Research interests - Greg Muller

The geometry of cluster algebras. For a (complex) cluster algebra $\mathcal{A}$, the affine scheme $X := Spec(\mathcal{A})$ has many interesting properties. Each cluster determines an embedding of an algebraic torus, and the union of these tori determines an open subscheme X' (called the *cluster manifold* in [GSV03]) whose ring of global functions is the upper cluster algebra $\mathcal{U}$. I am interested in the complement $X_d := X - X'$, about which very little seems to be known in general. For fixed $\mathcal{A}$, several basic questions are already non-trivial, even when $\mathcal{A}$ is finite-type:

- Is X_d empty?
- Is X_d smooth in X?
- What is the codimension of X_d in X?
- Does the Poisson structure on X' (as defined in [GSV03]) extend to X_d? For acyclic $\mathcal{A}$, this was shown to be true in [Mul].

Cluster algebras of surfaces and skein algebras. Given an oriented surface Σ (possibly with boundary $\partial\Sigma$) with a finite collection of marked points M, there are two related algebras associated to Σ.[1] The first is the cluster algebra $\mathcal{A}(\Sigma)$ of Σ, as defined in [GSV05] and [FST08] (and also the upper cluster algebra $\mathcal{U}(\Sigma)$). The second is the *Kauffman skein algebra* $\mathsf{Sk}(\Sigma)$, an algebra of formal products of arcs and loops together with a local relation (the definition involves a parameter q).[2] When $q = 1$, there are natural inclusions

$$\mathcal{A}(\Sigma) \subseteq \mathsf{Sk}^o(\Sigma) \subseteq \mathcal{U}(\Sigma)$$

where $\mathsf{Sk}^o(\Sigma)$ is a certain localization of $\mathsf{Sk}(\Sigma)$. When $M \subset \partial\Sigma$, I can show that $\mathsf{Sk}^o(\Sigma) = \mathcal{U}(\Sigma)$, and there is evidence that this is true for general M.

Cluster algebras of surfaces and character algebras. Following [FG06], one may define *decorated local systems* on Σ, by taking an $SL_2(\mathbb{C})$ local system on Σ and adding extra data at the M. To this moduli problem, one can associate an affine *character scheme* $Char(\Sigma)$ (which is the schemification of the moduli stack). For $M \subset \partial\Sigma$, it is possible to (non-canonically)[3] identify the ring of functions $\mathcal{O}Char(\Sigma)$ with the skein algebra $\mathsf{Sk}(\Sigma)$; and so a localization of $\mathcal{O}Char(\Sigma)$ may be identified with $\mathcal{U}(\Sigma)$. This provides a regular version of a birational result obtained in [FG06]. This work is joint with Peter Samuelson.

References

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[1] We assume there are 'enough' points in M.
[2] When there are marked points not on the boundary, it is also necessary to add tagged arcs, as in [FST08].
[3] This identification may be made canonical by replacing decorated local systems with *twisted decorated local systems*.