CYCLES ON MODULAR VARIETIES

Banff International Research Station

Trans Canada Pipelines Pavillion, room 201

10/30/2011-11/03/2011

Schedule

Monday

8:45–9:00	Brenda Williams (BIRS)
	Welcome to Banff Center
9:00–10:00	Victor Rotger (Univ. P. de Catalunya)
	Iterated integrals, diagonal cycles and rational points on elliptic curves
10:00–10:30	Coffee Break
10:30–11:30	Henri Darmon (McGill Univ.)
	Diagonal cycles and Euler system for real quadratic fields
11:30–1:30	Lunch
	Optional tour of Banff Center, 12:30-1:30
1:30–1:40	Group photo
1:40–2:40	Chung Pang Mok (McMaster Univ.)
	Introduction to the Gan–Gross–Prasad conjecture
2:40–3:00	Coffee Break
3:00–4:00	Wei Zhang (Columbia Univ.)
	The global Gan–Gross–Prasad conjecture for unitary groups
4:00–4:10	Break
4:10–5:10	Shou-Wu Zhang (Columbia Univ.)
	Gross–Zagier formula for Shimura curves and their triple products

Tuesday

9:00–10:00	Pierre Colmez (Univ. Paris 6)
	On the method of Shintani
10:00–10:30	Coffee Break
10:30–11:30	Denis Benois (Univ. de Bordeaux)
	Trivial zeros of p-adic L-functions at near central points
11:30–1:30	Lunch
1:30–2:30	Miljan Brakocevic (McGill Univ.)
	Special values of anticyclotomic Rankin–Selberg L-functions
2:30–2:50	Coffee Break
2:50–3:50	Wieslawa Niziol (Univ. of Utah)
	Semistable Conjecture via K-theory: the case of open varieties
3:50–4:00	Break
4:00–5:00	Discussion of Open Problems, led by Jordan Ellenberg (Univ. of Wisconsin)

Wednesday

9:00–10:00	Eyal Goren (McGill Univ.)
	Cycles on modular varieties in positive characteristic
10:00–10:20	Coffee Break
10:20–11.20	Jan Bruinier (TU Darmstadt)
	On the converse theorem for Borcherds products
11.20–11.30	Break
11:30–12:30	Tonghai Yang (Univ. of Wisconsin)
	CM cycles on a Shimura variety and their 'Faltings' height
12.30	Lunch and free afternoon

Thursday

9:00–10:00	François Brunault (ENS Lyon)
	Explicit p-adic regulators on K_2 of modular curves
10:00–10:30	Coffee Break
10:30–11:30	Massimo Bertolini (Univ. di Milano)
	A p-adic Beilinson formula and arithmetic applications
11:30–1:30	Lunch
1:30–2:30	Matteo Longo (Univ. di Padova)
	Quaternionic Darmon points and arithmetic applications
2:30–2:50	Coffee Break
2:50–3:50	Mirela Ciperiani (Univ. Texas, Austin)
	The divisibility of the Tate-Shafarevich group of an elliptic curve in the Weil-Chatelet group
3:50–4:00	Break
4:00–5:00	Kartik Prasanna (Univ. of Michigan)
	Periods of quaternionic Shimura varieties

Friday

9:00–10:00	Romyar Sharifi (Univ. of Arizona)
	Cyclotomic units and the cohomology of modular curves
10.00–10.20	Coffee Break
10:20–11:20	Adebisi Agboola (UC Santa Barbara)
	On special values of p-adic L-functions
11:20–11:30	Break
11:30–12:30	Glenn Stevens (Boston Univ.)
	Vanishing Cycles and the Jacquet-Langlands correspondence over the eigencurve
12:30	Lunch and departure

Abstracts

Adebisi Agboola (UC Santa Barbara)
On special values of p-adic L-functions

I shall discuss the relationship between the arithmetic of a large class of special values of the Katz two-variable p-adic L-function, and the structure of restricted Selmer groups. This can be used to obtain precise information about certain special values whose behaviour is as yet not known to be governed by the arithmetic of an underlying algebraic cycle.

Denis Benois (Univ. de Bordeaux)
Trivial zeros of p-adic L-functions at near central points

Using the theory of (φ, Γ)-modules we prove a Mazur-Tate-Teitelbaum style formula for p-adic L-functions of modular forms at near central points.

Massimo Bertolini (Univ. di Milano)
A p-adic Beilinson formula and arithmetic applications

Francois Brunault has established a p-adic Beilinson formula, relating the value at 2 of the p-adic L-function of a modular elliptic curve to a p-adic regulator defined in terms of modular units. His proof relies on the interpolation properties of Kato's Euler system and on Perrin-Riou's approach to p-adic L-functions. The goal of this talk is to explain an alternate approach to Brunault's formula, based on the direct evaluation of a p-adic L-function interpolating Rankin products of cusp forms and Eisenstein series at a point outside the range of classical interpolation. It will also be explained how this approach can be used in order to gain new insights in the properties of Kato's Euler system. If time permits, some comments on analogies and connections with previous talks on diagonal cycles will be made. This talk reports on work in progress with Henri Darmon.

Miljan Brakocevic (McGill Univ.)
Special values of anticyclotomic Rankin–Selberg L-functions

We present a proof of non-vanishing modulo p of central critical Rankin–Selberg L-values with anticyclotomic twists of l-power conductor, for a prime l different from p. The L-function is Rankin product of a cusp form and a theta series of arithmetic Hecke character of an imaginary quadratic field. The result is concerned with the case when the weight of the Hecke character is greater than that of the cusp form, so the L-value is essentially different in nature from the one in the landmark work of Vatsal and Cornut–Vatsal on the same theme.

Jan Bruinier (TU Darmstadt)
On the converse theorem for Borcherds products

Richard Borcherds constructed a lift from vector valued weakly holomorphic elliptic modular forms of weight $1 - n/2$ to meromorphic modular forms for the discriminant kernel subgroup $\Gamma(L)$ of the orthogonal group of an even lattice L of signature $(n, 2)$. The forms in the image of the lift have their zeros and poles on special divisors with known multiplicities. Since they have particular infinite product expansions, they are often called "Borcherds products".

We consider a given meromorphic modular form F for the group $\Gamma(L)$ whose zeros and poles lie on special divisors. The converse theorem then states that, under certain assumptions on L, the form F has to be the Borcherds lift of a weakly holomorphic modular form. We present some new results on this problem.

François Brunault (ENS Lyon)
Explicit p-adic regulators on K_2 of modular curves

The deep work of Bloch, Kato, Perrin-Riou and many others has made conceivable the formulation of Beilinson's conjectures on special values of L-functions in a purely p-adic setting. We will explain how to do this in the case of curves. Then, as a first step towards the conjecture, we will express the p-adic regulator of suitable elements in K_2 of modular curves in terms of the value at $s = 0$ of p-adic L-functions of modular forms. The proof builds on three ingredients : the construction by Kato of Euler systems for modular forms, the Perrin-Riou exponential map, and the proof by Colmez of the explicit reciprocity law for p-adic representations.

Mirela Ciperiani (Univ. Texas, Austin)
The divisibility of the Tate-Shafarevich group of an elliptic curve in the Weil-Chatelet group

In this talk I will report on progress on the following two questions, the first posed by Cassels in 1961 and the second considered by Bashmakov in 1974. The first question is whether the elements of the Tate-Shafarevich group are infinitely divisible when considered as elements of the Weil-Chatelet group. The second question concerns the intersection of the Tate-Shafarevich group with the maximal divisible subgroup of the Weil-Chatelet group. This is joint work with Jakob Stix.

Pierre Colmez (Univ. Paris 6)
On the method of Shintani

Henri Darmon (McGill Univ.)
Diagonal cycles and Euler system for real quadratic fields

The theory of "Stark-Heegner points" attached to cycles on $\mathcal{H}_p \times \mathcal{H}$ suggests the need for Euler systems controlling the arithmetic of (modular) elliptic curves over $\mathbb{Q}$ and their Mordell-Weil groups over ring class fields of *real quadratic fields*. We will describe a tentative program for constructing such an Euler system which rests on p-adic deformations of the diagonal cycles introduced in the lecture of Victor Rotger.

This talk is largely an account of work in progress with Victor Rotger, in which we will also attempt to describe the connection with earlier works of Bertolini-Dasgupta and Bertolini-Prasanna.

Eyal Goren (McGill Univ.)
Cycles on modular varieties in positive characteristic

This talk is meant as a survey talk, with some attempt to popularize questions concerning cycles in characteristic p on modular varieties. There are various stratifications of moduli spaces of abelian varieties; they produce natural families of cycles. I will survey some of the stratifications that had been studied and some of the key results, as well as the connection to p-adic modular forms. In general, the relation between these cycles and cycles arising from sub-Shimura varieties is not well-understood. Equally mysterious is the action of the Hecke algebra. I will present some of what we know and some questions that seem within reach.

Matteo Longo (Univ. di Padova)
Quaternionic Darmon points and arithmetic applications

Following works by H. Darmon, S. Dasgupta and M. Greenberg, I will explain how to construct Darmon-style local points on Jacobians of compact Shimura curves. After stating the relevant rationality conjectures for these points, I will describe results in this direction and offer an application to the Birch and Swinnerton-Dyer conjecture for elliptic curves over narrow ring class fields of real quadratic fields. These results have been obtained in collaboration with V. Rotger and S. Vigni.

Chung Pang Mok (McMaster Univ.)
Introduction to the Gan-Gross-Prasad conjecture

This is an introduction to the local and global Gan–Gross–Prasad conjecture. We will focus on the case of unitary groups. We will indicate the role of Arthur's work on endoscopic classification of representations on classical groups in the context of the local Gan–Gross–Prasad conjecture.

Wieslawa Niziol (Univ. of Utah)
Semistable Conjecture via K-theory: the case of open varieties

I will sketch a construction of Chern classes from p-adic higher K-theory into log-syntomic cohomology that allows to define a comparison morphism between p-adic etale cohomology and de Rham cohomology of semistable schemes over local rings of mixed characteristic.

Kartik Prasanna (Univ. of Michigan)

Periods of quaternionic Shimura varieties

The Tate conjecture (combined with the Jacquet-Langlands correspondence) predicts the existence of interesting algebraic cycles on products of quaternionic Shimura varieties over a totally real field. In the absence of knowing the existence of such cycles, one may still ask for concrete consequences of their existence, such as relations between periods. Such a concrete consequence was made precise in a conjecture of Shimura (from the early 80's) on relations between Petersson inner products of arithmetic automorphic forms, up to algebraic factors. This conjecture was proven by Michael Harris under a local assumption on the automorphic representation in question. In my talk, I will discuss

(i) A more precise form of Shimura's conjecture i.e. not just up to algebraic factors, but up to p-adic units (for a good prime p), and,

(ii) Work in progress (joint with Atsushi Ichino) that makes some progress on understanding this refined conjecture.

Victor Rotger (Univ. P. de Catalunya)

Iterated integrals, diagonal cycles and rational points on elliptic curves

The theme of this talk is the connection between the pro-unipotent fundamental group $\pi_1(X; o)$ of a pointed modular curve X, algebraic cycles, and special values of L-functions. The extension of mixed Hodge structures arising in the second stage in the lower central series of $\pi_1(X; o)$ gives rise to a supply of complex points on the Jacobian $\mathrm{Jac}(X)$ of X, indexed by Hodge cycles on the surface X^2.

I'll explain how these points can be computed in practice and how are related to the Abel-Jacobi image of Gross-Kudla-Schoen's modified diagonal in X^3, and to a construction due to S. Zhang. When combined with the recent formula obtained by X. Yuan, S. Zhang and W. Zhang, this yields a criterion, in terms of the leading terms of certain L-series attached to modular forms, for these points to be of infinite order.

This reports on a joint work with H. Darmon and I. Sols, and a work in progress with M. Daub, H. Darmon, S. Lichstenstein and W. Stein.

Romyar Sharifi (Univ. of Arizona)

Cyclotomic units and the cohomology of modular curves

We will give an overview of work on a relationship between cup products of cyclotomic p-units and modular symbols. Under mild assumptions, conjectures of the speaker and the speaker and McCallum regarding these cup product values have recently been proven by Fukaya and Kato. We will discuss the set-up of these conjectures, the work of Fukaya-Kato, and other aspects.

Glenn Stevens (Boston Univ.)

Vanishing Cycles and the Jacquet-Langlands correspondence over the eigencurve

Illusie's vanishing cycles sequence for ℓ-adic cohomology of modular curves is used to give a geometric construction of the Jacquet-Langlands correspondence between classical p-new modular eigenforms and quaternionic forms associated to a definite quaternion algebra that is ramified at p. We then explain how this geometric construction can be generalized to produce the Jacquet-Langlands correspondence over the Coleman-Mazur-Buzzard eigencurve. This is joint work with Michael Harris and Adrian Iovita.

Tonghai Yang (Univ. of Wisconsin)

CM cycles on a Shimura variety and their 'Faltings' height

In this talk, we will focus on Shimura variety of orthogonal type $(n, 2)$ although the same (and more) would be true for unitary type $(n, 1)$. We will define and explain Kudla's special cycles in two extreme cases: codimension 1 (divisors) and codimension n (CM cycles). They are both CM points when $n = 1$. In the case $n = 1$, Bruinier and Funke defined their Green functions systematically using regular theta lifting. Bruiner and I computed the values of the Green function at CM cycles, certain Rankin-Selberg L-function pop up automatically. That leads to a different proof of the Gross-Zagier formula. It also leads to an (conjectural) formula of the Faltings height of the CM cycle (w.r.t. certain metrized line bundle) in terms of the central derivative of the Rankin-Selberg L-function.

Shou-Wu Zhang (Columbia Univ.)

Gross–Zagier formula for Shimura curves and their triple products

In this talk I will explain a new formulation a Gross–Zagier formula for Shimura curves which was proved a few years ago with Xinyi Yuan and Wei Zhang. I will also report a work-in-progress with X. Yuan and W. Zhang about a conjecture of Gross and Kudla.

Wei Zhang (Columbia Univ.)

The global Gan–Gross–Prasad conjecture for unitary groups

We show the existence of smooth transfer for the relative trace formulae of Jacquet–Rallis. Together with the relevant fundamental lemma proved by Zhiwei Yun earlier, we confirm the global Gan–Gross–Prasad conjecture for unitary groups under some restrictions for the automorphic representations.