

Let  $U_q^-(w)$  be the quantum unipotent subgroup associated with a Weyl group element  $w$  of a symmetric Kac-Moody Lie algebra  $\mathfrak{g}$ . Kimura showed that it is compatible with Kashiwara-Lusztig's dual canonical base. By the work of Geiss-Leclerc-Schröer, it has a structure of a quantum cluster algebra. It is conjectured that the dual canonical base contains quantum cluster monomials.

Recall that Geiss-Leclerc-Schröer showed that Lusztig's dual semicanonical base contains cluster monomials. And a dual semicanonical base element corresponds to an irreducible component of Lusztig's lagrangian subvariety. Let  $\Lambda_b$  be an irreducible component, which corresponds to a cluster monomial  $m$ . By Kashiwara-Saito,  $\Lambda_b$  corresponds to a (dual) canonical base element  $b$ . I conjecture a following statement: Let  $b'$  be another dual canonical base element in  $U_q^-(w)$  and  $P_{b'}$  be the corresponding perverse sheaf. If its singular support  $SS(P_{b'})$  contains  $\Lambda_b$ , then  $b' = b$ .

Assuming this conjecture, I can prove that  $b$ , specialized at  $q = 1$ , is the given cluster monomial  $m$ , i.e., the dual semicanonical base element is the specialization of the canonical base element in this case. Probably with a little more effort, I can also prove that  $b$  is a quantum cluster monomial.