-\Delta u - \frac{\gamma}{|x|^2}u = \frac{u^{2^*(s)-1}}{|x|^s} \text{ on } \Omega \tag{3}$$

has positive solutions is closely related to whether the best constants in the Caffarelli-Kohn-Nirenberg inequalities are attained. Here $2^*(s) = \frac{2(n-s)}{n-2}$ and $s \in [0, 2)$. Brezis-Nirenberg type methods were used by C.S. Lin et al. to show that this is indeed the case when $\gamma < \frac{(n-2)^2}{4}$ under the condition that the mean curvature of the domain at 0 is negative. Their results extend previous work by Ghoussoub-Robert who dealt with the case $\gamma = 0$.

The case when $\frac{(n-2)^2}{4} \leq \gamma < \frac{n^2}{4}$ turned out to be quite delicate. A detailed analysis of the linear Hardy-Schrodinger operator $L\gamma$ performed recently by Ghoussoub-Robert surprisingly show that $\gamma = \frac{n^2-1}{4}$ is another critical threshold for the operator. While the C. S. Lin et al. results extend to the situation where $\gamma < \frac{n^2-1}{4}$, the interval $\gamma \in [\frac{n^2-1}{4}, \frac{n^2}{4})$ requires the introduction of a notion of "mass" in the spirit of Shoen-Yau for the Hardy-Schrödinger operator. The existence of solutions then depend on the sign of such a mass.

25. Title : Infinitely many bifurcations for rapidly growing nonlinearities

E.N.Dancer

Abstract: We consider the problem

-Laplacian u = rf(u) on D u=0 on the boundary of D,

where D is a bounded domain with smooth boundary in n-dimensional space.We are interested in positive solutions ,assume n is at least 3 and f grows faster than exponentially. We prove in many cases that if f is real analytic certain branches of solutions have infinitely many bifurcations.We also allow some other boundary conditions. We also discuss some related open problems if n=2

26. Title: Finite-time blow-up in the fully parabolic Keller-Segel system

Michael Winkler

Abstract: We study the Neumann initial-boundary value problem for the fully parabolic Keller-Segel system

$$(*) \quad \begin{cases} u_t = \Delta u - \nabla(u\nabla v), \ x \in \Omega, t > 0 \\ v_t = \Delta v - v + u, x \in \Omega, t > 0 \end{cases}$$

in a ball $\Omega \subset R^n$, where $n \geq 2$. This system forms the core of numerous models used in mathematical biology to describe the spatio-temporal evolution of cell populations governed by both diffusive migration and chemotactic movement towards increasing gradients of a chemical that they produce themselves. We demonstrate that in the case $n \geq 3$, for any prescribed $m > 0$ there exist radi-ally symmetric positive initial data $(u_0, v_0) \in C^0(\bar{\Omega}) \times W^{1,\infty}(\Omega)$ with $\int_\Omega u_0 = m$ such that the corresponding solution blows up in finite time. Moreover, by pro-viding an essentially explicit blow-up criterion it is shown that within the space of all radial functions, the set of such blow-up enforcing initial data indeed is large in an appropriate sense; in particular, this set is dense with respect to the topology of $L^p(\Omega) \times W^{1,2}(\Omega)$ for any $p \in (1, \frac{2n}{n+2})$. We moreover comment on a corresponding result for $n = 2$ which indicates that finite-time blow-up is a generic phenomenon also in this case, at least within the framework of radial solutions.

One focus of the presentation is on the method through which these results can be obtained. In contrast to previous approaches, it is based on a more elaborate use of the natural energy inequality associated with (*), e.g. in the case $n \geq 3$ involving an estimate of the form

$$\int_\Omega uv \leq C(\|\Delta v - v + u\|^{2\theta}_{L^2(\Omega)} + \|\frac{\nabla u}{\sqrt{u}} - \sqrt{u}\nabla v\|_{L^2(\Omega)}$$

which is valid with certain $C > 0$ and $\theta \in (0, 1)$ for a wide class of smooth positive radial functions $(u, v) = (u(x), v(x))$.

27. Title: Geometric methods for the existence of quasi-periodic solutions for ill-posed equations

Yannick Sire

We develop a method to construct some special classes of solutions for a priori ill-posed equations. The solutions in question are quasi-periodic (invariant KAM tori) and the method consists in a Nash-Moser iteration suitably designed. We apply it to several PDEs coming from fluid mechanics.

28. Title: Large mass boundary condensation patterns in the stationary Keller-Segel system

Angela Pistoia

Abstract: We consider the stationary Keller-Segel system from chemotaxis in a domain of the plane. We establish the existence of a solution which exhibits a sharp boundary layer along the entire boundary of the domain. This is a joint work with Manuel del Pino and Giusi Vaira.

29. Title: Convex compact sets in $\mathbb{R}^{N-1}$ give traveling fronts in $\mathbb{R}^N$ in cooperative diffusion systems

M. Taniguchi

Abstract:This paper studies traveling fronts to cooperative diffusion systems in $\mathbb{R}^N$ for $N \geq 3$. We consider $(N-2)$-dimensional smooth surfaces as boundaries of strictly convex compact sets in $\mathbb{R}^{N-1}$. We prove that there exists a traveling front associated with a given surface and that it is asymptotically stable for given initial perturbation. The associated traveling

fronts coincide up to phase transition if and only if the given surfaces satisfy an equivalence relation

30. Title: Bubbling Solutions for Chern-Simons Model in a Torus
Shusen Yan
Abstract: In this talk, I will present some results on the structure of the bubbling solutions for Chern-Simons model in torus. The results include the co-existence of bubbles, and the existence of a one to one map between the bubbling solutions and non-degenerate critical points of some functions. We also discuss the number of the critical points of those functions and their non-degeneracy by using the elliptic function theories.

31. Title: Desingularization of immersed self-shrinkers.
Xuan H. Nguyen
Abstract: In the 1990's, Kapouleas and Traizet constructed new examples of minimal surfaces by desingularizing the intersection of existing ones, such as catenoids and planes, with Scherk surfaces. The technique can be extended to prove the existence of new self-translating and self-shrinking surfaces under mean curvature flow. All of these construction so far involve the intersection of two surfaces. In this talk, we present some recent results about desingularization of immersed surfaces and discuss the challenges that arise.

32. Title: Harmonic approximation and improvement of flatness in singular perturbation problems
Kelei Wang
Abstract: Improvement of flatness type estimates play an important role in establishing various epsilon-regularity theorems, e.g. in the regularity theory of minimal surfaces and harmonic mappings. One main ingredient, which goes back to De Giorgi, is to approximate nonlinear problems by linear ones such as harmonic functions. In this talk I will discuss the generalization of this classical method to singular perturbation problems of semilinear elliptic equations, and applications of these results to the De Giorgi conjecture for several entire space problems.

33. Title: TBA
L. Hauswirth