Research Interests of Kentaro Nagao

Recently, I'm interested in the product of quantum dilogarithms associated to a mapping class.

Let (Q, W) be a QP and $\mathbf{k} = (k_1, \ldots, k_l)$ be a sequence of vertices. The QP $\mu_{\mathbf{k}}(Q, W)$ obtained by the successive mutations is derived equivalent to the original one. The equivalence is given by a torsion pair $(\mathcal{T}_{\mathbf{k}}, \mathcal{F}_{\mathbf{k}})$ of the module category of the Jacobi algebra of (Q, W). The generating series of the motivic DT type invariants of objects in $\mathcal{T}_{\mathbf{k}}$ is described as a product of quantum dilogarithms.

Assume that $\mu_{\mathbf{k}}(Q, W) = (Q, W)$ and the derived equivalence is identity functor. Then we have $\mathcal{T}_{\mathbf{k}} = 0$ and so the product of the quantum dilogarithm is 1. This is the quantum dilogarithm identity proved by Keller.

On the other hand, assume that $\mu_{\mathbf{k}}(Q, W) = (Q, W)$ but the derived equivalence is not identity functor. Then the product of the quantum dilogarithm is not 1 and difficult to compute explicitly, but commutes with the generating series of DT invatriants.

A typical example is obtained from a triangulation of a surface. For a triangulation, a quiver with a potential is associated and the mapping class group acts on the derived category. Then, for any mappling class the assciated product of dilogarithms commutes with the generating series of motivic DT invariants. In other word, the motivic DT series satisfies the constraints indueced from the mapping class group symmetry.

From this viewpoint, it will be impertant to study the product of quantum dilogarithms associated to a mapping class.

Given a mapping class group, we take the mapping torus to get a 3-manifold. It is natural to expect that there is a "TQFT-like" construction of an invariant of such a 3-manifold.

Recently, Tarashima and Yamazaki propose to study the trace of a mapping class in quantum Teichmuller theory. They checked that the asymptotic behaviors of the trace of a pseudo-Anosov mapping classes coincides with the volume of the mapping torus in some examples.

Kashaev's volume conjecture claims that the asymptotic behaviors of Jones polynomial of a knot coincides with the volume of the compliment of the knot. Tarashima-Yamazaki's proposal implies the product of quantum dilogarithms would play an important role in the volume conjecture.