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Title: *A new direction in classical harmonic analysis with applications to the hyperinvariant subspace problem*
Abstract: **Definitions.** Let X be a subspace of $L^0 = L^0[0, 2\pi]$. X is called measurable set determined (msd) if $f \in X$, $f = 0$ on a set of positive measure implies $f = 0$ a.e. X is called open set determined (osd) if $f \in X$, $f = 0$ on a non-empty open subset of $[0, 2\pi]$ implies $f = 0$ a.e. $E \subset Z$ is called measurable set determining (msd) if L^1_E is msd. (Notation: For B a 'natural' Frechet space contained in L^1, $B_E = \{f \in B : \hat{f}(n) = 0 \textit{ all } n \notin E\}$). A subset E of $\mathbb{N}$ is called summable if $\sum_{n \in E} 1/n < \infty$. It is called co-summable if $\mathbb{N} \setminus E$ is summable. For $E \subset \mathbb{N} \cup \{0\}$, $sE = E \cup -E$.

Theorem 1. (A) [R] If E is summable, then L^1_{sE} is msd.
(B) [Mandelbroit, 1935] If E is co-summable, then C^∞_{sE} is not osd.

Definition. Let $\beta : \mathbb{Z} \to \mathbb{R}^+$ satisfy
(i) $\beta(0) = 1$, $\beta(n) \geq 1$ and $\beta(-n) = \beta(n)$ for all n.
(ii) $\lim_{n\to\infty} \frac{\beta(n+1)}{\beta(n)} = 1$,

$$L^2_\beta = \{f \in L^2[0, 2\pi] : \Big(\sum_{n=-\infty}^{\infty} |\hat{f}(n)|^2 \beta_n^2 \Big)^{1/2} = \|f\|_{L^2_\beta} < \infty\}$$

Theorem 2. There exists a β as above such that letting $T = M_{e^{i\theta}}$ in L^2_β, then
(i) T is unitarily equivalent to $T^{-1} = M_{e^{-i\theta}}$ and similar to T^*,
(ii) $\|T^n\| = e^{\frac{n}{\log(n+1)}}$ for all $n \geq 1$,
(iii) L^2_β is msd.

The sequence (β_n) is defined as follows: For $n \geq 1$, $1 \leq j \leq n$,
(i) $\beta_{n^2+j} = e^{\frac{j}{\log(n+1)}}$
(ii) $\beta_{n^2+n+j} = e^{\frac{n+1-j}{\log(n+1)}}$
(iii) $\beta_{n^2} = 1$, $\beta_{-n} = \beta_n$.

Conjecture T and T^{-1} have no common non-trivial invariant subspace.