

Cycles on modular varieties

Finding solutions of polynomial equations using analytic and geometric methods is a classical and highly developed subject. On the other hand, methods for solving Diophantine equations, i.e., finding integer solutions of integral polynomial equations, are usually of an algebraic nature. Hinted at by Dirichlet's famous "class number formula", analytic methods for solving important mathematical equations, like those defining elliptic curves, have seen significant development in the second half of the twentieth century. The major tool allowing for this development is the theory of modularity, which connects the worlds of analysis, geometry and arithmetic. In this workshop, which attracted 44 participants from all over the world, explored a variety of state-of-the art modularity-based techniques for studying the solutions of diophantine equations.

The meeting began with a pair of lectures by Victor Rotger and Henri Darmon on the connections between three beautiful objects -- the de Rham fundamental group of the modular curve, the triple product L-function, and diagonal cycles. Put together, these objects yield explicitly computable points on elliptic curves, as well as new cases of the Birch and Swinnerton-Dyer conjecture in ranks zero and one. The constructions of these lectures fit into the framework of the Gross-Prasad conjectures, which were discussed in lectures by Chung-Pang Mok, Shou-Wu Zhang, and Wei Zhang, where the case of unitary groups was emphasized.

Much of Tuesday was concerned with the zeta and L-functions. Colmez lectured on constructions and analysis of Shintani zeta functions. Denis Benois presented his results on exceptional zeroes of p-adic L-functions of modular forms at near central points, and Mijian Brakocevic presented deep theorems on the nonvanishing of Rankin Selberg L-functions, results in the spirit of the famous theorem of Cornut and Vatsal. The day ended with a lively problem session moderated by Jordan Ellenberg that resulted in a list of important open problems that would have significant impact on the field. A summary of this session is available on the workshop's web site.

On Wednesday, we began with a lecture by Eyal Goren in which we took a step back and looked at the variety of cycle constructions that arise in different parts of the subject. The next two lectures, by Jan Bruinier and Tonghai Yang focused on the connections between the cycles discussed in Goren's lecture and Kudla's program relating Arakelov heights of cycles to derivatives of Eisenstein series.

Thursday morning was highlighted by lectures of Francois Brunault and Massimo Bertolini. Brunault discussed the connection between p-adic regulators and special values of p-adic L-functions of elliptic curves. Bertolini presented a p-adic Beilinson formula and made a connection between Kato's Euler system and the results of Darmon and Rotger presented earlier in the conference. The theme of Kato's Euler system was taken up again on Friday morning in Romyar Sharifi's talk on cyclotomic units and the cohomology of modular curves.

The workshop had a fantastic "vibe" and all participants communicated their enthusiasm to the organizers:

"The workshop was really excellent, with a focus that was narrow enough to provide real opportunities for research synergy but broad enough that one was not seeing "the same people, one always sees." The talks of Rotger and Darmon suggested a

research project that I could undertake with them, and with a postdoctoral collaborator, Kirsten Wickelgren.” (Jordan Ellenberg, University of Wisconsin)

“The workshop has exceeded my expectations. The program was carefully thought out, balanced current results with some background material and was of exceptionally high level altogether. A mixture of current results and recent work were reported which informed me and broadened my understanding of the current state of the field. I returned home inspired by the great mathematics I have learned. The facilities and staff were, as always, superb. It remains to congratulate BIRS and the organizers for once again putting together a great workshop.” (Eyal Goren, McGill University)