Research Statement on Cluster Algebras
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My current research interests in cluster algebras focus on cluster algebras from surfaces and connections to Teichmüller theory. I have also recently worked with REU students exploring cluster algebras related to integrable systems, such as those corresponding to Gale-Robinson sequences $x_n x_{n-k} = x_{n-r}x_{n-k+r} + x_{n-s}x_{n-k+s}$, and worked with Christian Stump developing software in SAGE for working with cluster algebras and quivers.

Regarding my research on cluster algebras from surfaces, I have been working with Ralf Schiffler and Lauren Williams to extend our previous work, "Positivity for cluster algebras from surfaces" [MSW11], which provided combinatorial formulas for cluster variables in terms of perfect matchings of graphs. The long term goal of this work is to provide positive canonical, also known as atomic bases (for example in the recent works of Cerulli and Dupont-Thomas) for such cluster algebras. Towards this end, Williams and I posted a recent paper to the arxiv, 1108.3382, connecting our graph theoretic formula to matrix product formulas in $PSL_2(\mathbb{R})$ related to previous work of Fock-Goncharov, and others. We work in the generality of principal coefficients, and write down explicit skein relations that correspond to multiplying together cluster variables in this context.

With Schiffler and Williams, we are now wrapping up a paper exhibiting a vector space basis for cluster algebras associated to unpunctured surfaces with principal coefficients, based on earlier work of Fock-Goncharov and Fomin-Shapiro-Thurston. In particular, we exhibit a bijection between $\mathbb{Z}^n$ and g-vectors associated to the basis elements to illustrate linear independence.

Work in this area has also led me to further study the associated geometric background. This past year, I met regularly with geometers and topologists in Minnesota, Ren Guo and Helen Wong, to compare constructions of quantum Teichmüller space to constructions of quantum cluster algebras.

I also recently gave a five lecture course on cluster algebras and Teichmüller theory at MSRI, `http://www.msri.org/web/msri/scientific/workshops/show/-/event/Wm550`. Stump and I have been writing software for SAGE, currently available through the SAGE Combinat queue at `http://combinat.sagemath.org/patches/`, for computations related to cluster algebras. Some of the code's features thus far include cluster mutations, recognizing the Dykin or Extended Dynkin type of a cluster algebra from its matrix or quiver, and computing a list of the exchange matrices or cluster variables mutation-equivalent to a given seed. A compendium describing these features is available on the arxiv, 1102.4844.