

Algebraic K-Theory and Equivariant Homotopy Theory

February 12-17, 2012

MEALS

*Breakfast (Buffet): 7:00–9:30 am, Sally Borden Building, Monday–Friday

*Lunch (Buffet): 11:30 am–1:30 pm, Sally Borden Building, Monday–Friday

*Dinner (Buffet): 5:30–7:30 pm, Sally Borden Building, Sunday–Thursday

Coffee Breaks: As per daily schedule, in the foyer of the TransCanada Pipeline Pavilion (TCPL)

*Please remember to scan your meal card at the host/hostess station in the dining room for each meal.

MEETING ROOMS

All lectures will be held in the new lecture theater in the TransCanada Pipelines Pavilion (TCPL). LCD projector and blackboards are available for presentations.

SCHEDULE

Sunday

- 16:00 Check-in begins (Front Desk - Professional Development Centre - open 24 hours)
Lecture rooms available after 16:00 (if desired)
- 17:30–19:30 Buffet Dinner, Sally Borden Building
- 20:00 Informal gathering in 2nd floor lounge, Corbett Hall (if desired)
Beverages and a small assortment of snacks are available on a cash honor system.

Monday

- 7:00–8:45 Breakfast
- 8:45–9:00 Introduction and Welcome by BIRS Station Manager, TCPL
- 9:00 Mike Hill – All Aboard the Good Time Express
- 10:00 Coffee Break, TCPL
- 10:30 Bjørn Dundas – Equivariant commutative ring spectra
- 11:30–13:00 Lunch
- 13:00–14:00 Guided Tour of The Banff Centre; meet in the 2nd floor lounge, Corbett Hall
- 14:00 Group Photo; meet on the front steps of Corbett Hall
- 14:30 Coffee Break, TCPL
- 15:00 Andrew Blumberg – Progress towards TC(MU)
- 16:00 Short Break
- 16:15 Dan Ramras – The K -theoretic assembly map, Rips complexes, and equivariant phantom maps
- 17:30–19:30 Dinner

Tuesday

- 7:00–9:00 Breakfast
- 9:00 Bert Guillou – G -spectra as presheaves of spectra
- 10:00 Coffee Break, TCPL
- 10:30 Peter May – Equivariant Perspectives
- 11:30–13:30 Lunch
- 13:30 Anna Marie Bohmann – Global equivariant homotopy theory
- 14:30 Coffee Break, TCPL
- 15:00 Stefan Schwede – Equivariant properties of the infinite symmetric product filtration
- 16:00 Short Break
- 16:15 Lars Hesselholt – Algebraic K-theory and reality
- 17:30–19:30 Dinner

Wednesday

7:00–9:00	Breakfast
9:00	Brooke Shipley – An algebraic model for free rational G -equivariant spectra
10:00	Coffee Break, TCPL
10:30	John Greenlees – An algebraic model for rational G -spectra
11:30–13:30	Lunch
13:30–17:30	Free time!
17:30–19:30	Dinner

Thursday

7:00–9:00	Breakfast
9:00	Doug Ravenel – The Gap theorem at 3
10:00	Coffee Break, TCPL
10:30	Angelica Osorno – Modeling stable n -types
11:30–13:30	Lunch
13:30	Mike Mandell – Localization sequences in THH
14:30	Coffee Break, TCPL
15:00	Gunnar Carlsson – Representation rings and K -theory
16:00	Short Break
16:15	John Lind – Equivariant A_∞ bundle theory
17:30–19:30	Dinner

Friday

7:00–9:00	Breakfast
9:00	Chris Douglas – TBA
11:30–13:30	Lunch
Checkout by 12 noon.	

** 5-day workshop participants are welcome to use BIRS facilities (BIRS Coffee Lounge, TCPL and Reading Room) until 3 pm on Friday, although participants are still required to checkout of the guest rooms by 12 noon. **

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ABSTRACTS

(in alphabetic order by speaker surname)

Speaker: Anna Marie Bohmann (Northwestern)

Title: *Global equivariant homotopy theory*

Abstract: Much recent work has shown that equivariant homotopy theory can give insight into the non-equivariant world. While concrete calculations focus on specific groups, many familiar objects in homotopy theory have (or we would like them to have) equivariant generalizations that feel "natural." One way of stating such naturality is by asking how these generalizations fit together across different groups of equivariance. Global equivariant homotopy theory is the study of spectra that vary naturally in the group of equivariance. Change of groups is an important tool in existing calculations, and one might hope that some calculations work globally and not just one group at a time. I will discuss the basic ideas of what we mean by "global" spectra and explain how these notions capture the naturalness we see in familiar spectra such as complex cobordism and K-theory.

Speaker: Bjørn Dundas (Bergen)

Title: *Equivariant commutative ring spectra*

Abstract: In this talk I will address the first question posed in the objectives of the conference in that I will discuss the interplay between the smash product of commutative ring spectra and equivariant structures. The talk will be partially foundational, building on the recent thesis of Martin Stolz, and partially directed towards concrete constructions displaying chromatic red shift.

Speaker: Bert Guillou (UIUC)

Title: *G-spectra as presheaves of spectra*

Abstract: We describe a model for the equivariant stable homotopy category as a category of enriched presheaves of spectra.

Speaker: Lars Hesselholt (Nagoya)

Title: *Algebraic K-theory and reality*

Abstract: This is joint work with Ib Madsen. Let $G = Gal(\mathbb{C}/\mathbb{R})$. Atiyah's Real K-theory gives rise to a G -equivariant spectrum KR whose underlying non-equivariant spectrum is equivalent to KU and whose spectrum of G -fixed points is equivalent to KO . By analogy with this construction, we associate to a pointed exact category with strict duality $(\mathcal{C}, T, 0)$ a G -equivariant spectrum $KR(\mathcal{C}, T, 0)$ that we call the Real algebraic K -theory of $(\mathcal{C}, T, 0)$. The underlying non-equivariant spectrum is equivalent to Quillen's algebraic K-theory spectrum $K(\mathcal{C}, 0)$, and the spectrum of G -fixed points is equivalent to the hermitian K -theory of $(\mathcal{C}, T, 0)$. The construction of $KR(\mathcal{C}, T, 0)$ is based on a new variant of Waldhausen's S-construction that we call the Real Waldhausen construction. To understand the G -equivariant homotopy type of $KR(\mathcal{C}, T, 0)$, we also introduce a G -equivariant spectrum $KR^\oplus(\mathcal{C}, T, 0)$ that we call the Real direct sum K -theory of $(\mathcal{C}, T, 0)$. The construction uses a variant of Segal's Γ -category construction that we call the Real Γ -category construction. A theorem of Shimakawa identifies the underlying G -infinite loop space of $KR(\mathcal{C}, T, 0)$ with the equivariant group-completion of the classifying pointed G -space $B(i\mathcal{C}, T, 0)$ of the pointed category with strict duality $(i\mathcal{C}, T, 0)$ given by the subcategory of isomorphisms. Our main theorem states that if \mathcal{C} is split-exact, then there is a canonical weak equivalence of G -spectra

$$KR^\oplus(\mathcal{C}, T, 0) \xrightarrow{\cong} KR(\mathcal{C}, T, 0).$$

We define the Real algebraic K-groups of $(\mathcal{C}, T, 0)$ to be the bi-graded family of equivariant homotopy groups

$$KR_{p,q}(\mathcal{C}, T, 0) = [S^{p,q}, KR(\mathcal{C}, T, 0)]_G$$

where $S^{p,q}$ is the (virtual) G-equivariant sphere $S^{\mathbb{R}^{p^2q}} \wedge S^{i\mathbb{R}^q}$. If (A, L, α) is a ring with antistructure, and if $(\mathcal{C}, T, 0)$ is the category finitely generated projective right A-modules with the induced duality structure, then the main theorem together with the theorem of Shimakawa identifies the groups $KR_{p,0}(\mathcal{C}, T, 0)$ with the Hermitian K-groups of (A, L, α) defined by Karoubi.

Speaker: John Lind (Johns Hopkins)

Title: *Equivariant A_∞ bundle theory*

Abstract: In recent years the foundations of multiplicative stable homotopy theory have been applied to unstable homotopy theory, giving monoidal models for A_∞ and E_∞ multiplications on topological spaces. Using this technology, we can set up a framework in which there is a good notion of a principal Π -bundle, where Π is a grouplike A_∞ space. This talk will explore the equivariant generalization in which a compact Lie group G acts on the A_∞ -space Π . There are also connections with a cocycle description of the group cohomology of G with coefficients in Π .

Speaker: Mike Mandell (Indiana)

Title: *Localization sequences in THH*

Abstract: (Joint work with Andrew Blumberg, preprint arXiv:1111.4003.) For a discrete valuation ring R with quotient field F and residue field k , you have a cofibration sequence of K-theory spectra

$$K(k) \rightarrow K(R) \rightarrow K(F).$$

The corresponding sequence in THH is not a cofibration sequence, but both the cofiber of the map $THH(k) \rightarrow THH(R)$ and the fiber of the map $THH(R) \rightarrow THH(F)$ have an interpretation in terms of the THH of Waldhausen categories. Thinking in terms of Waldhausen categories, we therefore get two cofibration sequences for THH ,

$$THH(k) \rightarrow THH(R) \rightarrow THH(F|R)$$

(first constructed by Hesselholt and Madsen) and

$$THH(\text{Spec}(k) \text{ on } \text{Spec}(R)) \rightarrow THH(R) \rightarrow THH(F)$$

(generalizing to THH a well-known exact sequence in Hochschild homology). The first arises by looking at enrichments by connective spectra and the second by looking at enrichments in non-connective spectra.

Speaker: J. Peter May (U Chicago)

Title: *Equivariant Perspectives*

Abstract: Let G be a finite group. We give an overview of some recent equivariant results, starting with a theorem that describes a model of the category of G -spectra as a category of enriched presheaves of spectra. The domain category is explicitly constructed by applying an infinite loop space machine to an elementary category of finite G -sets ‘enriched in permutative categories’. Ingredients and offshoots of the proof are new perspectives on 1. Classifying G -spaces for equivariant bundles. 2a. The definition and examples of genuine permutative G -categories and, 2b. more generally, the definition and examples of E_∞ G -categories. 3. Equivariant infinite loop space theory. 4a. The equivariant Barratt-Priddy-Quillen theorem and, as a corollary, 4b. the tom Dieck splitting theorem for suspension G -spectra. 5. Equivariant algebraic K-theory. 6. Pairings of permutative G -categories. The talk will explain how some of these ingredients are combined to prove the theorem. This is joint work with Bertrand Guillou and, in part, Mona Merling.

Speaker: Angelica Osorno (UChicago)

Title: *Modeling stable n -types*

Abstract: It is a classical result that groupoids model homotopy 1-types, in the sense that there is an equivalence between the homotopy categories, via the classifying space and fundamental groupoid functors. We extend this result to stable homotopy 1-types and Picard groupoids, that is, symmetric monoidal groupoids in which every object has a weak inverse. Using an algebraic description of Picard groupoids, we identify the Postnikov data associated to a stable 1-type; the abelian groups π_0 and π_1 , and the unique k -invariant. We also briefly explore the case for $n=2$, where we expect stable homotopy 2-types to be modelled by Picard bigroupoids. Joint with Niles Johnson

Speaker: Dan Ramras (New Mexico)

Title: *The K -theoretic assembly map, Rips complexes, and equivariant phantom maps*

Abstract: I'll discuss a new (partly conjectural) approach, based on Rips complexes, to the classical assembly map for the K -theory of group rings. In the last 20 years, a great deal of progress has been made by studying the assembly map via large-scale geometry. This requires some means for accessing the large-scale structure of the group G , and Rips complexes offer a canonical approach while simultaneously providing co-compact approximations to EG (for G torsion-free). For example, my recent joint work with Tessera and Yu uses Rips complexes, along with Carlsson's descent method, to establish injectivity of the assembly map for linear groups with finite classifying spaces. For groups with level-wise finite classifying spaces, work of Bökstedt-Hsiang-Madsen shows that the kernel of the assembly map is torsion. The ideas discussed here lead to a relationship between the kernel of the assembly map (for torsion-free linear groups with level-wise finite classifying spaces) and a notion of equivariant phantom maps.

Speaker: Brooke Shipley (UIC)

Title: *An algebraic model for free rational G -equivariant spectra*

Abstract: In this talk I will address the first question posed in the objectives of the conference in that I will discuss the interplay between the smash product of commutative ring spectra and equivariant structures. The talk will be partially foundational, building on the recent thesis of Martin Stolz, and partially directed towards concrete constructions displaying chromatic red shift.