ABSTRACTS of 12w5067 WORKSHOP
Monday, May 7, 2012

Idun Reiten:
McKay correspondence and higher Auslander and preprojective algebras

Abstract: This talk is based upon work with Claire Amiot and Osamu Iyama. By Auslander's McKay correspondence, the stable category of (maximal) Cohen - Macaulay modules over a simple singularity in dimension 2 is equivalent to the 1-cluster category of the path algebra of a Dynkin quiver. We discuss a method for constructing a similar type of equivalence between the stable category of Cohen-Macaulay modules over a Gorenstein singularity R and the generalized cluster category of some finite dimensional algebra, using higher Auslander algebras and higher preprojective algebras. Here Calabi - Yau algebras play a central role.

Wolfgang Ebeling:
Mirror symmetry between orbifold curves and cusp singularities with group action

Abstract: We consider an invertible polynomial in three variables together with a finite group of symmetries containing the exponential grading operator and the Berglund-Henningson-Hübsch transpose of this pair.

We show that this defines a mirror symmetry between orbifold curves and cusp singularities with group action. We define Dolgachev numbers for the orbifold curves and Gabrielov numbers for the cusp singularities with group action. We show that these numbers are the same and that the stringy Euler number of the orbifold curve coincides with the equivariant Milnor number of the mirror cusp singularity with respect to the dual group. We also give a formula for the variance of the spectrum of a cusp singularity with group action and relate it to the cohomology of the mirror orbifold curve.

This is joint work with A.~Takahashi.

Xiao-Wu Chen:
Some irreducible representations of Leavitt path algebras

Abstract: Recently, Paul Smith studied point modules over path algebras of finite quivers. These modules correspond to some "points" on certain non-commutative schemes. Inspired by Smith's results, we relate these modules to some irreducible modules over Leavitt path algebras. The latter might be described via algebraic branching systems.

Nathan Broomhead:
Dimer models and non-commutative crepant resolutions

Abstract: Dimer models, as studied in theoretical physics, can be used to produce non-commutative crepant resolutions (NCCRs) of all Gorenstein 3-fold affine toric singularities. In this talk I will give an introduction, with examples, to dimer models in this context. In particular I shall explain how to construct a non-commutative algebra and a toric singularity associated to a dimer model. I will then talk about the `consistency' condition that underlies the NCCR property.

Hagen Meltzer:
Rank two vector bundles and Happel-Seidel symmetry

Abstract: This is a report on joint work with Dirk Kussin and Helmut Lenzing.
We study stable categories of vector bundles on weighted projective lines **X.**
In particular we show that each indecomposable vector bundle of rank two is exceptional in the category of coherent sheaves and also in the stable category of vector bundles on **X.**
We use those rank two bundles for the construction of tilting objects in the stable category.

As an easy consequence we obtain equivalences of derived categories for certain Nakayama algebras given by equi-oriented finite quivers, equipped with all nilpotency relations of a fixed degree, which first have been observed by Happel and Seidel. Moreover we study some spontaneous series of those algebras.

Henning Krause:
Koszul, Ringel, and Serre duality for strict polynomial functors

Abstract: Strict polynomial functors were introduced by Friedlander and Suslin in their work on the cohomology of finite group schemes. More recently, a Koszul duality for strict polynomial functors has been established by Chalupnik and Touze. In my talk I will give a gentle introduction to strict polynomial functors (via representations of divided powers) and will explain the Koszul duality, making explicit the underlying monoidal structure which seems to be of independent interest.
Then I connect this to Ringel duality for Schur algebras and describe Serre duality for strict polynomial functors.

ABSTRACTS (Tuesday)

Lutz Hille: Exceptional and spherical modules over the Auslander algebra of k[T]/T^t

This is joint work with David Ploog.

The Auslander algebra A of the truncated polynomial ring k[T]/T^t appears in many contexts. In this note we focus on a chain of (-2)-curves on a rational surface and a certain category of sheaves associated with this configuration. It turns out that this category is equivalent to the category of modules over A. We also give a short overview on further applications.

In a joint work with Brüstle, Ringel and Röhrle we have classified all tilting modules of projective dimension at most one. We extend the classification to all spherical modules, all exceptional modules and eventually all rigid modules and all tilting modules. If time permits we also give an introduction to exceptional complexes and spherical complexes.

The final result is surprisingly easy to understand, the classification of all exceptional modules, this seems to be the crucial part, is obtained using certain diagrams.

Steffen Oppermann: n-representation infinite algebras

This talk is based on joint work with Martin Herschend and Osamu Iyama.

n-representation infinite algebras generalize representation infinite hereditary algebras in that they are defined by the existence of a higher dimensional version of preprojective and preinjective components (in the sense of Iyama's higher Auslander-Reiten theory). Motivated by classical representation theory of non-Dynkin quivers, it is natural to ask about a version of regular modules for n-representation infinite algebras. We suggest a definition, and point out how we can use a connection to non-commutative geometry (due to Minamoto) to better understand these n-regular modules.

Martin Herschend: Skew-group and higher preprojective algebras

This talk is based on joint work with Osamu Iyama and Steffen Oppermann. In higher dimensional Auslander-Reiten theory there is an analogue of representation infinite hereditary algebras called n-representation infinite. They can be characterized by the fact that their higher preprojective algebras are (n+1)-Calabi-Yau. Skew-group algebras of the polynomial algebra by finite subgroups H of SL_{n+1} are (n+1)-Calabi-Yau. By introducing a certain type of gradings of such Skew-group algebras I will show how they can be viewed as higher preprojective algebras for the case when H is abelian.

Paul Smith: The space of Penrose tilings as a non-commutative curve

The space X of Penrose tilings of the plane has a natural topology on it. Two tilings are equivalent if one can be obtained from the other by a translation. The quotient topological space X/~ is bad: every point in it is dense. The doctrine of non-commutative geometry is to refrain from passing to the quotient and construct a non-commutative algebra that encodes some of the data lost in passing to X/~. In this example (see Connes book for details) the relevant non-commutative algebra is a direct limit of products of matrix algebras. We will obtain this non-commutative algebra by treating R=k{x,y}/(y^2) as the homogeneous coordinate ring of a non-commutative curve. The category of quasi-coherent sheaves on this non-commutative curve is defined to be the quotient category

QGr(R)=Gr(R)/Fdim(R), the quotient of the category of graded R-modules modulo the full subcategory of modules that are the sum of their finite-dimensional submodules. We show that QGr(R) is equivalent to Mod(S) the category of modules over an infinite dimensional simple von Neumann regular ring S. The norm closure of S is the C^*-algebra that Connes associates to X/~. We will discuss algebraic analogues of various topological features of X/~. For example, the non-vanishing of extension groups between simple modules is analogous to the fact that every point in X/~ is dense (which is analogous to the fact that any finite region of one Penrose tiling appears infinitely often in every other tiling).

There is also a quiver Q such that QGr(kQ) is equivalent to QGr(R), and by a result of Xiao-Wu Chen the singularity category $D^b_{sing}(kQ/kQ_{\ge 2})$ is equivalent to mod(S) made into a triangulated category in a natural way.

Sefi Ladkani: Mutation classes of quivers with constant number of arrows and derived equivalences

We characterize the quivers with the property that performing arbitrary sequences of Fomin-Zelevinsky mutations does not change their number of arrows. We also show that to each such quiver there is a naturally associated potential such that performing arbitrary sequences of QP mutations does not change the derived equivalence class of the corresponding Jacobian algebra. Thus, for these quivers (and potentials), mutation at ANY vertex behaves quite like the BGP reflection both combinatorially and algebraically.

It turns out that these quivers arise from ideal triangulations of certain marked bordered surfaces. Most of the associated Jacobian algebras are finite-dimensional and gentle, but some of them are infinite-dimensional and locally gentle. The latter resemble the 3-Calabi-Yau algebras despite not being so.

The derived categories of some of these algebras can be arranged in sequences. Moreover, there is also a covering by an infinite quiver with potential arising from triangulations of an infinity-gon.

Ragnar-Olaf Buchweit: Maximal Cohen-Macaulay Modules and Orlov's Theorem I and II

One of the deepest results in the theory of maximal Cohen-Macaulay modules surely is Orlov's theorem from 2005 that relates the stable category of graded such modules over a not necessarily commutative graded Gorenstein algebra A to the derived category of coherent sheaves on the underlying (virtual) projective scheme.

In these talks we explain this result and the ingredients of its proof, showing how it defines for any graded A-module an octahedron of complexes of such modules that connects projective geometry and maximal Cohen-Macaulay approximations.

We will discuss in some detail the case of graded matrix factorizations, or graded maximal Cohen-Macaulay modules, over (quasi-)homogeneous hypersurface and complete intersection rings.

ABSTRACTS (Wednesday - Friday)

Hiroyuki Minamoto: Ampleness of two-sided tilting complexes and Fano algebras

In my talk I give an exposition on ampleness of two-sided tilting complexes and Fano algebras. In particular I will show by example that certain property of Serre functor captures representation theoretic property. I will also explain that there is a strong relation between Fano algebras and AS-regular algebras (joint work with I. Mori). If time permit I will discuss the relation between higher dimensional A-R theory and graded coherentness of higher preprojective algebras.

David Favero: Variation of Geometric Invariant Theory for Derived Categories

Given a quasi-projective algebraic variety X, with the action of a linear algebraic group G, there are various (birational) incarnations of the quotient X/G coming from a choice of a G-equivariant ample line bundle. As we vary this choice, there is a semi-orthogonal relationship between the derived categories of the resulting quotients, A and B. Furthermore, if (X, w) is a Landau-Ginzburg model, and w is a G-invariant section of a line bundle on X, then the same holds for "coherent sheaves on" (A, w) and $B, w)$ (categories of matrix factorizations/categories of singularities/stable derived categories). As a special case, one can reproduce a theorem of Orlov relating categories of coherent sheaves for complete intersections in projective space to the graded category of singularities of the cone, a theorem of Herbst and Walcher demonstrating an equivalence of derived categories between "neighboring" Calabi-Yau complete intersections in toric varieties, and two theorems of Kawamata; one concerning behavior of derived categories of algebraic varieties under simple toroidal flips, the other stating that the derived category of coherent sheaves on any smooth toric variety has a full exceptional collection (in the projective case). If time permits, I will also discuss the relationship with Kuznetsov's homological projective duality.

Izuru Mori: Fixed subalgebras, skew group algebras and endomorphism algebras of AS-regular algebras

AS-regular algebras are the most important class of algebras studied in noncommutative algebraic geometry. They are the noncommutative analogues of the polynomial algebra and graded Morita equivalent to higher preprojective algebras by the recent joint work with Minamoto. Suppose that a finite cyclic group G acts on an AS-regular algebra S. In this talk, we will relate the fixed subalgebra of S by G, the skew group algebra of S by G, the endomorphism algebra of S over S^G, and the higher preprojective algebra obtained by McKay quiver of G. If time permits, we will also try to explain the relationship to the resolution of the quotient singularity of the quantum projective space associated to S.

Kazushi Ueda: Mirror symmetry for K3 surfaces and singularities

This talk is based on joint works with Masahiro Futaki, Masanori Kobayashi, and Makiko Mase.

We take an example of the E_{12} singularity and discuss relations between the graded singularity category studied by Kajiura-Saito-Takahashi and Lenzing-de la Pena, the derived category of coherent sheaves on the K3 surface obtained as the compactification of the Milnor fiber and a spherical collection on it studied by Ebeling-Ploog, and the derived category of coherent sheaves on the corresponding weighted projective space which admits a description in terms of a Beilinson-type quiver, with application to mirror symmetry in mind.

Michael Wemyss: Cluster tilting via birational geometry

There is now a well-developed conjectural picture that relates certain aspects of birational geometry in dimension three to noncommutative algebras, which in turn are related to cluster tilting and maximal rigid objects in a particular triangulated category. I will explain some of the connections, and in particular the geometric significance of rigid objects.

Markus Schmidmeier: Operations on arc diagrams and degenerations for invariant subspaces of linear operators" vortragen

We study geometric properties of varieties associated with invariant subspaces of nilpotent operators. There are reductive algebraic groups acting on these varieties. We give dimensions of orbits of these actions. Moreover, a combinatorial characterization of the partial order given by degenerations is described. This is a report about a joint project with Justyna Kosakowska from Torun.

Charles Paquette: Some results on the Auslander-Reiten theory for the representations of infinite quivers

In this talk, I will present some results on the Auslander-Reiten theory for the representations of (strongly locally finite) infinite quivers with relations. It is possible to get all the almost split sequences of $\mathrm{rep}(Q, I)$ when I is a nice relation ideal. When there are no relations, we have a complete description of the Auslander-Reiten components of the category of finitely presented representations of Q. Representations of infinite quivers (with or without relations) have many applications. They could, for example, be related to the first test problem of this workshop.

Dirk Kussin: Triangle singularities: Orlov's theorem and Auslander bundles

This is about joint work with Helmut Lenzing and Hagen Meltzer. For triangle singularities $S = k[X, Y, Z]/(X^a + Y^b + Z^c)$, graded by the abelian group $\mathbf{L} = \mathbf{L}(a, b, c)$, there is the $\mathbf{L}$-graded version of Orlov's theorem which shows the relation between the triangulated categories $D^b(\mathrm{coh}\mathbf{X})$ and $\underline{\mathrm{vect}}\mathbf{X} = \mathbf{D}^{\mathbf{b},\mathbf{L}}_{\mathbf{Sg}}(\mathbf{S}) = \underline{\mathrm{CM}}^{\mathbf{L}}\mathbf{S}$; here, the simple graded S-module k corresponds to an Auslander bundle in

$\underline{\mathrm{vect}}\mathbf{X}$. We construct the cuboid tilting object in $\underline{\mathrm{vect}}\mathbf{X}$. The associated simple modules over this cuboid form a complete exceptional sequence of $\underline{\mathrm{vect}}\mathbf{X}$ consisting of Auslander bundles. In case $(a, b, c) = (2, 3, p)$ we find even a *strong* complete exceptional sequence consisting of Auslander bundles. Assuming negative Gorenstein parameter (wild case), Orlov's theorem leads to the question how "well" in general the "canonical configuration" can be extended to a complete exceptional sequence.

Colin Ingalls:Rationality of Brauer-Severi Varities of Sklyanin Algebras

Iskovskih's conjecture states that a conic bundle over a surface is rational if and only if the surface has a pencil of rational curves which meet the discriminant in 3 or fewer points, (with one exceptional case). We generalize Iskovskih's proof that such conic bundles are rational, to the case of projective space bundles of higher dimension. The proof involves maximal orders and toric geometry. As a corollary we show that the Brauer-Severi variety of a Sklyanin algebra is rational.

Atsushi Takahashi: Classical mirror symmetry between orbifold projective lines and cusp singularities We report on our recent study on the mirror symmetry between orbifold projective lines and cusp singularities. After preparing some necessary notations and terminologies, we discuss the homological mirror symmetry, an equivalence of triangulated categories. Then, we talk about the classical mirror symmetry, an isomorphism of Frobenius manifolds from Gromov–Witten theory for an orbifold projective line and the one from the unfolding of the cusp singularity.

Lidia Angeleri-Hügel: Large tilting modules and quasi-coherent sheaves

The category of quasi-coherent sheaves over the projective line can be viewed as the heart of a t-structure induced by the "largest" infinite dimensional cotilting module over the Kronecker algebra A, and the Serre localizations of this heart correspond bijectively to the equivalence classes of infinite dimensional cotilting A-modules. A similar phenomenon occurs in more general contexts. We will discuss how to exploit this in connection with classification problems. The talk will be based on joint work with Javier Sanchez and on work in progress with Dirk Kussin.

Osamu Iyama: Cluster tilting for Cohen-Macaulay modules

I will discuss cluster tilting for maximal Cohen-Macaulay modules.A nice class of (non-commutative) Gorenstein algebras withcluster tilting modules is constructed from n-representationinfinite algebras. We give a generalization of Auslander's algebraicMcKay correspondence between cluster categories and stable categoriesof maximal Cohen-Macaulay modules.