RESEARCH DESCRIPTION

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My focus is on the connection between *cluster algebras* and Lusztig's *canonical basis.*

Given an acyclic quiver Q, one associates two algebras: the path algebra $\mathbb{C}Q$ and the preprojective algebra Λ of Q. Representation theory is concerned with the study of the module categories $\mathrm{mod}(\mathbb{C}Q)$ and $\mathrm{mod}(\Lambda)$. Out of these one can construct two triangulated categories which categorify cluster algebras: the cluster category $\mathcal{C}_Q$ which is an orbit category of the bounded derived category of $\mathrm{mod}(\mathbb{C}Q)$, see [1], and the stable module category $\underline{\mathrm{mod}}(\Lambda)$ of the selfinjective algebra Λ. Both $\mathcal{C}_Q$ and $\underline{\mathrm{mod}}(\Lambda)$ are Calabi-Yau of dimension two. The cluster category $\mathcal{C}_Q$ is triangle equivalent to the stable category $\underline{\mathcal{C}}_w$ of a suitable subcategory $\mathcal{C}_w$ of $\mathrm{mod}(\mathbb{C}Q)$ which is parametrized by a particular Weyl group element of length $2n$, where $n = |Q_0|$, see [2].

We are interested in the associated cluster algebra $\mathcal{A}(\mathcal{C}_w)$. Let $\mathfrak{g} = \mathfrak{n} \oplus \mathfrak{h} \oplus \mathfrak{n}_-$ be the corresponding Kac-Moody Lie algebra. The quantization $\mathcal{A}(\mathcal{C}_w)_q$ is then a quantum cluster algebra; it can be realized as an integral form of a subalgebra $U_q(w) \subset U_q(\mathfrak{n})$ of the quantized universal enveloping algebra of $\mathfrak{n}$. The algebra $U_q(w)$ was introduced by Lusztig [6]. It is constructed via braid automorphisms and it admits several bases. Among these are the various (non-canonical) Poincaré-Birkhoff-Witt bases, and moreover, there is a geometrically constructed canonical basis.

Instead of Lusztig's geometric approach, Leclerc [5] uses the quantum shuffles introduced by Rosso [7] to obtain a combinatorial description of Lusztig's canonical basis.

My own work in this circle of ideas is to investigate whether quantum cluster variables belong to the dual of Lusztig's canonical basis (with respect to Kashiwara's bilinear form) which we confirm [3, 4] in the case when Q is either the Kronecker quiver or an alternating quiver of type A_n. In both cases we give explicit recursions for the quantum cluster variables. Our proof features Leclerc's combinatorial description of the dual canonical basis and the explicit form of the straightening relations among the generators of $U_q(w)$. For $Q = A_n$ we also explore the combinatorics of the recursions in terms of quantum shuffles and alternating permutations.

References

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