Groups, graphs and stochastic processes (11w5141)

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1 Goals and an Overview of the Field

This workshop aimed to bring together experts and young researchers in the following fields: asymptotic group theory, ergodic theory, L^2-cohomology, geometric group theory, percolation, random walks, 3-manifold theory, analytic graph theory, free probability and logic. A common object of interest was unimodular random networks (in the language of probability), measure preserving actions of a countable group (in the language of group theory) or graphings (in the language of graph theory). Each field investigates this object from a different angle, and although there has already been considerable interaction, the organizers felt that learning each other's math in more depth would be productive.

The main topics of the workshops were:

- Graph convergence, both Benjamini-Schramm and local-global
- Stochastic processes on Cayley graphs
- Finite approximation of infinite graphs and processes
- L^2 cohomology, Rank gradient and cost of measure preserving actions
- Random walks on groups and compression

Limits of finite graphs. Benjamini and Schramm introduced a natural topology on bounded-degree finite graphs which has since become a central object of study in many areas of mathematics. Two graphs are close if they agree with high probability in a neighborhood of a randomly chosen vertex, called the root. This space has a natural compactification and a limit of convergent graph sequences is always a unimodular random network, that is, a probability distribution on rooted graphs that stays the same if we move the root (see work of Aldous and Lyons).

Groups come into the game at this point; for instance, if the limit is concentrated on one graph, then it must be vertex transitive. If the limit is not one point, then one gets something close to a measure-preserving group action. This connection allows one to use ergodic theory to understand finite graphs and has been succesfully used by Szegedy, Elek, Schramm and others. Energy also moves in the other direction, as some of the known phenomena on finite graphs translate to unimodular random networks and even to general ergodic theory. An example for the last is the recent theorem of Abert and Weiss that for a countable group, every free measure preserving action weakly contains its Bernoulli actions. The analogous theorem is trivial for finite graphs. Much more is expected in both ways.

Another very recent, largely unexplored area is local-global convergence, that has been defined by Hatami, Lovasz and Szegedy. Groups again come into the picture, as cowering towers of graphs, in particular, graph sequences coming from chains of subgroups of finite index are local-globally convergent. On the other hand, the corresponding notion has been recently introduced in ergodic theory by Kechris as weak containment by measure preserving actions.

Stochastic processes on Cayley graphs. One can also get unimodular random networks starting directly from groups. A finitely generated group with a generating set gives rise to a so-called Cayley graph. This graph carries a lot of information on the group and one can use geometric methods to exploit this. This area of investigation started with the work of Gromov, who characterized groups with Cayley graphs of polynomial growth.

Probabilists are interested in Cayley graphs because they are a rich source of natural homogeneous (vertex transitive) test spaces. Now one can look at natural random subgraphs of the Cayley graph. An example is edge percolation, another is a random spanning forest. In both cases, the resulting distribution will be invariant under the group action, so one gets a unimodular random network. Again, there is an interesting interplay between group invariants and the behaviour of natural stochastic processes on the Cayley graph and energy flows both ways on these connections. For instance, by the work of Lyons, the most natural definition of the first L^2 Betti number of a group is through free spanning forests. One can also study other invariant subgraphs of Cayley graph, for instance, Lyons and Nazarov recently proved that every nonamenable bipartite Cayley graph admits a factor of i.i.d. perfect matching.

Finite approximation. A group is residually finite, if the intersection of its subgroups of finite index is trivial. Many of the most interesting groups, including lattices in linear Lie groups, fall into this category. There is no strong general infinite group theory, as one can construct weird examples that defy any reasonable conjecture. However, residual finiteness is a restriction that allows one to build such a theory. For instance, by work of Zelmanov, finitely generated residually finite groups of finite exponent are finite. Another class of groups with a well-built theory are amenable groups.

It is easy to see, that for both residually finite and amenable groups, their Cayley graphs can be obtained as the Benjamini-Schramm limit of finite graphs, that is, they are sofic. It is an exciting question whether every Cayley graph (or even every unimodular random network) is sofic. A positive answer would have a far-reaching consequence on group theory, as some of the machinery invented for residually finite or amenable groups can be used for sofic groups as well. Note that the famous Connes embedding problem asks the same question, just instead of finite permutations, we want to approximate the group by finite dimensional matrices, using the trace norm.

Similarly, one can try to approximate invariant processes on infinite Cayley graphs using processes on sequences of finite graphs converging to the Cayley graph in the Benjamini-Schramm sense.

Rank, L^2 cohomology and cost. Another invariant under intense investigation is the so-called cost, introduced by Levitt and analyzed in depth by Gaboriau. The cost is a rank-type invariant for countable measure preserving actions. It comes up naturally in probability on Cayley graphs, as it derives from the expected degree of invariant connected random subgraphs and in asymptotic group theory, as, by work of Abert and Nikolov the cost of a profinite action can be expressed in terms of the growth of rank on the corresponding subgroup chain. The main question in this direction is whether the cost depends on the action, or is a group invariant (fixed price problem). This question has natural translations in both group theory, percolation theory and finite graph theory. For finite graphs, the task would be to understand how many edges one can erase from a large graph to stay bi-Lipshitz to it in the graph metric. Logicians, including Kechris or Hjorth are also interested in the cost, as it is a natural orbit equivalence invariant on measure preserving equivalence relations.

While the cost is not understood satisfactorily, L^2 theory is in much better shape. Lück approximation implies that the first L^2 Betti number equals the growth of homology on a normal chain with trivial intersection, at least for finitely presented groups. Gaboriau has recently introduced L^2 Betti numbers of measure preserving equivalence relations and showed that they are actually group invariants. That is, fixed price does hold homologically and one can hope that this will help attacking the general problem.

Random walks and compression. The structure of graphs and unimodular random networks can also be understood by studying the properties of random walks on them. An example is Kesten's theorem – a Cayley graph is amenable if and only if the return probabilities of random walks decay exponentially. A similar characterization via percolation is still open: it is widely believed that a graph is amenable if there is a parameter for which Bernoulli percolation has infinitely many infinite clusters.

Similarly, a Cayley graph supports bounded harmonic functions if and only if the random walk has positive speed – whether this property depends on the graph or only the group is a central open question. The more general connection between the asymptotic rate of escape and group-theoretic properties is also an object of recent scrutiny. Peres recently gave a simple elegant proof that random walks on groups are at least $n^{1/2}$ away after n steps. Rate of escape of walks has also been connected to Hilbert compression and other geometric properties of graphs and groups by work of Naor and Peres.

2 Recent Developments and Open Problems

We list some recent examples where ideas from one subject were applied in a ground-breaking way in another:

- Gaboriau's work on distinguishing ergodic equivalence relations by developing a measure-theoretic analogue of L^2 Betti numbers;
- Lackenby's seminal work on asymptotic invariants of finitely presented groups;
- Lück Approximation and its follow-ups;
- Abert and Nikolov's work on profinite actions connected the fixed price problem to the 'rank vs. Heegaard genus problem' in 3-manifold theory;
- Gaboriau and Lyons solved the measurable version of von Neumann's problem using percolation on transitive graphs;
- Osin constructed a finitely generated non-amenable torsion group, using cost;
- Kechris re-proved fixed price for free groups (originally a theorem of Gaboriau that used treeings) using that the cost is monotonic to weak containment and the Abert-Weiss result;
- Abert and Weiss showed that for a countable group, every free measure preserving action weakly contains its Bernoulli actions
- Monod and Epstein have used minimal spanning forests to partially solve the Diximier problem that asks whether non-amenable groups are necessarily non-unitarizable;
- Bowen has introduced a new entropy notion for actions of non-amenable groups, using sofic approximations.

Major open problems in the area include:

- Is every group sofic? That is, is every Cayley graph the limit of finite graphs?
- Does every non-hyperfinite measure preserving equivalence relation admit a free action of a free group?
- Is it true, that every invariant process on the 3-regular tree that is approximable on every sequence of finite graphs Benjamini-Schramm converging to the infinite tree, lies in the weak closure of factor of i.i.d. processes?
- Does the rank gradient depend on the chain, assuming it approximates the ambient group?
- Is the cost of a free action an invariant of the group?
- Is it true that the speed of random walk being positive does not depend on the generating set?
- What is the essential girth of finite Ramanujan graphs?
- Does every nonamenable Cayley graph admit a factor of i.i.d. perfect matching?

3 People at the workshop

The following people participated in the workshop. We put a * next to people who gave a presentation. The schedule and list of talks can be found at www.birs.ca/workshops/2011/11w5141/Schedule11w5141.pdf.

1. Abert, Miklos, Renyi Institute of Mathematics
2. * Amir, Gidi, Bar-Ilan University
3. * Babai, Laszlo, University of Chicago
4. * Bowen, Lewis, Texas A&M
5. * Brieussel, Jeremie, Neuchatel University
6. Candellero, Elisabetta, Graz University of Technology
7. Csoka, Endre, Eötvös Loránd University
8. * Elek, Gabor, Renyi Institute
9. * Friedman, Joel, University of British Columbia
10. * Gilch, Lorenz, Graz Technical University
11. * Glasner, Yair, Ben Gurion University of the Negev
12. * Grabowski, Łukasz, Goettingen University
13. Gurel-Gurevich, Ori, UBC
14. * Harangi, Viktor, Renyi Institute
15. Hegedus, Pal, Central European University
16. * Kaimanovich, Vadim, University of Ottawa
17. * Karlsson, Anders, University of Geneva
18. Kassabov, Martin, Cornell University
19. * Lee, James, University of Washington
20. Li, Xiang (Janet Lisha), University of Toronto
21. * Lippner, Gabor, Harvard
22. * Lyons, Russell, Indiana University
23. Matter, Michel, University of Geneva
24. * Mineyev, Igor, University of Illinois at Urbana-Champaign
25. * Nachmias, Asaf, MIT
26. Navas, Andres, Universidad de Santiago de Chile
27. * Peres, Yuval, Microsoft Research
28. Pete, Gabor, University of Toronto
29. * Saloff-Coste, Laurent, Cornell University
30. * Smirnova-Nagnibeda, Tatiana, University of Geneva

31. Stewart, Andrew, University of Toronto
32. * Szegedy, Balazs, University of Toronto
33. Thompson, Russ, Cornell University
34. * Timar, Adam, Hausdorff Center for Mathematics, Bonn University
35. Virag, Balint, University of Toronto
36. Weinberger, Shmuel, University of Chicago
37. * Woess, Wolfgang, Technische Universität Graz
38. * Young, Robert, University of Toronto at Scarborough

4 Presentation Highlights

The following is the list of presentations given at the workshop.

Speaker: Gideon Amir (Bar Ilan University)

Title: Liouville property of automaton groups

Abstract: Many classical fractals, such as the Sierpinski gasket, and Julia sets of polynomials can be described through groups generated by finite automata. Automaton groups also provide a rich source of examples (such as Grigorchuk group of intermediate growth) and play an important role in geometric group theory. In this talk we will show a phase transition in the Liouville property of automaton groups, and deduce that all automaton groups with activity growth of degree up to 1 are amenable. A key ingredient is the analysis of random walks on the Schreier graphs of these groups: We will discuss how further understanding of the structure of these Schreier graphs, and in particular estimatinng the resistances between vertices may be used to further our knowledge of automaton groups.

This talk is based on joint works (some in progress) with O. Angel and B. Virag.

Speaker: Laszlo Babai (University of Chicago)

Title: Asymptotic characterization of finite vartex-transitive graphs with bounded Hadwiger number via rooted limits

Abstract:

Speaker: Lewis Bowen (Texas A&M)

Title: Entropy for sofic group actions

Abstract: In 1958, Kolmogorov defined the entropy of a probability measure preserving transformation. Entropy has since been central to the classification theory of measurable dynamics. In the 70s and 80s researchers extended entropy theory to measure preserving actions of amenable groups (Kieffer, Ornstein-Weiss). My recent work generalizes the entropy concept to actions of sofic groups; a class of groups that contains for example, all subgroups of GL(n,C). Applications include the classification of Bernoulli shifts over a free group, answering a question of Ornstein and Weiss.

Speaker: Gabor Elek (Renyi Institute, Budapest)

Title: Groups and graph limits

Abstract: Convergence of finite graphs was introduced by Benjamini and Schramm. Finer notion of convergence was studied by Bollobas and Riordan. We survey these basic notions of convergence and the respective limit constructions; measurable graphings and unimodular networks.

Speaker: Joel Friedman (UBC)

Title: Fancy Linear Algebra and the Hanna Neumann Conjecture

Abstract: In this talk we describe a proof of the Hanna Neumann Conjecture of the 1950's based on linear algebra, and an invariant of a collection of linear maps that we call the "maximum excess." The maximum excess has a number of remarkable properties, and can be viewed as an analogue of the L^2 Betti numbers defined by Atiyah. Our linear algebra can be intuitively understood in terms of a very simple type of sheaf cohomology theory on graphs; however, as we shall show, our proof can be given without cohomology theory or any heavy-handed techniques from sheaf theory.

Speaker: Yair Glasner (Ben Gurion University)

Title: A probabilistic Kesten theorem and counting closed circles in graphs

Abstract: We give explicit estimates between the spectral radius and the densities of short cycles for finite d-regular graphs. This allows us to show that the essential girth of a finite d-regular Ramanujan graph G is at least $c \log \log |G|$.

We prove that infinite d-regular Ramanujan unimodular random graphs are trees. Using Benjamini-Schramm convergence this leads to a rigidity result saying that if most eigenvalues of a d-regular finite graph G fall in the Alon-Boppana region, then the eigenvalue distribution of G is close to the spectral measure of the d-regular tree.

We also show that for a nonamenable invariant random subgroup H, the limiting exponent of the probability of return to H is greater than the exponent of the probability of return to 1. This generalizes a theorem of Kesten who proved this for normal subgroups.

Speaker: Lukasz Grabowski

Title: Turing machines, graphings and the Atiyah problem

Abstract: It is shown that all non-negative real numbers are l^2-Betti numbers. The main new idea is embedding Turing machines into integral group rings. The main tool developed generalizes known techniques of spectral computations for certain random walk operators to arbitrary operators in groupoid rings of discrete measured groupoids.

Speaker: Lorenz Gilch (Graz)

Title: Branching Random Walks on Free Products of Groups

Abstract: In this talk we will consider discrete-time branching random walks on free products of groups, which can be described in the following way. An initital particle starts at some vertex of the free product. At each instant of time, each particle produces in a first stage some offspring according to an offsping distribution and in a second stage each of the offspring particles moves independently to a neighbour element in the free product. That is, each particle performs its own independent single random walk from its place of birth. We investigate the phase of branching random walks, where the branching process survives and where the process vacates each finite subset almost surely after finite time. The purpose of this talk is to describe the boundary to which the particle cloud moves. We give an explicit phase transition criterion in order to describe the set of ends of the Cayley graph of the free product, where the branching random walk accumulates. Furthermore, we give an explicit formula for the box-counting dimension and the Hausdorff dimension of the boundary set (in comparision to the dimensions of the boundary of the whole free product), which is reached by the branching random walk.

Speaker: Viktor Harangi (Renyi Institute, Budapest)

Title: Low moments are sofic

Abstract: We explicitly describe the possible pairs of triangle and square densities for regular ifnite simple graphs. We also prove that every r-regular unimodular random graph can be approximated by r-regular infinite graphs with respect to these densities. As a corollary one gets an explicit description of the possible pairs of the third and fourth moments of the spectral measure of r-regular unimodular random graphs.

Speaker: Vadim Kaimanovich (Ottawa)

Title: Finite approximations of invariant measures

Abstract: Although the notion of an invariant measure is usually formulated for a single transformation or in the presence of a (semi)group action, the theory of foliations provided a first example where it was done without these assumptions (holonomy invariant measures, Plante 1975). Soon afterwards Feldman and Moore (1977) defined the notion of invariance with respect to a discrete equivalence relation. As it was noticed by the author (1998), the space of rooted locally finite graphs has a natural ”root moving” equivalence relation, so that one can talk about invariant measures on this space as well.

A similar notion of invariance for measures on graphs (”unimodular measures”) was later developed by probabilists (these two notions coincide for graphs with trivial group of isomorphisms). For any finite graph the uniform distribution on the set of its vertices is unimodular. Benjamini and Schramm (2001) showed that any weak limit of a sequence of such measures is also unimodular, which naturally leads to the question about the measures which can be obtained in this way (Aldous and Lyons, 2007). In particular, for Cayley graphs existence of such an approximation is equivalent to the group being sofic. Elek (2010) by using an earlier idea of Bowen (2003) proved that any unimodular measure on the space of rooted trees with uniformly bounded vertices is finitely approximable.

In this talk we show that if an invariant measure on the space of rooted graphs is such that the ”root moving” equivalence relation is amenable with respect to this measure, then the measure is finitely approximable. The proof is based on ergodic considerations. Namely, amenability (i.e., hyperfiniteness) of the equivalence relation allows one to apply the martingale convergence theorem to obtain the approximation in question.

Speaker: Anders Karlsson (Royal Institute of Technology, Sweden)

Title: Spanning forests, heat kernels, and Epstein zeta values

Abstract: I will describe rather detailed asymptotics of the number of spanning trees, and rooted spanning forests for discrete tori, Z^d/AZ^d as the integral matrix A tends to infinity. In dimension $d = 2$ this was known from the statistical physics literature. In these asymptotics several interesting constants appear, often special values of Epstein zeta functions associated to a limiting torus. In particular, the determinant of the Laplacian of the limiting torus appears in the spanning tree case. The methods involve heat kernel analysis on the graph tori and the manifold tori. Joint work with G. Chinta and J. Jorgenson.

Speaker: James Lee (University of Washington)

Title: Rate of escape and harmonic functions

Abstract: We prove that on any infinite, connected, transitive graph, the random walk escapes at rate at least $t^{1/2}$ after t steps. Following Erschler, we use non-constant equivariant harmonic mappings to reduce the problem to a study of L^2-valued martingales. For the case of discrete, amenable groups, we present a new construction of such harmonic mappings based on the heat flow from a Folner set.

Speaker: Gabor Lippner (Harvard University)

Title: Nodal domains on graphs

Abstract: We show how to discretize Cheng's result on bounding the multiplicity of Laplacian eigenvalues by the genus. The proof is based on a discrete version of the Courant Nodal Domain theorem. Though certain versions of this theorem have been around, they are insufficient for our application of bounding multiplicities. Extending ideas of Davies et al, we prove two new forms of the Nodal Domain Theorem, one for bounded degree graphs and another for 3-connected graphs with bounded genus.

Speaker: Russell Lyons (Indiana)

Title: From probability to measured group theory

Abstract: We explain the differing viewpoints of probability theory on the one hand and measured group theory on the other. This involves passing from random processes on, say, Cayley graphs to random rooted graphs to graphings of measured equivalence relations. We illustrate with an outline of joint work with Gaboriau of the proof that every non-amenable group contains a "measurable free subgroup" of rank 2.

Speaker: Igor Mineyev (University of Illinois Urbana-Champaigne)

Title: Groups, graphs and the Hanna Neumann Conjecture

Abstract: The Hanna Neumann Conjecture asserts a specific upper bound on the rank of the intersection of two finitely generated subgroups in a free group. Walter Neumann proposed a strengthened version of the Hanna Neumann Conjecture (SHNC). We will present a proof of SHNC in terms of groups and graphs. We will also discuss trees, flowers, forests, gardens, and leafages. There is an analytic version of this proof using Hilbert modules which allows for generalizations of SHNC to complexes.

Speaker: Asaf Nachmias

Title: Is the critical percolation probability local?

Abstract: We show that the critical probability for percolation on a d-regular non-amenable graph of large girth is close to the critical probability for percolation on an infinite d-regular tree. This is a special case of a conjecture due to O. Schramm on the locality of p_c. We also prove a finite analogue of the conjecture for expander graphs

Speaker: Yuval Peres (Microsoft Research)

Title: Embedding groups in Hilbert space and rate of escape of random walks.

Abstract: A metric space X has Markov type 2 if for any stationary reversible finite hidden Markov chain taking values in X, the second moment of the distance from the starting point grows at most linearly in the number of steps. Since Hilbert space L_2 has Markov type 2 (K. Ball 1992) this can be used to bound from below the distortion of any embedding into L_2 of a space where Markov chains can escape faster, e.g. the hypercube. We derive from this a general inequality that bounds the compression exponent for embeddings of infinite amenable groups via the escape exponent for random walks. The inequality is sharp in lamplighter type groups (Talk based on joint work with Assaf Naor).

Speaker: Laurent Saloff-Coste (Cornell University)

Title: Random walks driven by low moment measures

Abstract: On finitely generated groups, we consider symmetric probability measures satisfying some natural moment conditions and prove lower bounds for the probability of return to the starting point after n steps. For instance, on a polycyclic group of exponential volume growth and for a symmetric measure with finite first moment, we prove that the probability of return after n step is bounded below by $\exp(-cn^{1/2})$. The proofs involve the notion of trace, comparison of Dirichlet forms and other tools from functional analysis.

Speaker: Tatiana Smirnova-Nagnibeda (Geneva)

Title: Abelian sandpile model, unimodular rooted random networks and self-similar groups

Abstract: We propose to consider the Abelian sandpile model on unimodular random rooted networks and address in this context the problem on criticality of the model. This viewpoint allows us to exhibit a multitude of examples of infinite graphs where we prove rigorously criticality of the ASM.

Speaker: Balazs Szegedy

Title: Higher-order Fourier Analysis

Abstract: In a famous paper Timothy Gowers introduced a sequence of norms U(k) defined for functions on abelian groups. He used these norms to give quantitative bounds for Szemeredi's theorem on arithmetic progressions. The behavior of the U(2) norm is closely tied to Fourier analysis. In this talk we present a generalization of Fourier analysis (called k-th order Fourier analysis) that is related in a similar way to the U(k+1) norm. Ordinary Fourier analysis deals with homomorphisms of abelian groups into the circle group. We view k-th order Fourier analysis as a theory which deals with morphisms of abelian groups into algebraic structures that we call "k-step nilspaces". These structeres are variants of structures introduced by Host and Kra (called parallelepiped structures) and they are close relatives of nil-manifolds. Our approach has two components. One is an uderlying algebraic theory of nilspaces and the other is a variant of ergodic theory on ultra product groups. Using this theory, we obtain inverse theorems for the U(k) norms on arbitrary abelian groups that generalize results by Green, Tao and Ziegler. As a byproduct we also obtain an interesting limit theory for functions on abelian groups in the spirit of the recently developed graph limit theory.

Speaker: Adam Timar (Vienna)

Title: Approximating Cayley graph versus Cayley diagrams

Abstract: We construct a sequence of finite graphs that weakly converge to a Cayley graph, but there is no labelling of the edges that would converge to the corresponding Cayley diagram. This is related to the question whether soficity of a group depends only on the Cayley graph and not on its orientation and labelling by the generators.

Speaker: Robert Young (University of Toronto)

Title: Pants decompositions for random surfaces

Abstract: Every hyperbolic surface has a decomposition into a union of three-holed spheres glued along their boundaries, which is called a pants decomposition. The length of a pants decomposition gives one way to describe the geometric complexity of a surface: it describes how difficult it is to break the surface into simple pieces. Bers proved that every hyperbolic surface of genus g has a pants decomposition which has length bounded by a constant that depends only on g, and one natural question is how this constant depends on g. In this talk, we will give a brief introduction to some techniques from the geometry of hyperbolic surface, describe some results on pants decompositions of surfaces, and present some recent work showing that random surfaces are difficult to decompose into pairs of pants. This talk is joint work with Larry Guth and Hugo Parlier.

Speaker: Wolfgang Woess (Technical University, Graz)

Title: On the spectrum of lamplighter random walks and percolation clusters

Abstract: Let G be a finitely generated group and X its Cayley graph with respect to a finite, symmetric generating set S. Furthermore, let H be a finite group. We consider the lamplighter group over G with group of "lamps" H. This is the wreath product of G with H. We show that the spectral measure (Plancherel measure) of the "switch–walk–switch" random walk on the lamplighter group coincides with the expected spectral measure (integrated density of states) of the random walk with absorbing boundary on the cluster of the group identity for Bernoulli site percolation on X with parameter $p = 1/|H|$. The return probabilities of the lamplighter random walk coincide with the expected (annealed) return probabilites on the percolation cluster. In particular, if the clusters of percolation with parameter p are almost surely finite then the spectrum of the lamplighter group is pure point. This generalizes results of Grigorchuk and Zuk, resp. Dicks and Schick regarding the case when G is infinite cyclic.

(Joint work with F. Lehner and M. Neuhauser, published in Mathematische Annalen 342 (2008) 69-89.)

5 Scientific Progress Made

The greatest progress of the meeting was to make the attending researchers of various areas meet and talk to each other. Participants learned each other's angles on the topic of the workshop. Some new questions and directions also emerged from the talks.