Spectral theory of Laplace and Schrödinger operators
July 28 –August 2, 2013

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday
*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday
*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday
Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)
Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the lecture theater in the TransCanada Pipelines Pavilion (TCPL). An LCD projector, a laptop, a document camera, and blackboards are available for presentations.

SCHEDULE

You are welcome to schedule lectures as you see fit, as long as you adhere to the meal times (noted above), coffee break start and end times (noted below) and take into account the "welcome" on Monday morning, the Banff Centre tour at 1:00 pm, and the group photo at 2:00 pm every Monday afternoon.

Please email your finalized schedule and abstracts to BIRS Station Manager birsmgr@birs.ca by Thursday morning before your arrival (at the latest) in order to allow for printing and posting to the website.

You are also encouraged to e-mail the schedule to your participants. BIRS provides the option of an electronic mail list in order to facilitate communications with your participants. When you login to the Organizer Interface at https://www.birs.ca/orgs, you will be prompted to create an electronic mail list for your workshop. Click "Yes" to create one and receive instructions, or "No" to decline. If you would like more information about our electronic mail lists, please e-mail help@birs.ca.

Sunday
16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
17:30–19:30 Buffet Dinner, Sally Borden Building
20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and a small assortment of snacks are available on a cash honor system.

Monday

7:00–8:45	Breakfast
8:45–9:00	Introduction and Welcome by BIRS Station Manager, TCPL
9:00–9:40	**Dmitry Jakobson** (McGill University)
	Nodal sets in conformal geometry
10:00–10:20	Coffee Break
10:20–11:00	**Yaiza Canzani** (Harvard University)
	Local geometry of the nodal sets of Laplace eigenfunctions
11:10–11:30	**Stefan Steinerberger** (University of Bonn)
	A geometric uncertainty principle and Pleijel's estimate
11:30–13:00	Lunch
13:00–14:00	Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
14:00	Group Photo; meet in foyer of TCPL (photograph will be taken outdoors so a jacket might be required).
14:10–15:00	Coffee Break
15:00–15:40	**Leonid Friedlander** (University of Arizona)
	Generic multiplicities of eigenvalues for families of operators
15:50–16:30	**Gregory Berkolaiko** (Texas A&M University)
	Nodal count and Dirac points in the spectrum of graphene graphs
16:50–17:30	*Discussion and open problems session on Geometry of Eigenfunctions*
	Moderator: **Dmitry Jakobson** (McGill University)
17:30–19:30	Dinner

Tuesday

7:00–9:00	Breakfast
9:00–9:50	**Antoine Henrot** (Institut Elie Cartan and École des Mines de Nancy)
	Recent advances in isoperimetric inequalities for eigenvalues
10:00–10:20	Coffee Break
10:20–11:00	**Alessandro Savo** (Università di Roma la Sapienza)
	On the spectrum of the Hodge Laplacian and the John ellipsoid
11:10–11:30	**Alexandre Girouard** (Université Laval)
	Sharp inequalities for Steklov eigenvalues
11:30–13:30	Lunch
14:00–15:00	Coffee Break
15:00–15:40	**Jochen Denzler** (University of Tennessee)
	Existence and Regularity in the Oval Problem
15:50–16:30	**Pedro Freitas** (University of Lisbon and TU Lisbon)
	Behaviour of optimisers for the kth eigenvalue of the Laplacian
16:50–17:30	*Discussion and open problems session on Geometry of Eigenvalues*
	Moderator: **Evans Harrell** (Georgia Tech)
17:30–19:30	Dinner

Wednesday

9:00–9:50	**Nilima Nigam** (Simon Fraser University)
	Numerical analysis of spectral problems, validated numerics, and proof
10:00–10:20	Coffee Break
10:20–11:00	**Michael Levitin** (University of Reading)
	Complex spectra of self-adjoint linear operator pencils
11:10–11:30	**Lotfi Hermi** (University of Arizona)
	An isoperimetric inequality of Saint-Venant-type for a wedge-like membrane
11:30–13:30	Lunch
	Free Afternoon
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00–9:50	**Ari Laptev** (Imperial College and Mittag–Leffler Institute)
	Recent improvements of Berezin-Li-Yau type inequalities
10:00–10:20	Coffee Break
10:20–11:00	**Rupert Frank** (Caltech)
	Stability estimates for the lowest eigenvalue of a Schrödinger operator
11:10–11:30	**Leander Geisinger** (Princeton University)
	Delocalization of eigenvectors on random regular graphs
11:30–13:30	Lunch
14:00–15:00	Coffee Break
15:00–15:40	**Bernard Helffer** (Université Paris-Sud)
	On magnetic wells in the semi-classical limit
15:50–16:30	**Michael Loss** (Georgia Tech)
	Nonlinear flows and rigidity results on compact manifolds
16:50–17:30	*Discussion and open problems session on Mathematical Physics*
	Moderator: **Dirk Hundertmark** (Karlsruhe Institute of Technology)
17:30–19:30	Dinner

Friday

9:00–9:20	**Ram Band** (University of Bristol and Technion)
	Universality of the momentum band density of periodic graphs
9:30–9:50	**Peter Herbrich** (Dartmouth College)
	On Inaudible Properties of Broken Drums - Isospectrality with Mixed Dirichlet-Neumann Boundary Conditions
10:00–10:20	Coffee Break
10:20–10:40	**Francesco Chiacchio** (Università degli Studi di Napoli)
	Sharp lower bounds for eigenvalues of linear and nonlinear Neumann problems
10:45-11:05	**Jeffrey Langford** (Bucknell University)
	Some New Comparison Results in Balls and Shells
11:10 – 11:30	**Bartłomiej Siudeja** (University of Oregon)
	Homogeneity of the ground state for fractional Laplacian on cones
11:30–13:30	Lunch
Checkout by	
12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to check out of the guest rooms by 12 noon. **

Spectral theory of Laplace and Schrödinger operators
July 28–August 2, 2013

ABSTRACTS
(in alphabetic order by speaker surname)

Speaker: **Ram Band** (University of Bristol and Technion)
Title: *Universality of the momentum band density of periodic graphs*
Abstract: The momentum spectrum of a periodic network (quantum graph) has a band-gap structure. We investigate the relative density of the bands or, equivalently, the probability that a randomly chosen momentum belongs to the spectrum of the periodic network. We show that this probability exhibits universal properties. More precisely, the probability to be in the spectrum does not depend on the edge lengths (as long as they are generic) and is also invariant within some classes of graph topologies. Based on a joint work with Gregory Berkolaiko.

Speaker: **Gregory Berkolaiko** (Texas A&M University)
Title: *Nodal count and Dirac points in the spectrum of graphene graphs*
Abstract: The n-th eigenfunction of a graph has at least $n-1$ zeros. In general, this bound is achieved arbitrarily high in the spectrum. Using Neumann-Dirichlet interlacing inequalities for star graphs, we show that a family of simple graphs consisting of two vertices with E edges between them have an anomalous nodal count: the n-th eigenfunction will always have at least n zeros (for $n > 1$). Using a link between the nodal count and the behavior of eigenvalues under magnetic perturbation, we connect this nodal anomaly with existence of Dirac points in the dispersion relation of graphene-type periodic graphs. We further discuss existence of Dirac point in lattices with less than honeycomb symmetry and the "nodal" mechanism of their disappearing.

Based on joint work in progress with R. Band and T. Weyand.

Speaker: **Yaiza Canzani** (Harvard University)
Title: *Local geometry of the nodal sets of Laplace eigenfunctions*
Abstract: Let (M, g) be a compact real analytic surface with no boundary, and let H denote a closed analytic curve in M. Write φ_λ for the eigenfunctions of the Laplacian Δ_g with eigenvalue λ^2. When M is the two-torus and H has non-vanishing curvature (Burgain-Rudnick) or when M is an arithmetic surface and H is a geodesic circle (Jung), it has been shown that H is a *good* curve in the sense that $\|\varphi_\lambda\|_{L^2(H)} \geq e^{-C\lambda}$ for some $C > 0$ and all $\lambda > \lambda_0$. In these cases it was proved that

$$\#\{\varphi_\lambda^{-1}(0) \cap H\} = O(\lambda). \qquad (*)$$

In this talk we show that the bound $(*)$ holds on the general surface (M, g) provided H is a good curve. This is joint work with John Toth.

Speaker: **Francesco Chiacchio** (Università degli Studi di Napoli)
Title: *Sharp lower bounds for eigenvalues of linear and nonlinear Neumann problems*
Abstract: We prove some sharp lower bounds for the first nontrivial Neumann eigenvalue $\mu_1(\Omega)$ for the p-Laplace operator in a Lipschitz, bounded domain Ω of R^n. Our estimates are asymptotically sharp, at least for $n = p = 2$. In particular we show that equality is achieved along a sequence of rhombi as the acute angle goes to zero. (Joint work with B. Brandolini and C. Trombetti.)

Speaker: **Jochen Denzler** (University of Tennessee)
Title: *Existence and Regularity in the Oval Problem*

Abstract: The oval problem asks to determine, among all closed loops in $\mathbf{R}^n$ of fixed length, carrying a Schrödinger operator $\mathbf{H} = -\frac{d^2}{ds^2} + \kappa^2$ (with curvature κ and arclength s), those loops for which the principal eigenvalue of $\mathbf{H}$ is smallest. A 1-parameter family of ovals connecting the circle with a doubly traversed segment (digon) is conjectured to be the minimizer. Whereas this conjectured solution is an example that proves a lack of compactness and coercivity in the problem, it is proved in this talk (via a relaxed variation problem) that a minimizer exists; it is either the digon, or a strictly convex planar analytic curve with positive curvature. While the Euler-Lagrange equation of the problem appears daunting, its asymptotic analysis near a presumptive singularity gives useful information; a strong variation style necessary condition for minimizers, as well as geometric explicit descent steps are further ingredients.

Speaker: **Rupert Frank** (Caltech)
Title: *Stability estimates for the lowest eigenvalue of a Schrödinger operator*
Abstract: In 1961, Keller asked to minimize the lowest eigenvalue among all Schrödinger operators with potentials having a fixed L^p norm. This problem is equivalent to a Sobolev interpolation inequality and, using results about the latter, one can show that there is a unique optimal potential (up to translations and rescaling). Our main result is that if a potential has an almost optimal eigenvalue then it is close (in a quantified sense) to an optimal potential.

The talk is based on joint work with E. Carlen and E. Lieb.

Speaker: **Pedro Freitas** (University of Lisbon and TU Lisbon)
Title: *Behaviour of optimisers for the kth eigenvalue of the Laplacian*
Abstract: We consider the behaviour of shapes optimising the kth eigenvalue of the Laplace operator under different boundary conditions, together with the corresponding asymptotics of optimal eigenvalues.

For several families of domains with Dirichlet boundary conditions, such as rectangles with fixed area, or general planar domains with fixed perimeter, we show that the optimal domain approaches that which satisfies optimality for the corresponding geometric isoperimetric inequality as k goes to infinity (i.e. the square and the disk in the cases mentioned above).

Speaker: **Leonid Friedlander** (University of Arizona)
Title: *Generic multiplicities of eigenvalues for families of operators*
Abstract: Let $A(t)$ be a smooth family of operators. I am interested in three types of families: $A(t) = \Delta + q(t)$, $A(t) = \Delta_{g(t)}$, and $A(t) = \Delta_{\Omega(t)}$. Here Δ is the Laplacian either on a Riemannian manifold or in a domain, q is a potential, g is a Riemannian metric, and Ω is a domain; $t = (t_1, \ldots, t_k)$ is a vector of parameters. The question is what are possible spectral degenerations that generically occur in such families. In his paper "Modes and quasi-modes", Arnold suggested that the situation is the same as in the case of symmetric matrices. So far, it has been proven for $k = 0$ (no parameters) by Albert and Uhlenbeck, $k = 1$ by Teitel; I proved it for $k = 2$.

Speaker: **Leander Geisinger** (Princeton University)
Title: *Delocalization of eigenvectors on random regular graphs*
Abstract: We consider a random regular graph of fixed degree d with n vertices. We study spectral properties of the Laplace operator and of random Schroedinger operators on such a graph as n tends to infinity. We prove that the integrated density of states on the graph converges to the integrated density of states on the infinite regular tree and we give uniform bounds on the rate of convergence. Based on related estimates for the Green function we derive results about delocalization of eigenvectors.

Speaker: **Alexandre Girouard** (Université Laval)
Title: *Sharp inequalities for Steklov eigenvalues*
Abstract: In this talk I will survey sharp inequalities for the eigenvalues of the Dirichlet-to-Neumann operator of Euclidean domains and surfaces.

Speaker: **Bernard Helffer** (Université Paris-Sud)
Title: *On magnetic wells in the semi-classical limit*
Abstract: Considering the Schroedinger operator with magnetic field in $\mathbb{R}^2$ and assuming that the magnetic field never vanishes and admits a unique local minimum $b_0 > 0$, we would like to revisit in the semi-classical limit the analysis of the spectrum in some interval $]-\infty, h(b_0 + \epsilon)]$ for some $\epsilon > 0$. The previous papers by Helffer-Mohamed, Helffer-Kordyukov were only treating the ground-state energy or a finite (independent of the semi-classical parameter) number of eigenvalues. Two approaches will be discussed: one using Grushin's problem and another developed quite recently by N. Raymond and San Vu Ngoc.

Speaker: **Antoine Henrot** (Institut Elie Cartan and École des Mines de Nancy)
Title: *Recent advances in isoperimetric inequalities for eigenvalues*
Abstract: Since 2006 (and the publication of my book on this topic), several very interesting advances have been obtained in the field of isoperimetric inequalities for eigenvalues. In this talk, I will mainly describe these progress for problems like $\min\{\lambda_k(\Omega)\}$ or $\max\{\lambda_k(\Omega)\}$ where $\lambda_k(\Omega)$ denotes the k-th eigenvalue of the Laplacian with various boundary conditions (Dirichlet, Neumann, Robin, Steklov) and Ω is an open set in $\mathbb{R}^N$ subject to various geometric constraints (area, perimeter, diameter). I will also point out what appears to be some challenging open problems.

Speaker: **Peter Herbrich** (Dartmouth College)
Title: *On Inaudible Properties of Broken Drums - Isospectrality with Mixed Dirichlet-Neumann Boundary Conditions*
Abstract: The talk will address the inverse spectral problem for manifolds with mixed Dirichlet-Neumann boundary conditions. Various families of isospectral pairs have been found using the transplantation method, which we formulate in terms of graph theory. This allows for a computer-aided search of such families, a representation-theoretic characterization of transplantability, and the generating of new transplantable pairs from given ones. The talk will finish with examples showing that the Dirichlet spectrum of a manifold with boundary does not determine whether it is connected, and that a connected orbifold can be Dirichlet isospectral to a connected manifold.

Speaker: **Lotfi Hermi** (University of Arizona)
Title: *An isoperimetric inequality of Saint-Venant-type for a wedge-like membrane*
Abstract: For a wedge-like membrane, Payne and Weinberger proved in 1960 an isoperimetric inequality for the fundamental eigenvalue which in some cases improves the classical isoperimetric inequality of Faber-Krahn. In this work, we introduce "relative torsional rigidity" for this type of membrane and prove new isoperimetric inequalities in the spirit of Saint-Venant, Pólya-Szegő, Payne, Payne-Rayner, Chiti, and Talenti, which link the eigenvalue problem with the boundary value problem in a fundamental way. (Joint work with A. Hasnaoui, University of Tunis, El Manar.)

Speaker: **Dmitry Jakobson** (McGill University)
Title: *Nodal sets in conformal geometry*
Abstract: This is joint work with Y. Canzani, R. Gover and R. Ponge. We study conformal invariants that arise from nodal sets and negative eigenvalues of conformally covariant operators, which include the Yamabe and Paneitz operators. We describe applications to curvature prescription problems. We establish a version in conformal geometry of Courant's Nodal Domain Theorem. We also show that on any manifold of dimension $n \geq 3$, there exist many metrics for which our invariants are nontrivial. We prove that the Yamabe operator can have an arbitrarily large number of negative eigenvalues on any manifold of dimension $n \geq 3$.

Speaker: **Jeffrey Langford** (Bucknell University)
Title: *Some New Comparison Results in Balls and Shells*
Abstract: In this talk, we begin by discussing some of the classical comparison results, starting with Talenti's Theorem and the analogous result for Steiner symmetrization. We then discuss Neumann comparison

results, including a conjecture of Kawohl, and end with some new results in balls and shells involving cap symmetrization.

Speaker: **Ari Laptev** (Imperial College and Mittag-Leffler Institute)
Title: *Recent improvements of Berezin-Li-Yau type inequalities*
Abstract: We shall present a number of recent results related to spectral inequalities for Dirichlet Laplacians in domains of finite and infinite measure.

Speaker: **Michael Levitin** (University of Reading)
Title: *Complex spectra of self-adjoint linear operator pencils*
Abstract: A self-adjoint linear operator pencil is an operator family of the form $A - \lambda B$, where A, B are self-adjoint operators. Its spectrum is defined as a set of complex values λ such that $A - \lambda B$ is not invertible. If either A or B is sign-definite, then the spectrum of the operator pencil is usually real. If it's not the case, the spectrum may be complex, and the behaviour of eigenvalues may be quite interesting - they often lie on (or inside) a small set of curves in the complex plane.

We illustrate these phenomena using two rather distinct examples — a one-dimensional Dirac operator pencil, and a matrix pencil, — and additionally analyse the dependencies of their spectra on the parameters of the problems.

Speaker: **Michael Loss** (Georgia Tech)
Title: *Nonlinear flows and rigidity results on compact manifolds*
Abstract: This talk is about a certain class of non-linear PDEs on a compact connected Riemannian manifolds without boundary. The problem is to prove that there are no solutions other than the constant function. These rigidity results yield sharp Sobolev type inequalities. While some of the results date back to the 90-ies, a new perspective has emerged in the last five years. The idea is to use porous media or fast diffusion flows that yield relatively straightforward proofs for such rigidity results. This is joint work with Jean Dolbeault and Maria Ésteban.

Speaker: **Nilima Nigam** (Simon Fraser University)
Title: *Numerical analysis of spectral problems, validated numerics, and proof*
Abstract: The use of numerical analysis, careful scientific computing in conjunction with analysis to prove theorems is emerging as an interesting area of mathematics.

Except for very specific problems and domains, we do not have closed-form expressions for the eigenvalues and eigenfunctions of a given elliptic operator. If we wish to establish conjectures regarding these eigenpairs the use of approximation and numerical computation could be useful. However, a careful distinction is to be made between what we consider a proof and what we consider compelling numerical evidence.

An illustrative example is the location of eigenvalues of a generic $N \times N$ matrix with rational entries. For $N > 5$, there is no algorithm involving only a finite number of the usual arithmetic operations of addition, subtraction, multiplication, division and kth roots on rationals which can yield exact eigenvalues. In practice, we compute a finite number of terms in some sequence which provably converges to the true eigenvalue. We cannot assert any given term of the sequence is the true eigenvalue, but we may have provable bounds on the error. In addition, if we use finite precision calculations (on computers), we have to carefully account for the effects of rounding. Under what circumstances can we use such a numerically computed eigenvalue as part of a proof?

In this talk, we'll discuss the solution of elliptic eigenproblems from the numerical analysis and scientific computing standpoint. Numerical analysts are concerned with three distinct steps when computing these eigenpairs. First, we examine eigenfunctions by their representation in some (usually orthogonal) basis. Truncation of this expansion after N terms gives some approximation to the true eigenfunction. We try to prove rigorous error estimates and convergence theorems concerning such approximations.

Next, if we truncate the representation in this basis after a fixed number of terms N, the coefficients of the representation are found by studying some discrete system. This requires algorithms from numerical linear algebra. We try to understand convergence and approximation errors for this discrete system. Until this stage, everything is proved rigorously and under the assumption of exact arithmetic. We finally implement the algorithms and examine the effects of using finite precision in the combined algorithm.

We discuss what, in this context, may constitute validated numerics. We also discuss how careful computations may be used as parts of a proof. We illustrate these ideas through some model problems.

Speaker: **Alessandro Savo** (Università di Roma la Sapienza)
Title: *On the spectrum of the Hodge Laplacian and the John ellipsoid*
Abstract: In this talk, we focus on the first eigenvalue of the Hodge Laplacian acting on differential p-forms of a compact, convex Euclidean domain. We give upper and lower bounds of these eigenvalues in terms of the ellipsoid of maximal volume included in the domain and we observe some consequences. We also discuss an isoperimetric inequality for the Hodge-Laplace eigenvalues involving the volume of p-dimensional plane sections for each $p = 1, ..., n$; this inequality somewhat generalizes the classical Payne-Weinberger inequality (when $p = 1$) and the Faber-Krahn inequality (when $p = n$).

Speaker: **Bartłomiej Siudeja** (University of Oregon)
Title: *Homogeneity of the ground state for fractional Laplacian on cones*
Abstract: We discuss the classical Dirichlet Laplacian on cones first, concentrating on the zero eigenvalue and the positive eigenfunction. Next we generalize this to the fractional case. Finally, homogeneity exponents for classical and fractional cases are compared, and some fractional results are proved (informally).

Speaker: **Stefan Steinerberger** (University of Bonn)
Title: *A geometric uncertainty principle and Pleijel's estimate*
Abstract: Pleijel's estimate is not sharp because one cannot cover the Euclidean plane with discs of identical radius. We quantify our inability to do so. This yields an improved Pleijel estimate akin to recent work of Bourgain.