Noise Driven Solutions of Schooling Fish and a A Cellular Automata Model for Biofilm Growth with Surface Flow

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Initial equations

Birnir (2007) analyzed the following equations:

$$\begin{pmatrix} \cos(\phi_k(t+\Delta t)) \\ \sin(\phi_k(t+\Delta t)) \end{pmatrix} = \frac{1}{N}\sum_{j=1}^{N} \begin{pmatrix} \cos(\phi_j(t)) \\ \sin(\phi_j(t)) \end{pmatrix} \tag{1}$$

Note that equation (1) is the same for all k. It is then clear that the direction of each fish in each time iteration becomes the average of all the directions.

Polar coordinates and inertia

In polar coordinates we have

$$z_k = r_k e^{i\theta_k} \tag{2}$$

and

$$\dot{z}_k = v_k e^{i\phi_k} \tag{3}$$

and we add inertia $\beta = 1/\alpha$ to the latter equation:

$$\beta \ddot{z}_k + \dot{z}_k = v_k e^{i\phi_k} \tag{4}$$

Equations with inertia

Birnir arrived at the following equations for the speeds and direction angle:

$$\dot{v}_k = \frac{\alpha}{N^2} \sum_{i=1}^{N} v_i \sum_{j=1}^{N} \cos(\phi_j - \phi_k) - \alpha v_k \tag{5}$$

$$v_k \dot{\phi}_k = \frac{\alpha}{N^2} \sum_{i=1}^{N} v_i \sum_{j=1}^{N} \sin(\phi_j - \phi_k) \tag{6}$$

and for the position:

$$\dot{r}_k = v_k \cos(\phi_k - \theta_k) \tag{7}$$

$$r_k \dot{\theta}_k = v_k \sin(\phi_k - \theta_k), \tag{8}$$

Solutions

Birnir (2007) found several solutions.

i) *migratory solutions*, $\phi_k = \Phi$

ii) *stationary solutions*, $\phi_k = \omega_k$

for all k, where ω_k is the k-th root of unity. The speeds behave as follows:

i) $v_k \to \nu$

ii) $v_k \to 0$

We can actually find a family of stationary solutions.

Order parameter

Now, we introduce the usual order parameter,

$$re^{i\psi} := \frac{1}{N}\sum_{j=1}^{N} e^{i\phi_j} \tag{9}$$

where $r(t) \in [0,1]$ measures the coherence of the population and $\psi(t) \in]-\pi,\pi]$ is the average phase.

Simplified equations

$$\dot{v}_k = \frac{\alpha}{N} \sum_{i=1}^{N} v_i r \cos(\psi - \phi_k) - \alpha v_k \tag{10}$$

$$v_k \dot{\phi}_k = \frac{\alpha}{N} \sum_{i=1}^{N} v_i r \sin(\psi - \phi_k). \tag{11}$$

With $\bar{v} := \frac{1}{N} \sum_{i=1}^{N} v_i$ the above equations simplify even further:

$$\dot{v}_k = \alpha \bar{v} r \cos(\psi - \phi_k) - \alpha v_k \tag{12}$$

$$v_k \dot{\phi}_k = \alpha \bar{v} r \sin(\psi - \phi_k). \tag{13}$$

Behavior of average speed

Now, by summing up the equations for $\dot{v}_k$ and using identities from the order parameters, we can arrive at the following equations:

$$\begin{aligned} \dot{\bar{v}} &= \alpha \bar{v} \left(r \frac{1}{N} \sum_{k=1}^{N} \cos(\psi - \phi_k) - 1 \right) \\ &= \alpha \bar{v} (r^2 - 1). \end{aligned} \tag{14}$$

Similarly, equation for the direction angle turns into

$$v_k \dot{\phi}_k = \alpha \bar{v} r \sin(\psi - \phi_k). \tag{15}$$

Future work

We have now shown that the system of equations tends to a stationary solution unless the particles are perfectly aligned!

- There exists a whole family of solutions with $v_k \to 0$.
- The system included no random noise.
- Want to add noise to the system and investigate whether we can obtain the same structure of solutions with $\bar{v} = \nu > 0$.

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Part II

Biofilms

Joint work with

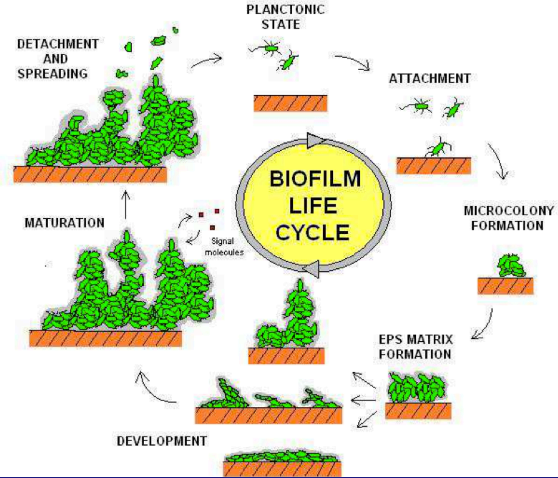
Prof. Ana Carpio and David Rodriguez

Biofilms

Biofilms appear in various forms:

- Deadly diseases (cystic fibrosis, legionellosis. . .)
- Infections in artificial joints, pacemakers, catheters, . . .
- Cause erosion on aircraft fuselage or metallic structures
- Contaminate water or food supplies

Life cycle


PLANCTONIC STATE
DETACHMENT AND SPREADING
ATTACHMENT
BIOFILM LIFE CYCLE
MATURATION
Signal molecules
MICROCOLONY FORMATION
EPS MATRIX FORMATION
DEVELOPMENT

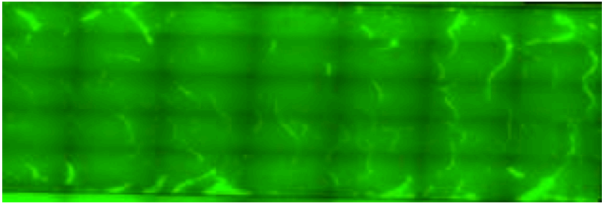
Model setting

Use a cellular automata model for the biofilm growth

- Bacteria grows in a rectangular pipe
- Water flows with a controllable Reynolds number
- Influx of nutrients controlled
- Discrete in time and space

Pseudomonas putida

We observe the species *Pseudomonas putida* and try to obtain its behavior.



(Experiments done by David Rodriguez at UCM)

Model

The following mechanism affect the behavior of cells:

- Cell division and spreading
- Cell erosion at surface due to shear stress
- Production of extracellular polymeric substances (EPS)
- Influx of cells which adhere to the surface

Determined at each time step by a set of simple probabilistic rules.

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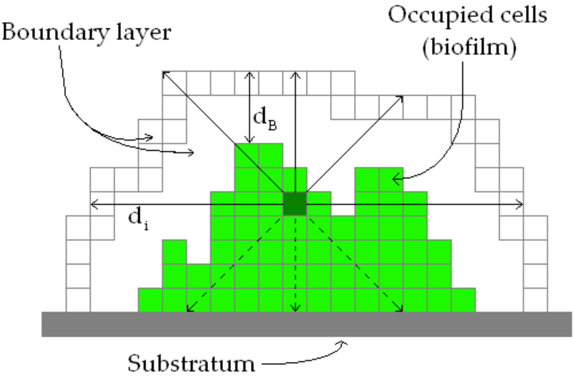
Biofilm structure



Cell division and nutrient levels

We calculate the nutrient levels, c, of each cell according to the following equation:

$$c(cell) = \left(\sqrt{C} - \sqrt{\frac{k}{2D} \left[\frac{1}{M} \sum_{i=1}^{M} \frac{1}{d_i^2} \right]^{-1}} \right)^2 \tag{16}$$

where C is the nutrient levels of the incoming flow.
(From Hermanowicz 2001)

EPS

A cell starts to produce EPS according to the following probability:

$$P_{eps} = R(R_e)\frac{1}{1+c} \tag{17}$$

where R is an increasing function of the Reynolds number, modeling the effect of the incoming flow.

- If a cell produces EPS then it does not divide.
- Also affects the biofilm's cohesion, which makes it resistant to erosion.

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Cell division

Cells divide according to the (non dimensionalized) Monod law:

$$P_{rep} = \frac{c}{1+c} \tag{18}$$

The new cell pushes cells in the direction of shortest distance to the boundary layer.

Cell erosion

Cells on the surface can erode due to the flow with probability P_e:

$$P_e = \frac{1}{1 + \frac{\sigma}{\tau}} \tag{19}$$

Here $\tau(\mathit{cell})$ denotes the flow force on the cell and $\sigma(\mathit{cell})$ is the cohesion of the surrounding biofilm. (Hermanowicz 2001))

* Both factors are determined by the structure of the neighboring cells. See appendix for details.

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Influx of cells

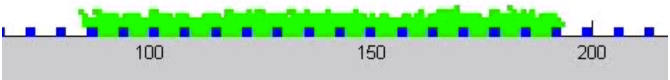
The influx of water carries cells, which can adhere to the substratum.

- N is the number of cells carried by the flow

We let νN number of cells adhere to random locations on the biofilm or substratum.

Rugosity

We also investigated the effect of rugous surfaces.



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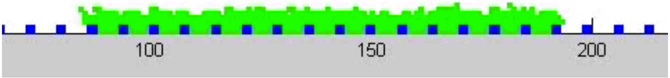

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Patterns

With varying parameter values, the model is able to reproduce observed behavior

- flat biofilm
- streamers
- ripples
- mushrooms

Movies

4 movies

Thank you!

Cell erosion

The force exerted on a cell due to the flow, $\tau(cell)$, is calculated according to

$$\tau = R(R_e)\,(1 - \beta\chi_1)\,(1 - f) \tag{20}$$

where

$$f = \frac{1}{17}\sum_{i=2}^{8} \omega_i \chi_i. \tag{21}$$

The dimensionless factor β measures the vertical erosion due to the flow. The functions χ_i are 1 if neighbor i is present, 0 otherwise. The ω_i's are weights.

Cell cohesion

The cohesion of the neighborhood of each cell is calculated as

$$\sigma = \frac{\sigma_0}{8} \sum_{i=1}^{8} \sigma_i, \tag{22}$$

where σ_0 is a parameter (here 1), and

$$\sigma_i = \begin{cases} 0 & \text{if cell } i \text{ is present} \\ \alpha & \text{if cell } i \text{ is present but does not produce EPS matrix} \\ 1 & \text{if cell } i \text{ produces EPS matrix} \end{cases} \tag{23}$$

For now, we let $\alpha = 1/2$.

References

- Birnir, B. 2007. *An ODE model of the motion of pelagic fish* Journal of Statistical Physics, 128: 535-568
- Einarsson, B., Rodriguez, D., Carpio, A. *Pattern formation in biofilms at increasing Reynolds numbers* (Submitted)
- Rodriguez, D., Einarsson, B., Carpio, A. *Biofilm growth on rugous surfaces.* (Submitted)