Sequential (Interactive) Testing in High Dimensions

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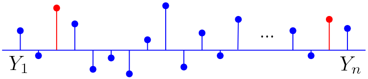
BIRS 2012 - Interactive Information Theory
Joint work with Rob Nowak

Problem Setup

- Consider a support set $\mathcal{S} \subset \{1, ..., n\}$ with $|\mathcal{S}| = s \ll n$ and the random variables

$$Y_i \sim \begin{cases} P_0 & i \notin \mathcal{S} \\ P_1 & i \in \mathcal{S} \end{cases} \qquad i = 1, ..., n$$



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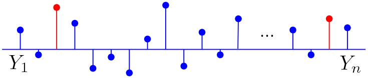

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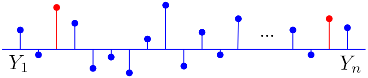

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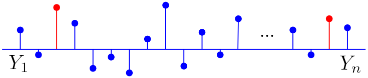

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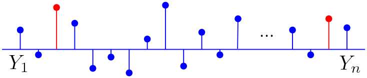

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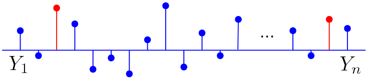

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- **This talk:** find relationship between (n, s, m) such that $\mathbb{P}(\hat{\mathcal{S}} = \mathcal{S}) \to 1$.

Related work

A. Wald and J. Wolfowitz
Optimum character of the sequential probability ratio test. 1948. 1-dimensional simple binary hypothesis test

E. Posner
Optimal Search Procedures. 1963. AWGN, $s = 1$, SPRT

J. Haupt, R. Castro, and R. Nowak
Distilled Sensing: Adaptive Sampling for Sparse Detection and Estimation. 2010. AWGN, $Y_{i,j} = x_i + \gamma_{i,j}^{-1/2} W_{i,j}$, FDP/NDP

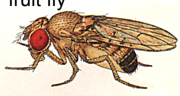
L. Lai, H. Vincent Poor, Y. Xin, and G. Georgiadis
Quickest Search Over Multiple Sequences. Trans. on Info Theory. 2011. Find one element in $\mathcal{S}$

E. Bashan, G. Newstadt, and A. Hero
Two-Stage Multi-Scale Search for Sparse Targets. 2011. AWGN, two stage procedure

A. Tajer, R. Castro
Adaptive Spectrum Sensing for Agile Cognitive Radios. 2010.
Spectrum Sensing

Motivation – Gene knock-out studies in biology

fruit fly



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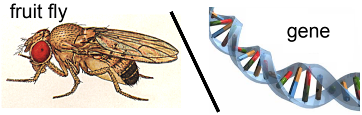

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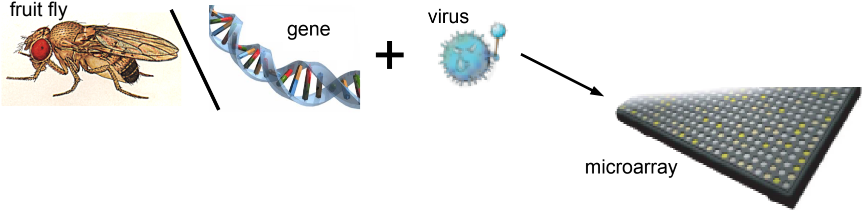

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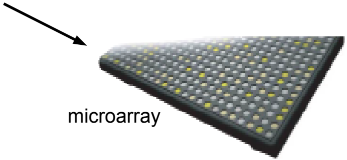

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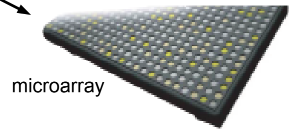

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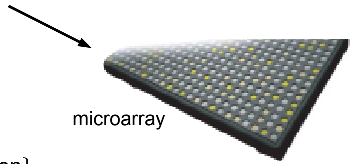

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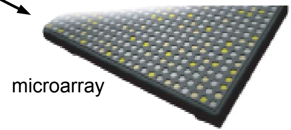

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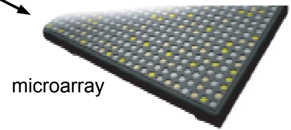

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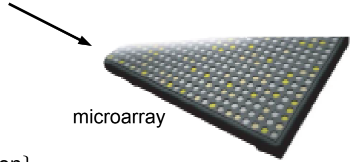

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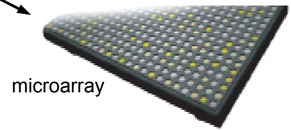

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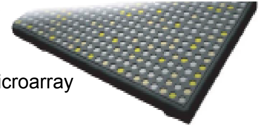

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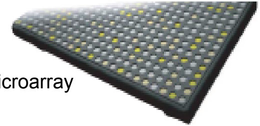

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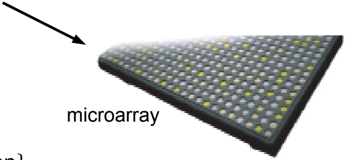

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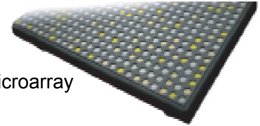

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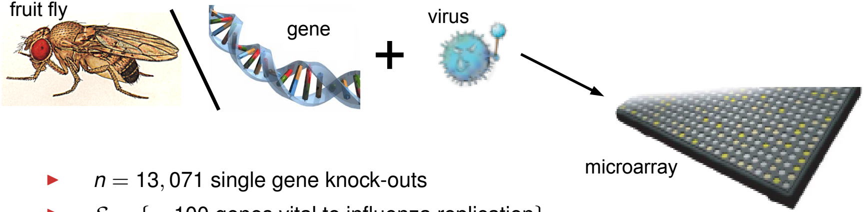

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What can we do with limited knowledge of distributions?

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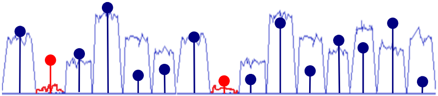

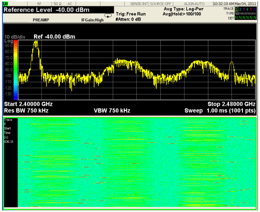

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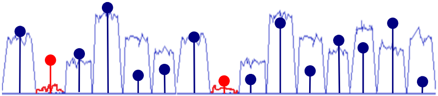

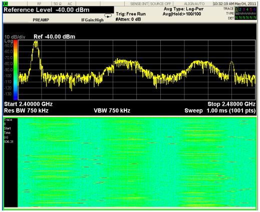

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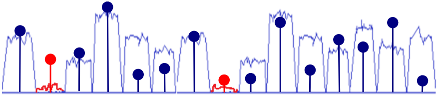

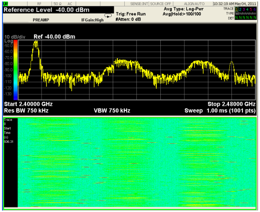

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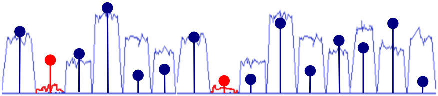

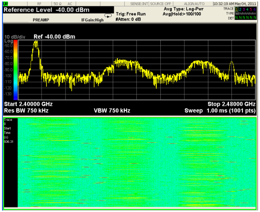

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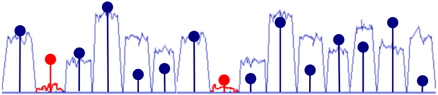

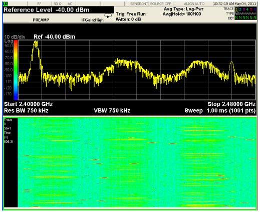

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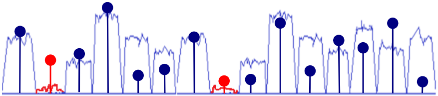

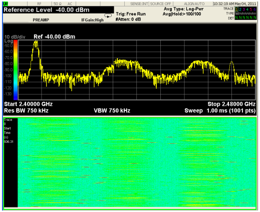

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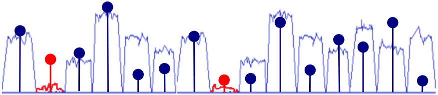

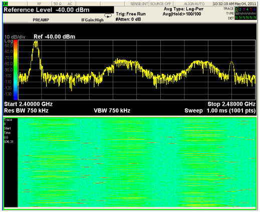

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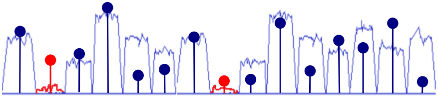

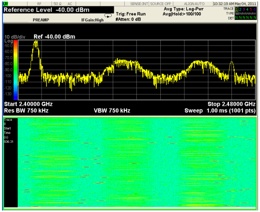

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2. *Sequential*
   - sequential thresholding or coordinate-wise SPRT [A. Tajer 2010, W. Zhang 2010, M. Malloy 2011]

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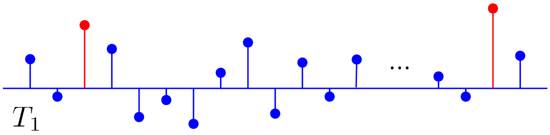

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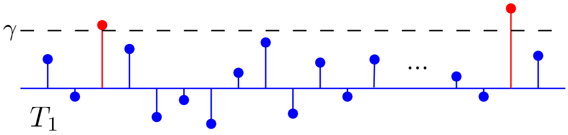

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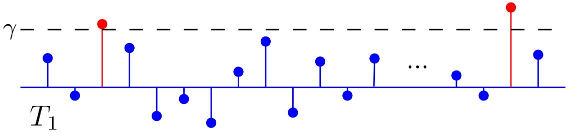

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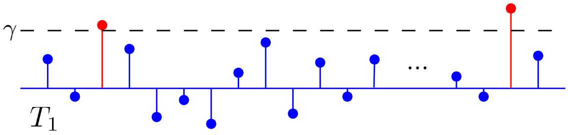

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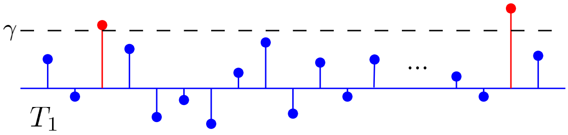

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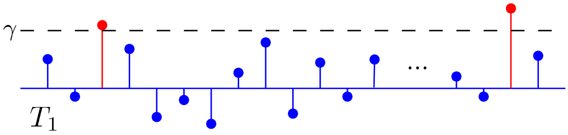

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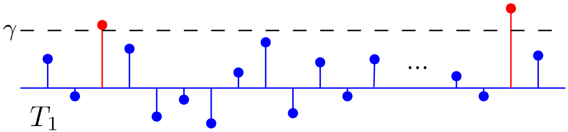

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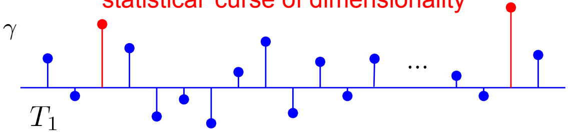

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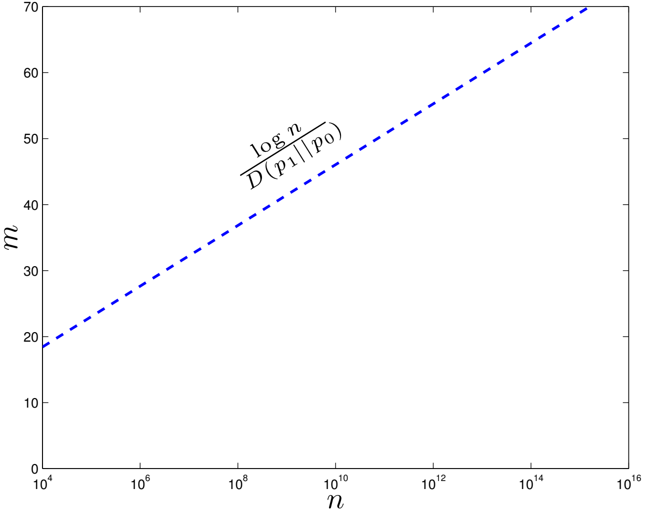

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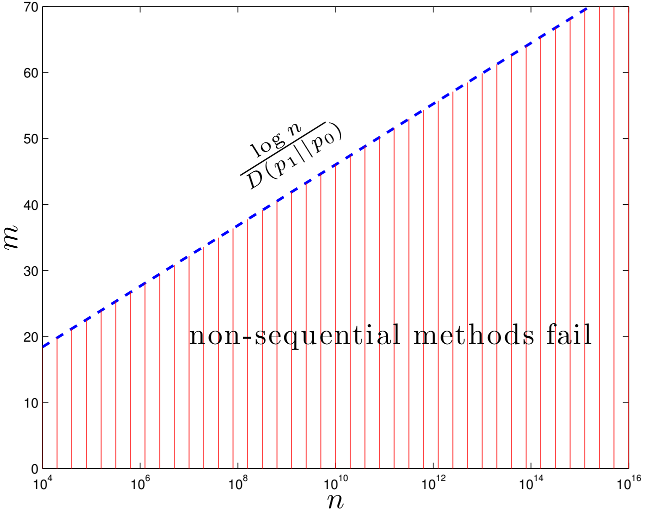

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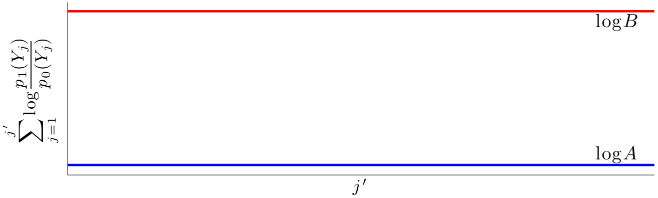

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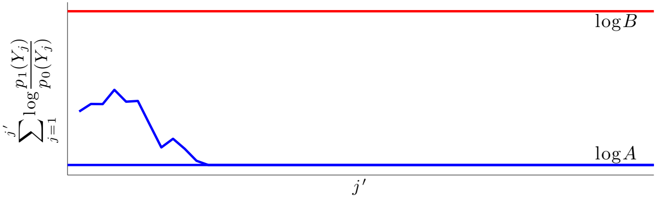

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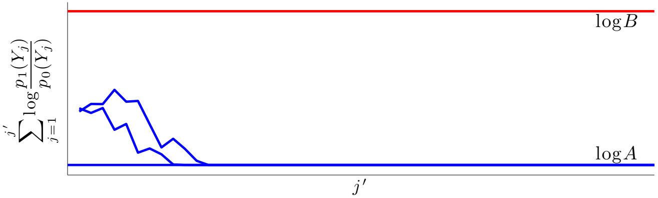

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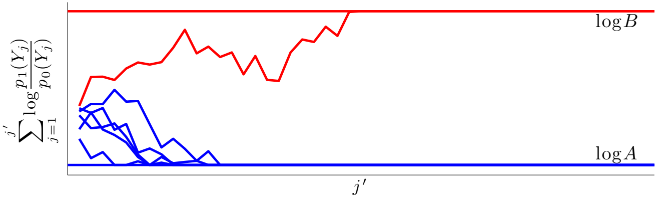

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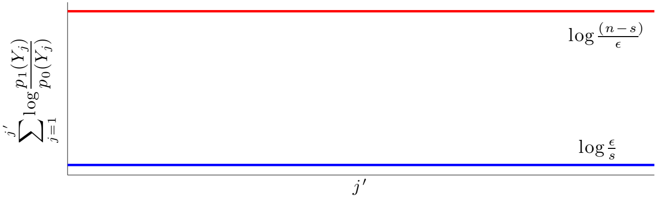

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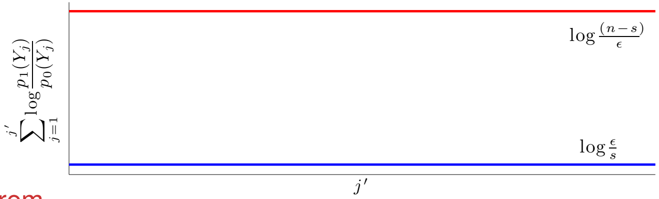

Theorem

Set $A = \frac{\epsilon}{2s}$, $B = \frac{2(n-s)}{\epsilon}$. The SPRT recovers $\mathcal{S}$ with probability

$$\mathbb{P}(\hat{\mathcal{S}} = \mathcal{S}) \geq 1 - \epsilon$$

and requires fewer than

$$m \leq (1 + \epsilon_0) \frac{\log s + \log \epsilon^{-1}}{D(P_0 || P_1)} \quad (1)$$

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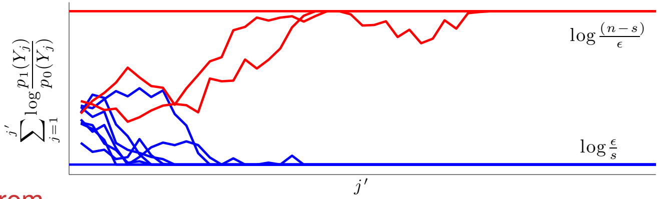

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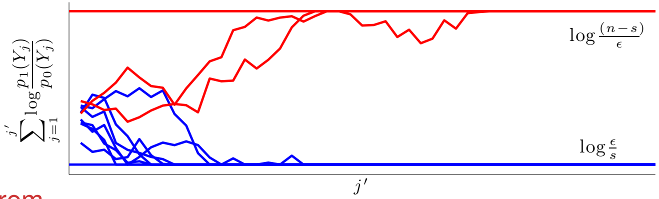

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There exist thresholds A and B such that the SPRT recovers $\mathcal{S}$ exactly if

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Results

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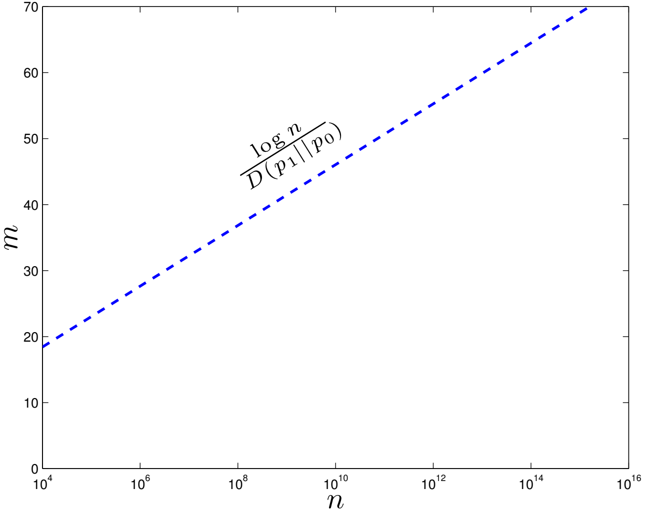

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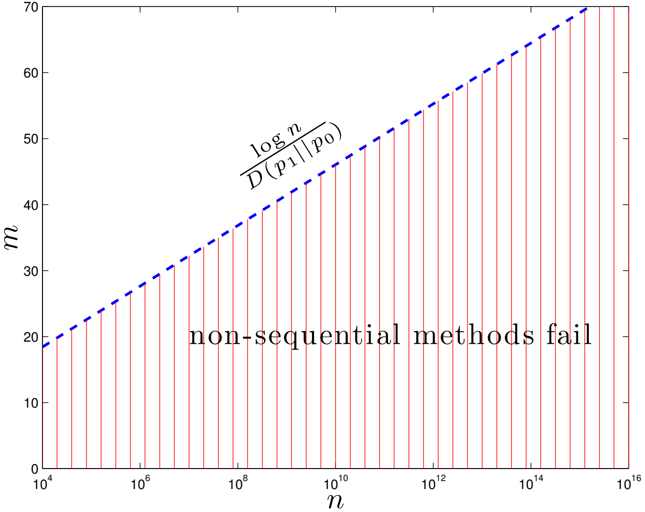

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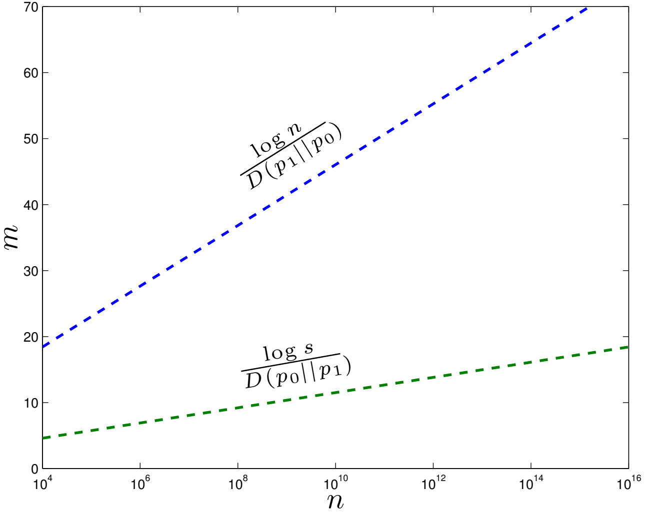

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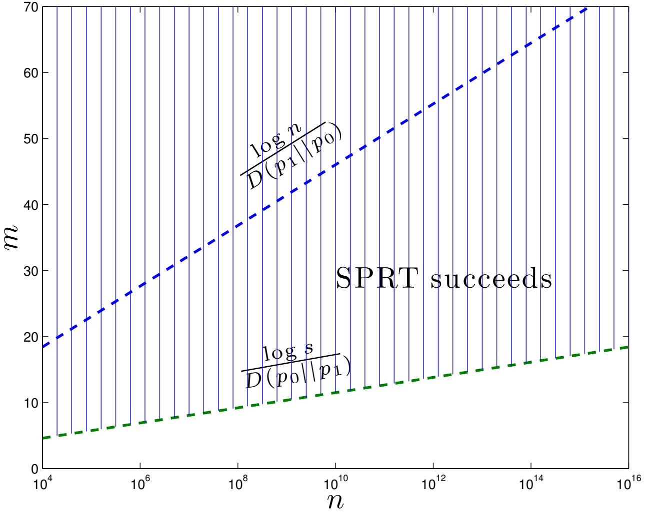

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Sketch of coordinate-wise lower bound

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- *1)* for $s \ll n$

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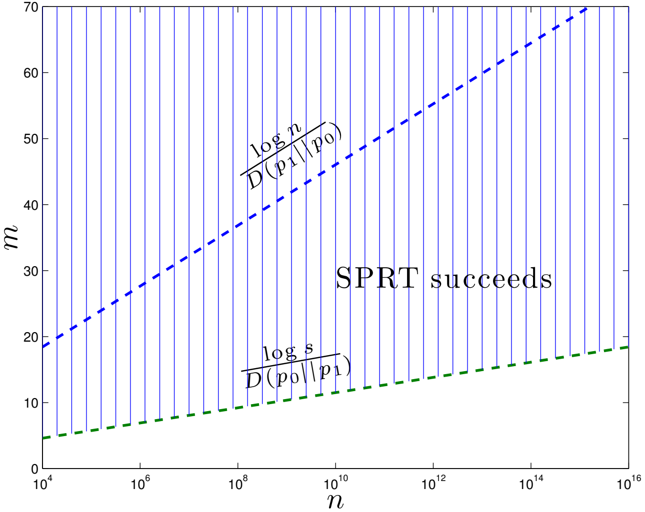

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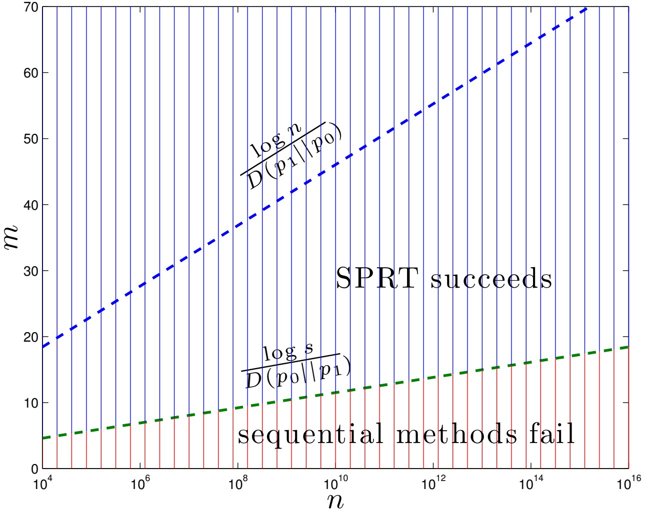

SPRT Implementation issues

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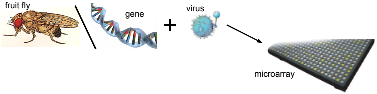

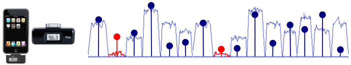

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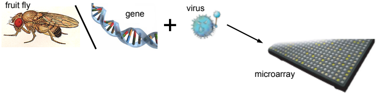

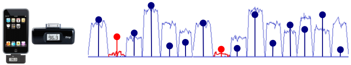

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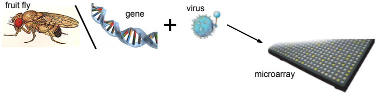

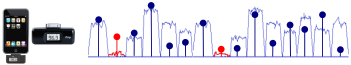

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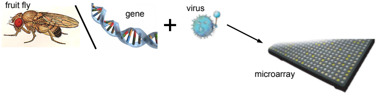

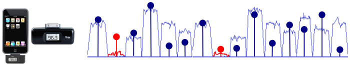

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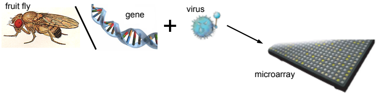

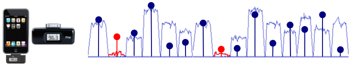

- SPRT not optimal for composite tests: consider $P_0 \sim \mathcal{N}(0, 1)$ and $P_1 \sim \mathcal{N}(\mu, 1)$, $\mu > 0$ unknown

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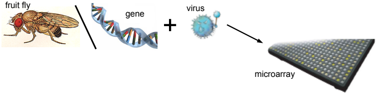
SPRT Implementation issues



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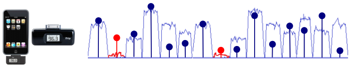

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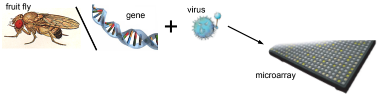

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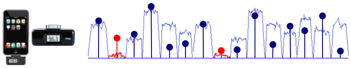

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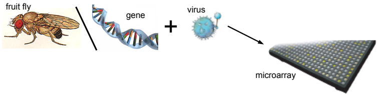

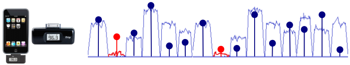

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- thresholds depend on μ, number of samples
- What can we do without knowledge of P_1 or s?

Sequential Thresholding

Input:

- $K \approx \log n$ measurement passes
- threshold $\gamma : \mathbb{P}_0 \left(T^{(m/2)} \leq \gamma \right) = \frac{1}{2}$

Sequential Thresholding

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Sequential Thresholding

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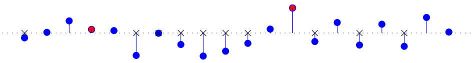
$k = 1$



1) sample each index $\frac{m}{2}$ times

$$\mathbb{E}\left[\sum_{i=1}^{n} J_i\right] \approx \frac{mn}{2}$$

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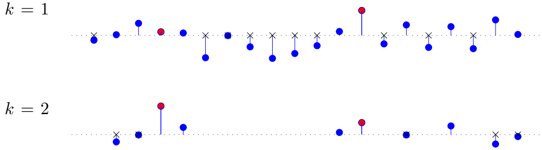

1) sample each index $\frac{m}{2}$ times

2) re-measure only indices above threshold

$$\mathbb{E}\left[\sum_{i=1}^{n} J_i\right] \approx \frac{mn}{2} + \frac{mn}{4}$$

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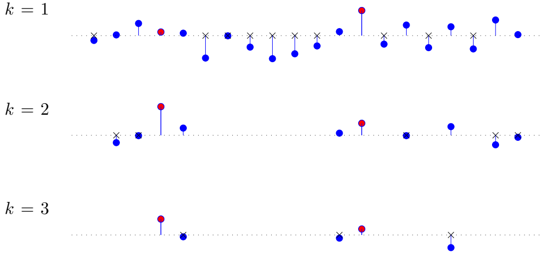

1) sample each index $\frac{m}{2}$ times

2) re-measure only indices above threshold

3) repeat

$$\mathbb{E}\left[\sum_{i=1}^{n} J_i\right] \approx \frac{mn}{2} + \frac{mn}{4} + \frac{mn}{8}$$

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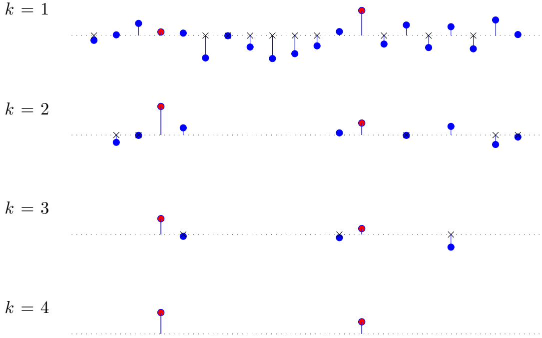

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4) ...

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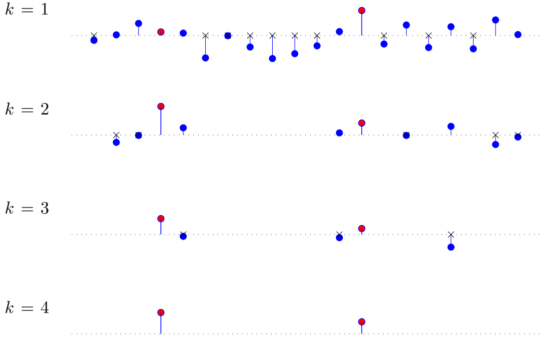

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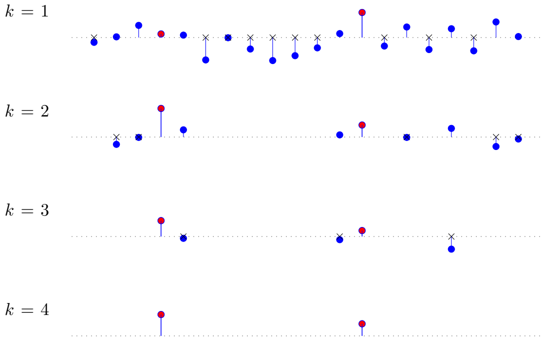

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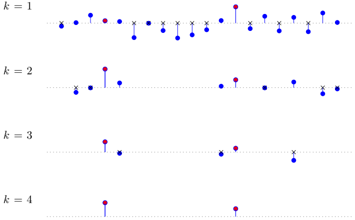
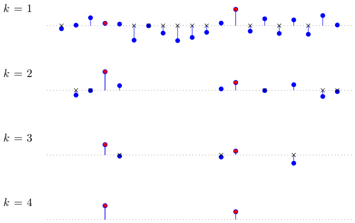
After $K \approx \log n$ passes, return remaining indices!

Controlling Family Wise Error Rates



Controlling Family Wise Error Rates

$$\mathbb{P}(\hat{\mathcal{S}} \neq \mathcal{S}) \quad = \quad \mathbb{P}\left(\left\{\bigcup_{i \notin \mathcal{S}} \bigcap_{k=1}^{K} T_{i,k} \geq \gamma\right\} \cup \left\{\bigcup_{i \in \mathcal{S}} \bigcup_{k=1}^{K} T_{i,k} \leq \gamma\right\}\right)$$



Controlling Family Wise Error Rates

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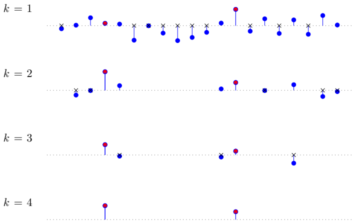

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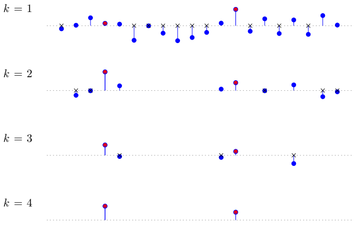

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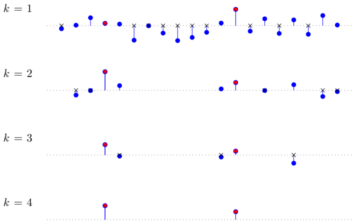

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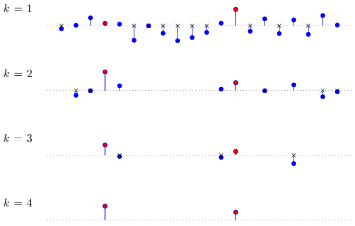

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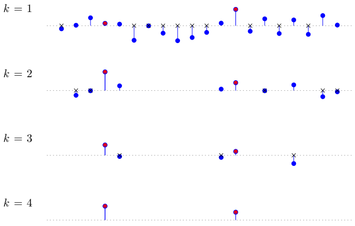

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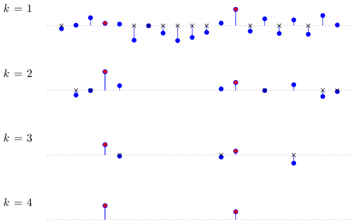

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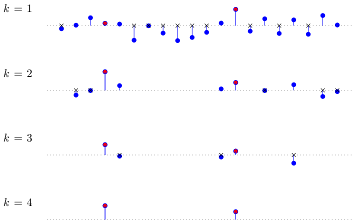

Theorem

Sequential Thresholding succeeds in exactly recovery of $\mathcal{S}$ if

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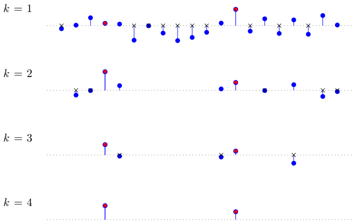
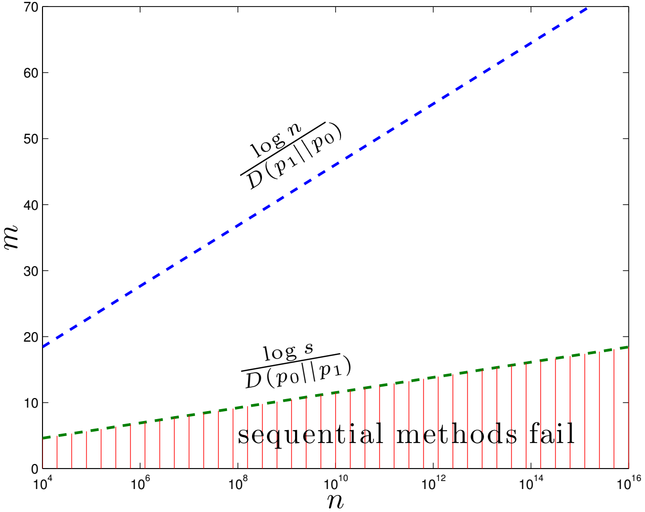
Note: For certain levels of sparsity, ST is asymptotically optimal!



Results

$s = n^{\frac{1}{4}}$ $\qquad$ $D(P_0||P_1) = \frac{1}{2}$.



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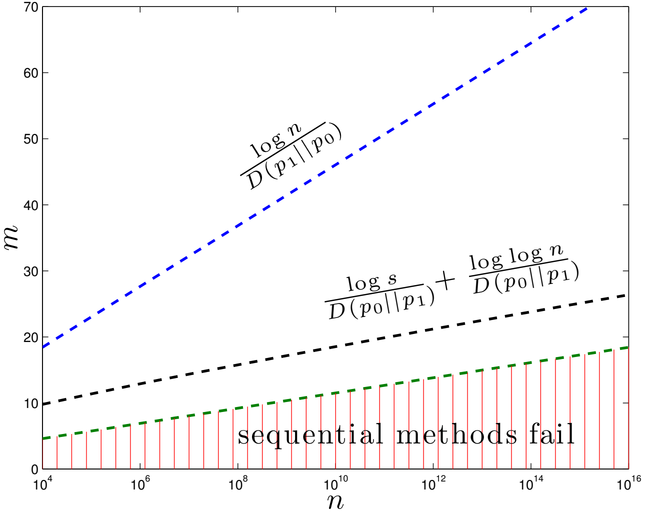

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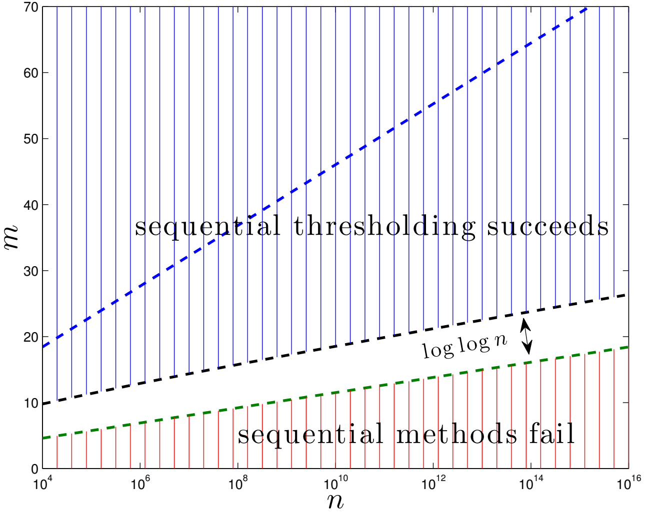
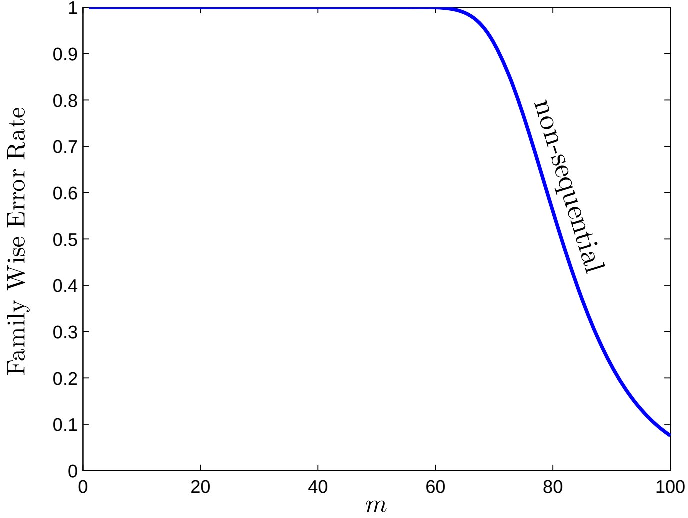


Results

$n = 10^9$ $\qquad s = n^{\frac{1}{4}}$ $\qquad D(P_0||P_1) = \frac{1}{2}.$



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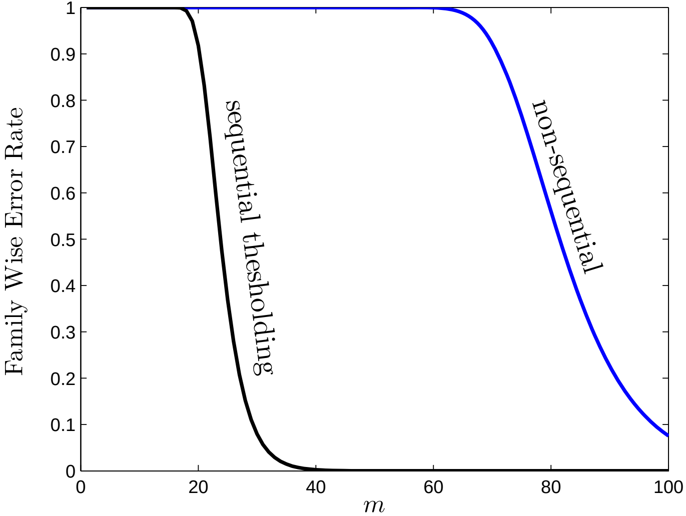

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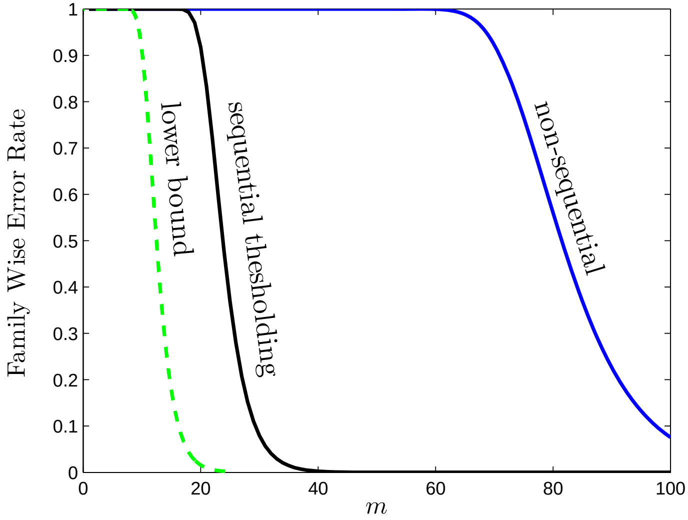

Conclusion

Remaining questions: can procedures remove doubly logarithmic gap without full knowledge of distributions?

For further reading:

M. Malloy, R. Nowak
Sequential Analysis in High Dimensional Multiple Testing and Sparse Recovery.
ISIT 2011.

M. Malloy, R. Nowak
On the limits of Sequential Testing in High Dimensions.
Asilomar 2011.