

# **Environmental Heterogeneity in Continuous-Space Continuous-Time Models**

Otso Ovaskainen

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# Outline of the talk

1. Animal movement in heterogeneous space
2. Evolution of dispersal in heterogeneous space

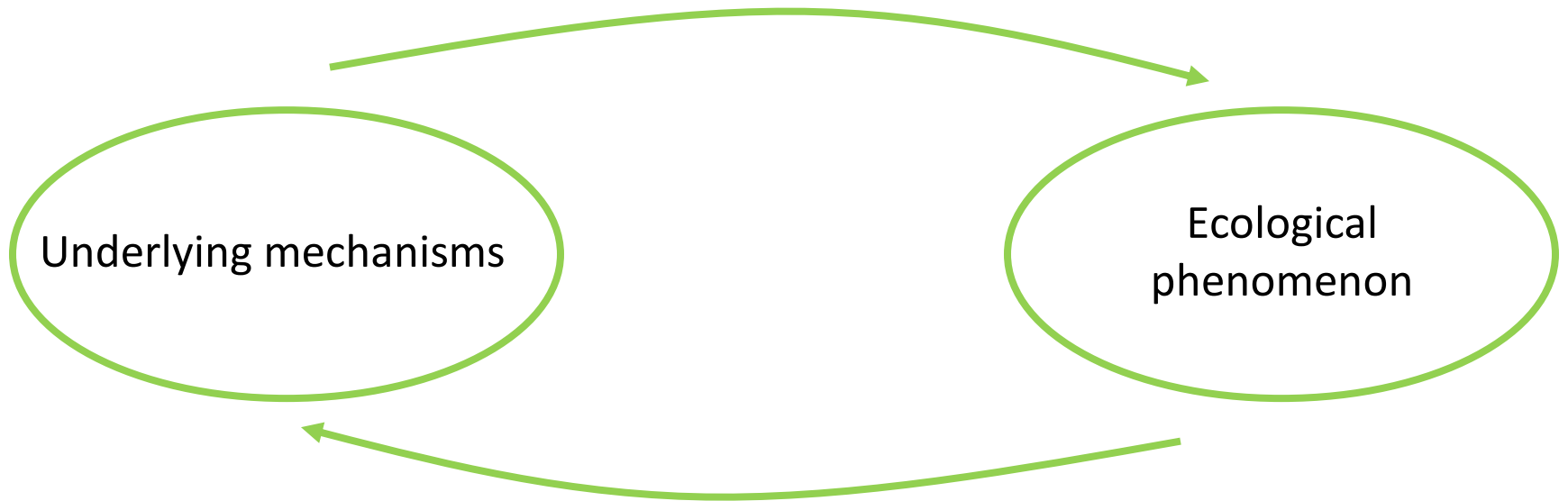
Where is the butterfly heading?



# Approaches in ecological modelling

## **Mathematical modelling.**

Aim: to understand causal relationships at the general level



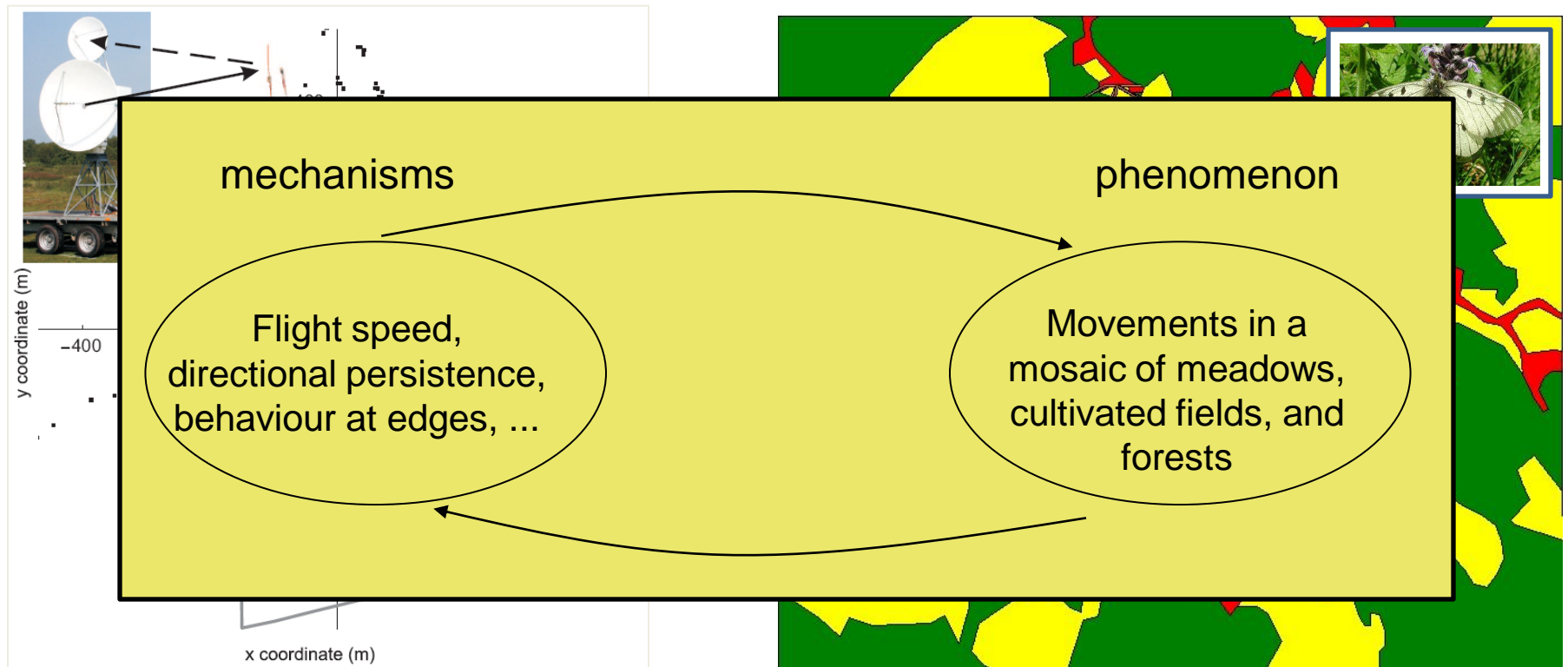
## **Statistical modelling.**

Aim: to find out factors shaping empirical data

# Empirical approaches for studying butterfly movements

Harmonic radar

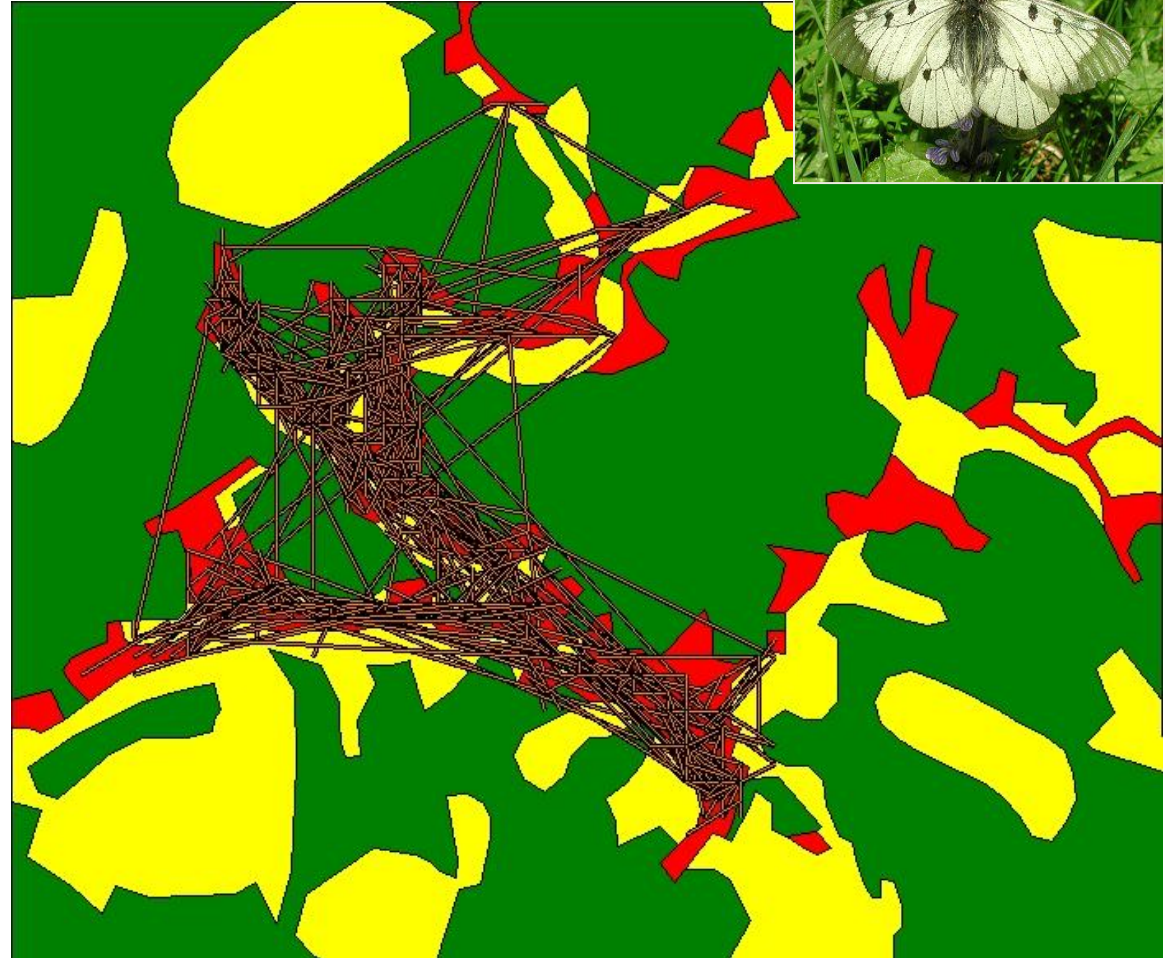
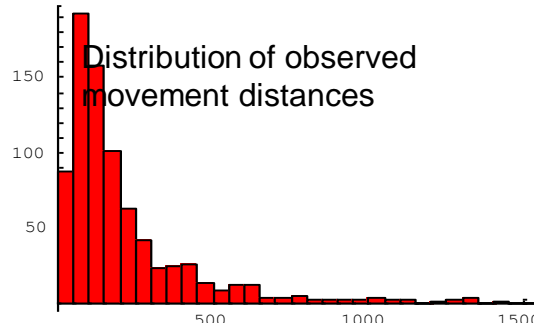
Mark-recapture



Ovaskainen et al. 2009 (PNAS)

Ovaskainen et al. 2008 (American Naturalist)

# What is the movement rate of a butterfly?



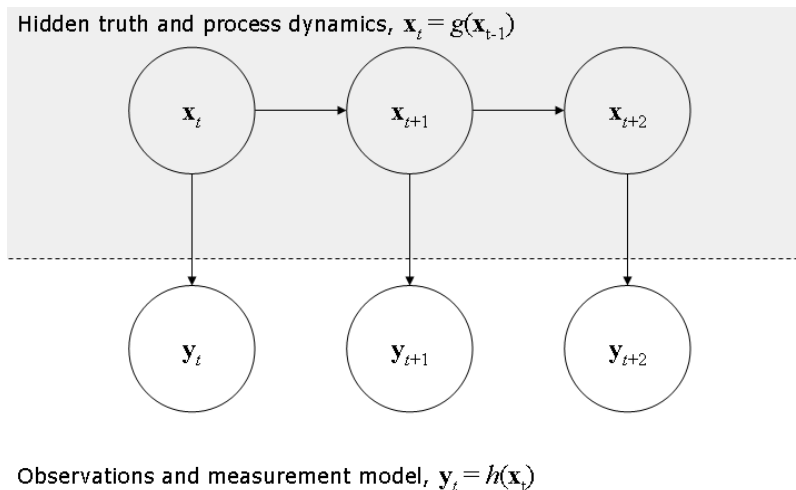
Mark-recapture data depends on

- i) the properties of the species
- ii) the structure of the landscape
- iii) the design of the study

# Combining the "mathematical" and "statistical" approaches

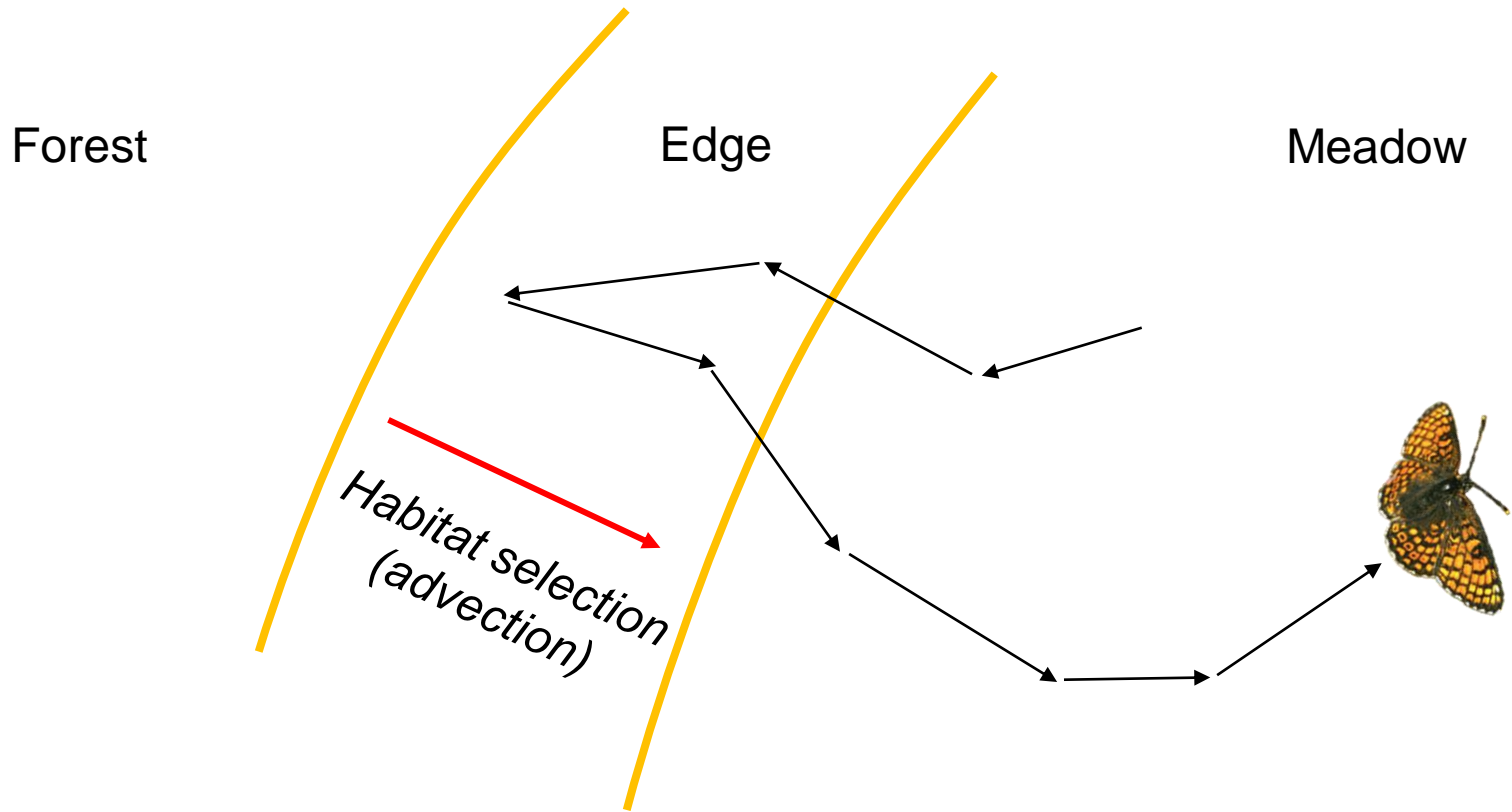
State-space models, Hidden Markov models, process based models...

Biological process of interest = process model, "mathematical modelling"



The way data are collected = observation model, "statistical modelling"

# Simple model for butterfly movement: random walk + habitat selection

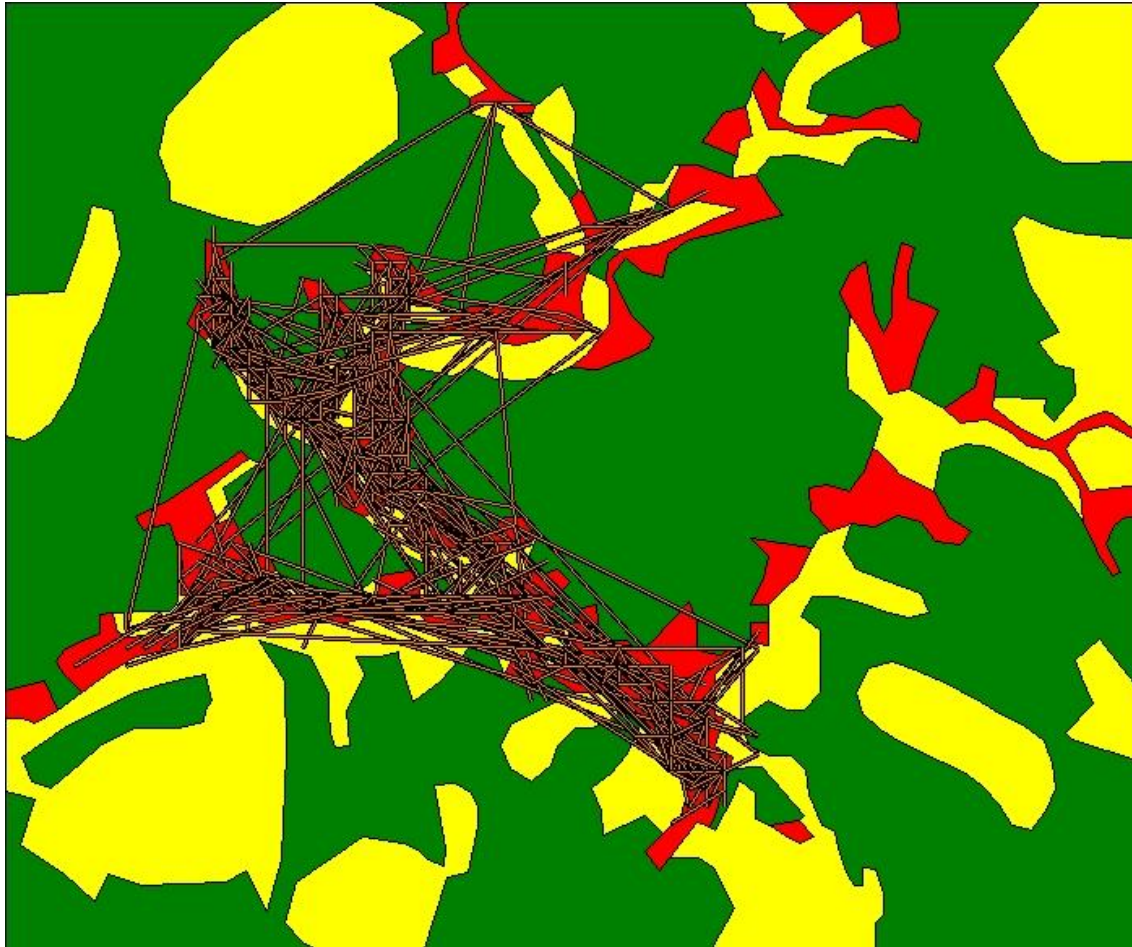


Time evolution of the probability density  $v(\mathbf{x}, t)$  for the individual's location:

$$\frac{\partial v}{\partial t} = Lv, \quad Lf(\mathbf{x}) = \sum_{i,j} \partial_{ij} [a_{ij}(\mathbf{x})f(\mathbf{x})] + \sum_i \partial_i [b_i(\mathbf{x})f(\mathbf{x})] - c(\mathbf{x})f(\mathbf{x}).$$

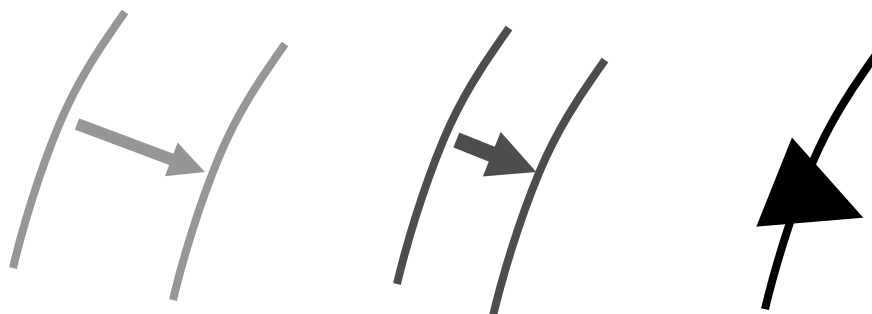


Edges can be narrow compared to the dimensions of the landscape elements



# Linear landscape elements 1/3: edge-mediated behaviour

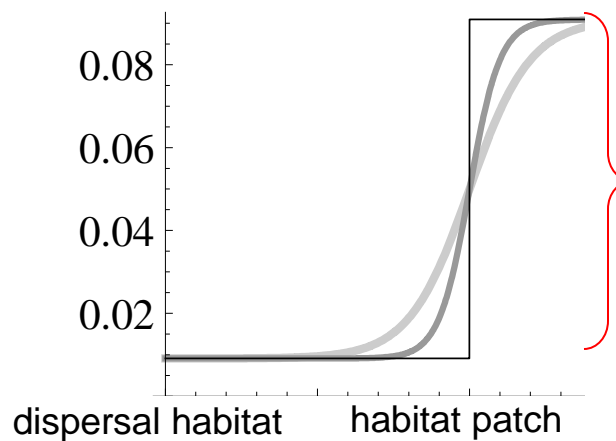
*1-dimensional approximation of the 2-dimensional model*



Matching condition:  
discontinuous probability  
density, continuous flux



Probability density

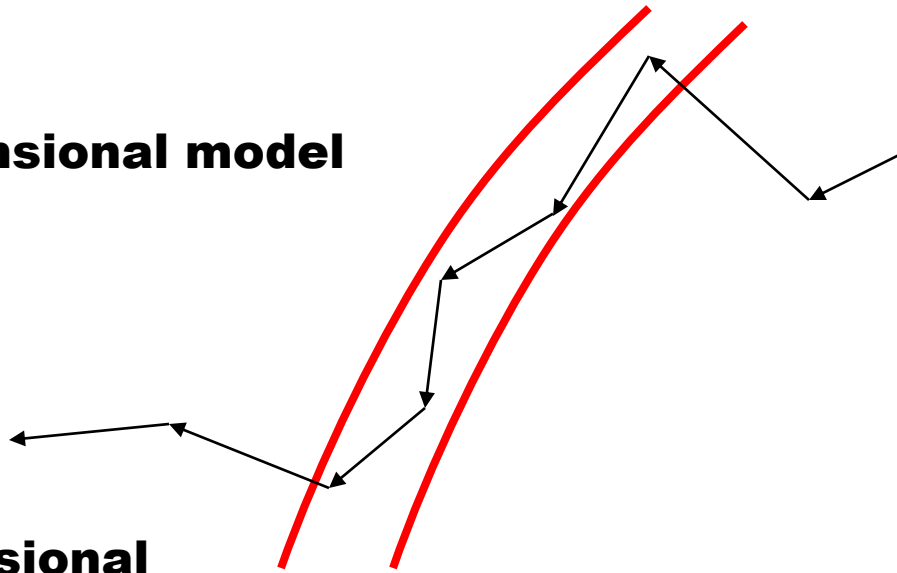


Relative difference  $k$   
is called the habitat  
selection parameter

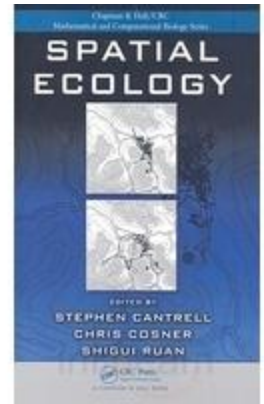
(Ovaskainen & Cornell, Journal  
of Applied Probability 2003)

# Linear landscape elements 2/3: corridors

**2-dimensional model**

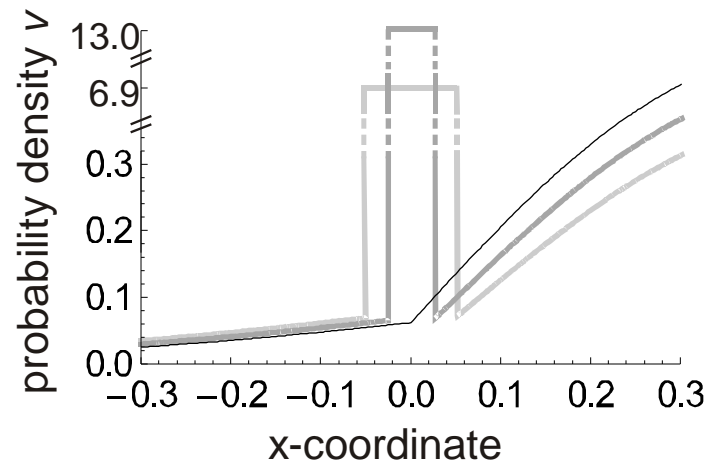
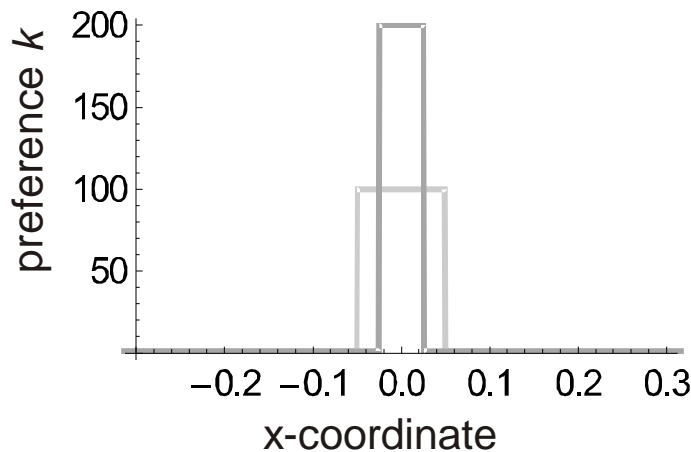


**1-dimensional approximation**



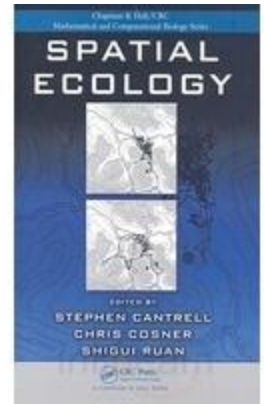
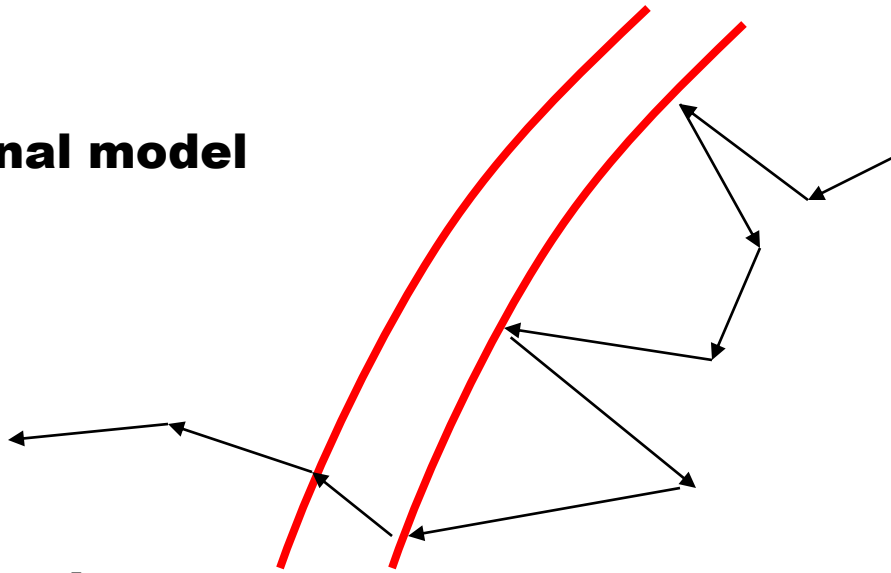
Ovaskainen & Crone (2010)

Matching condition: a non-zero probability that the individual is exactly in line representing the corridor



# Linear landscape elements 3/3: barriers

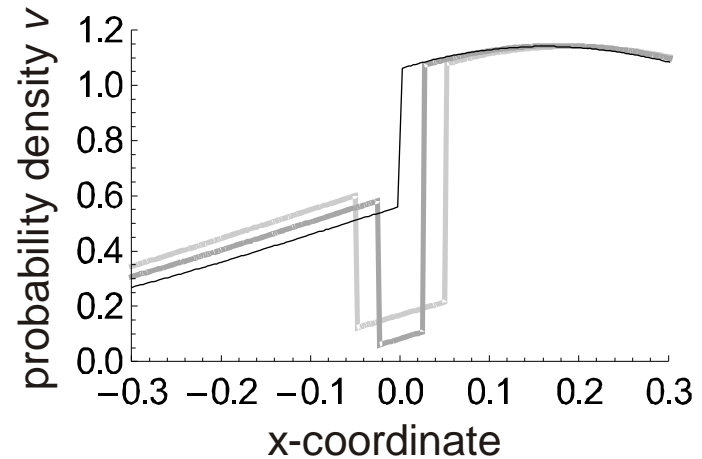
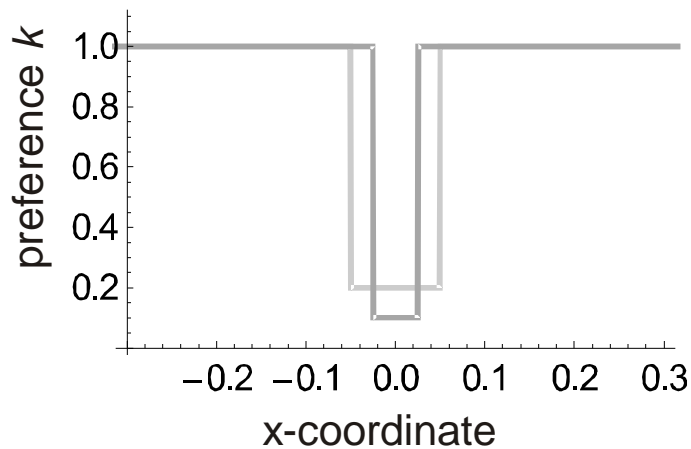
**2-dimensional model**



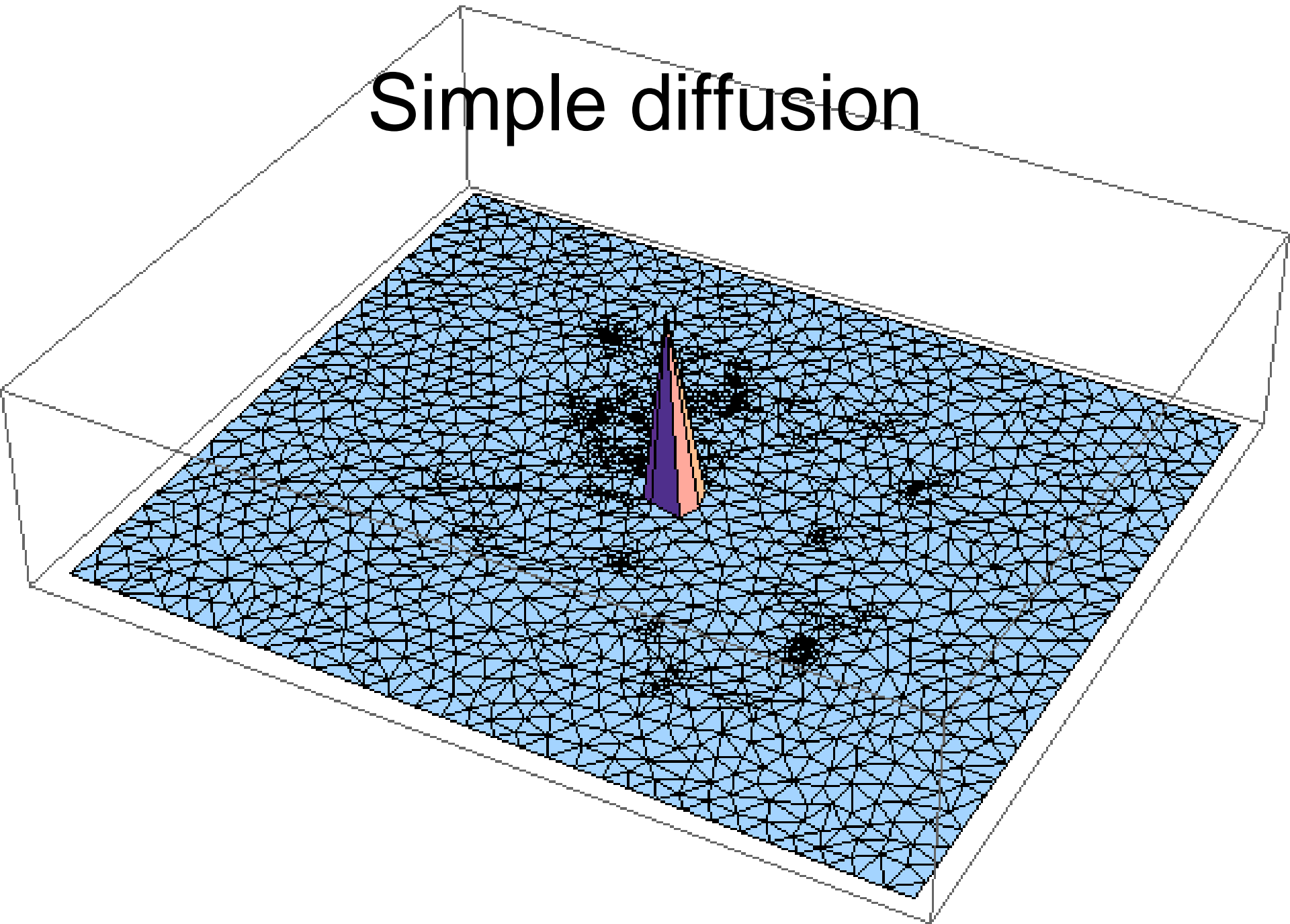
Ovaskainen & Crone (2010)

**1-dimensional approximation**

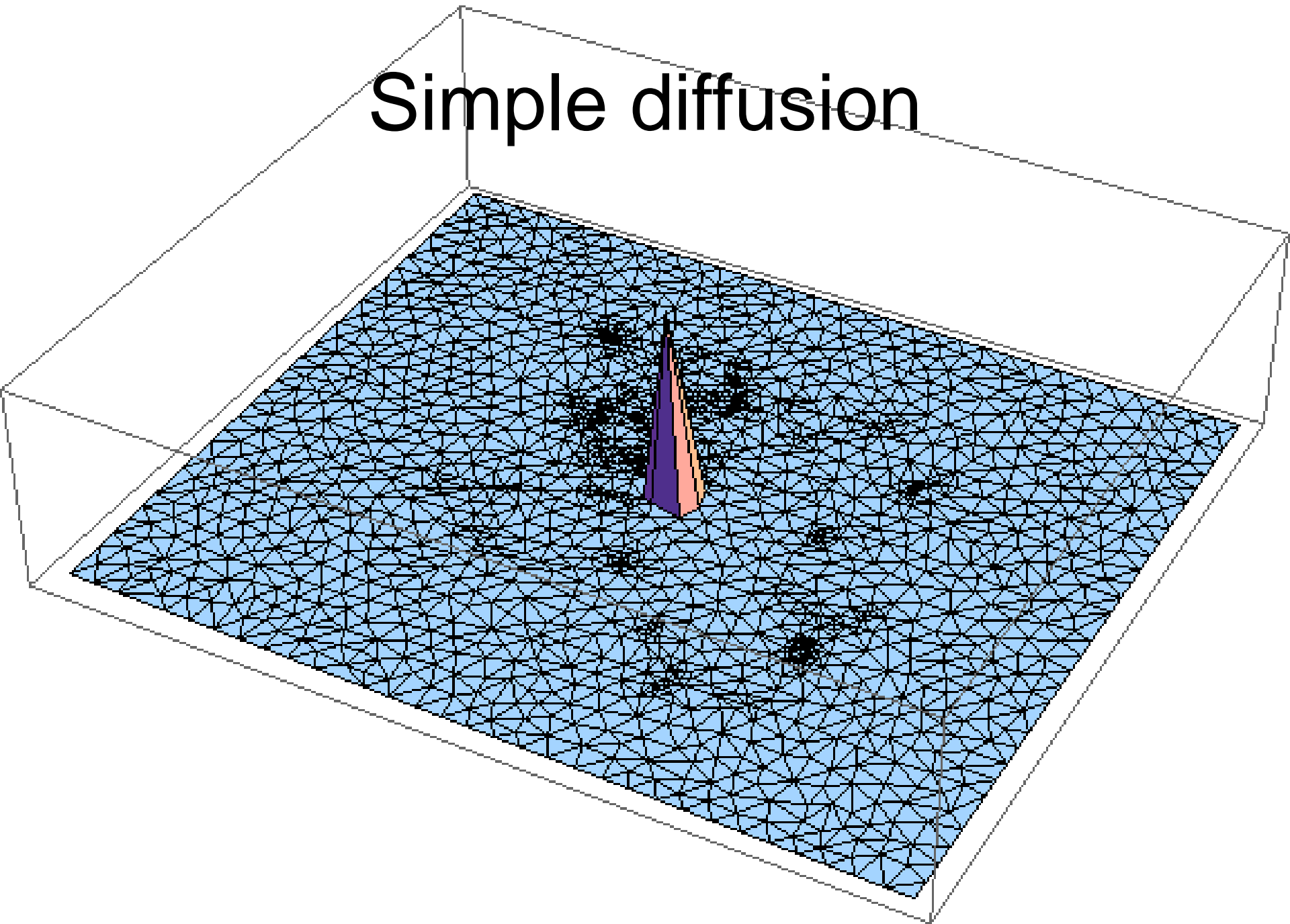
Matching condition: discontinuity in probability density averages out with a time delay



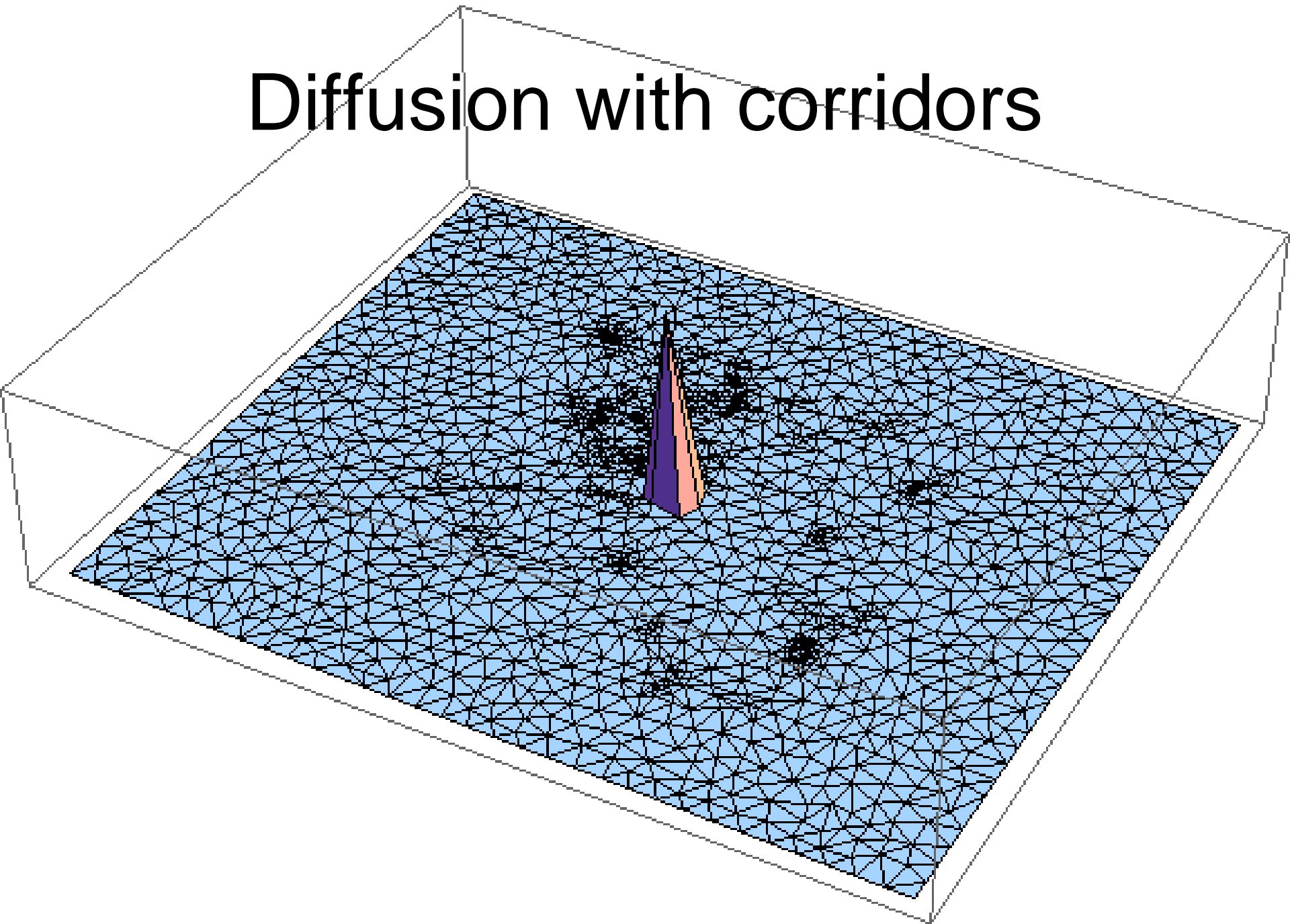
# Simple diffusion



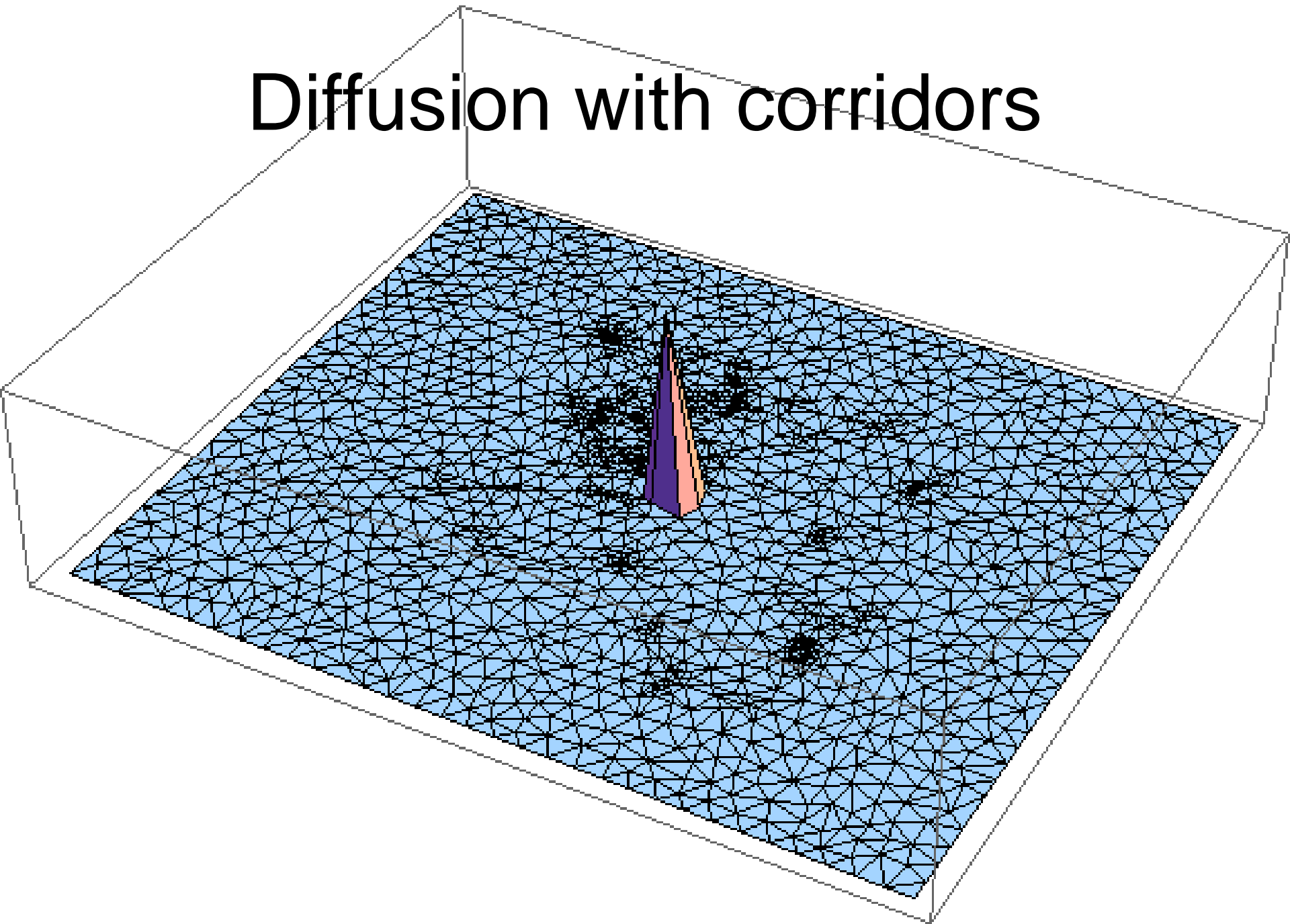
# Simple diffusion



# Diffusion with corridors

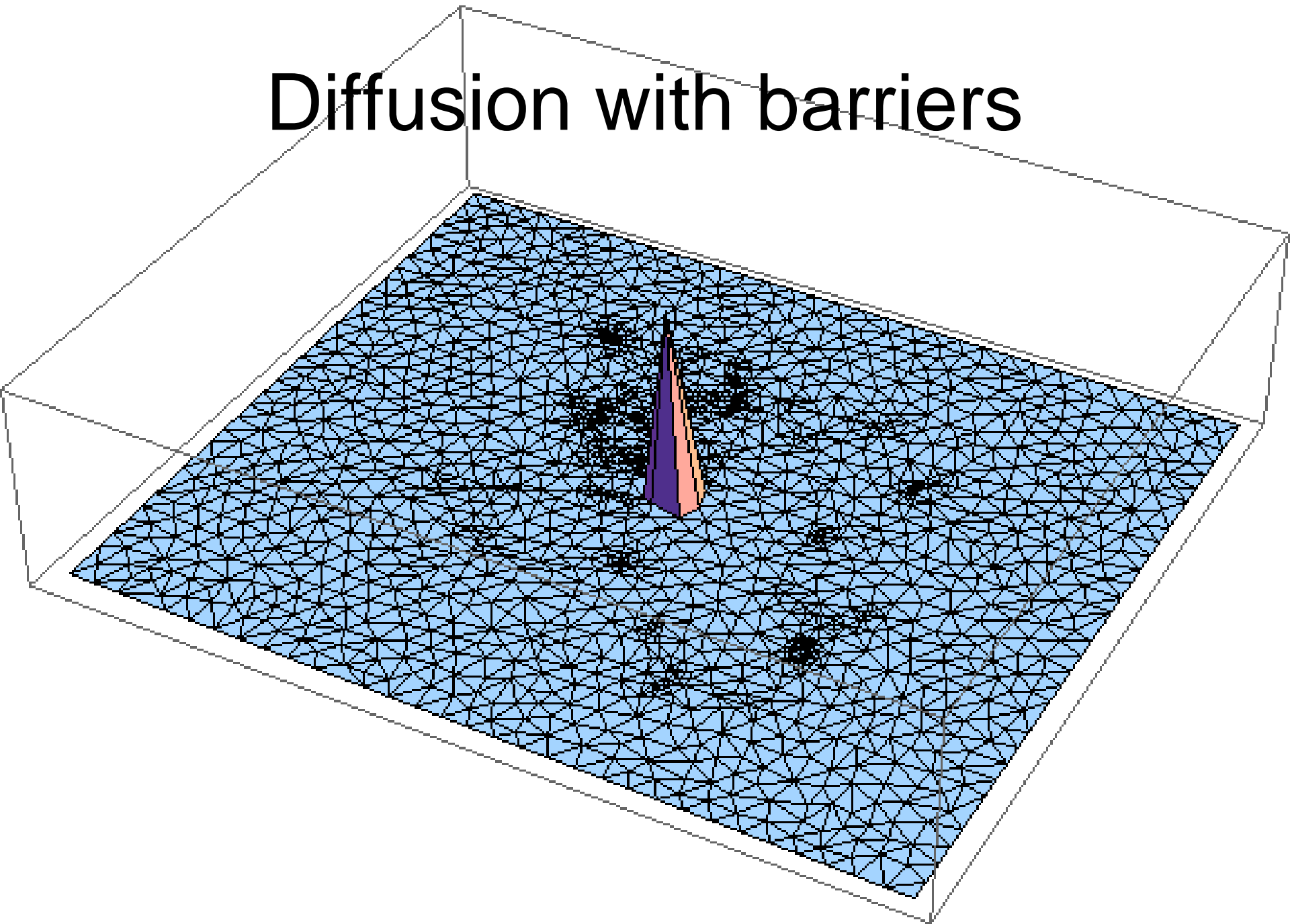


# Diffusion with corridors

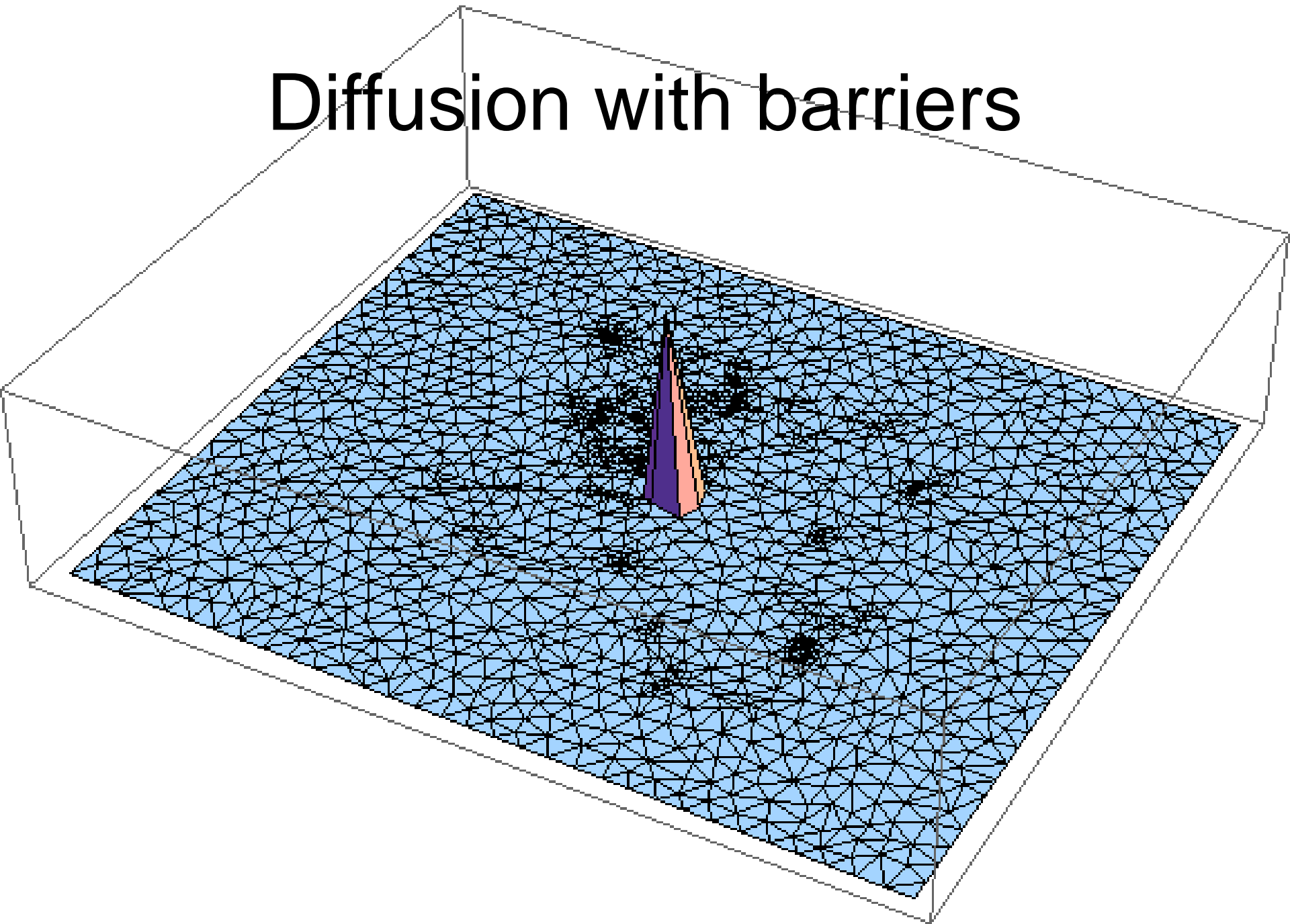




# Diffusion with barriers



# Diffusion with barriers



# Building biological assumptions in diffusion models: hypothetical movements in a mountainous landscape

Preference depends on altitude

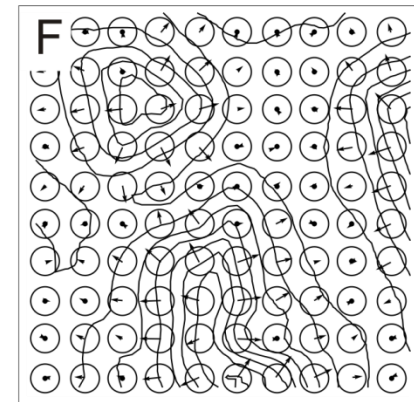
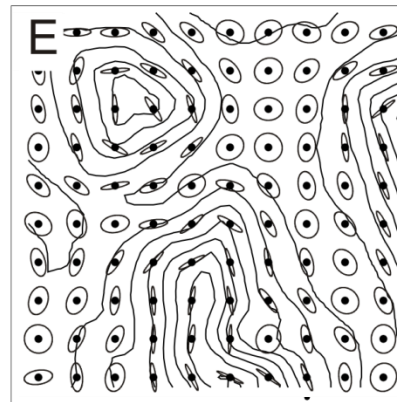
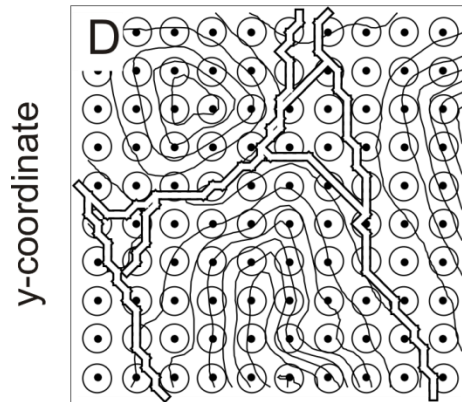
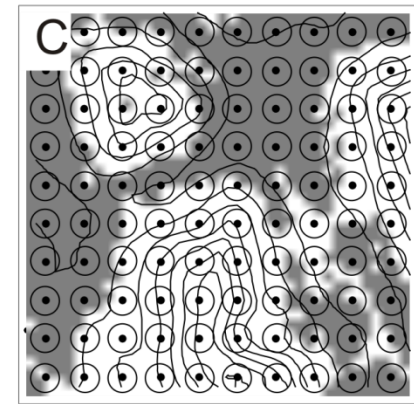
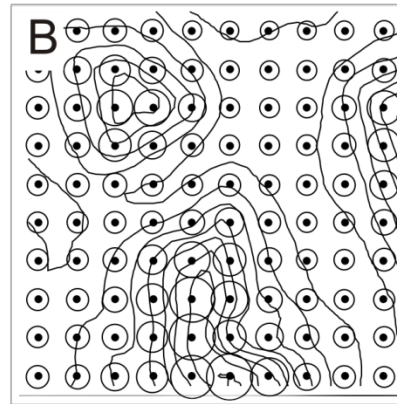
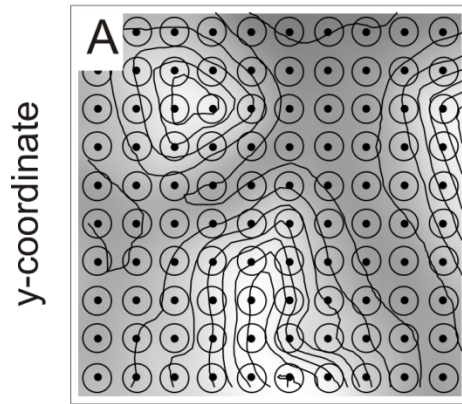
Diffusion depends on altitude

Preference for forest over rocks

Colour: habitat preference (the darker the better)

Ellipse: diffusion

Arrow: advection



x-coordinate

x-coordinate

x-coordinate

y-coordinate

y-coordinate

Roads as corridors

Anisotropy depends on slope

Advection downhill



# Model predictions

Preference depends on altitude

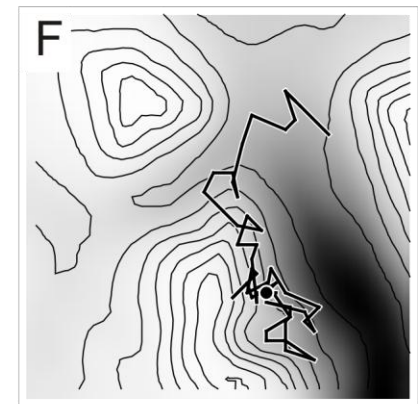
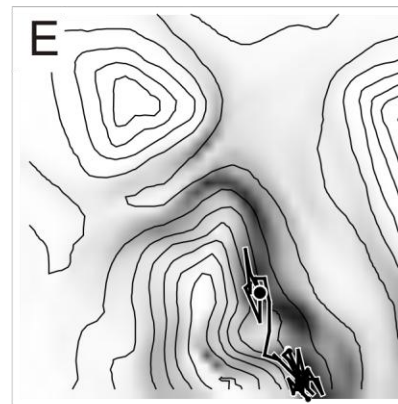
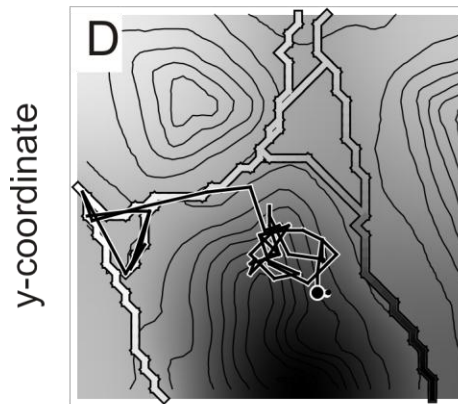
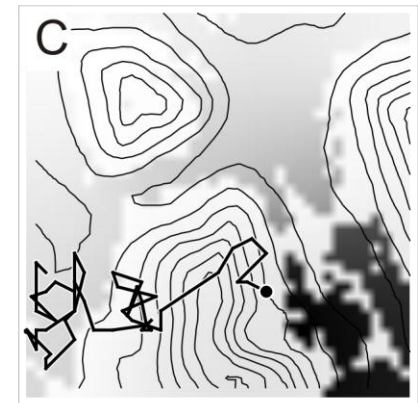
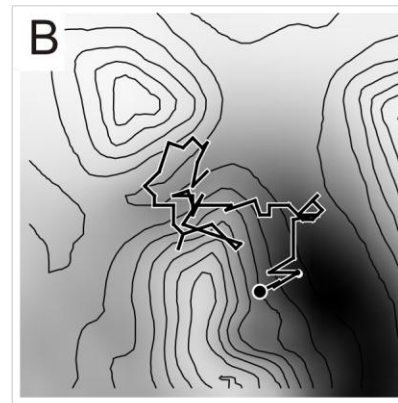
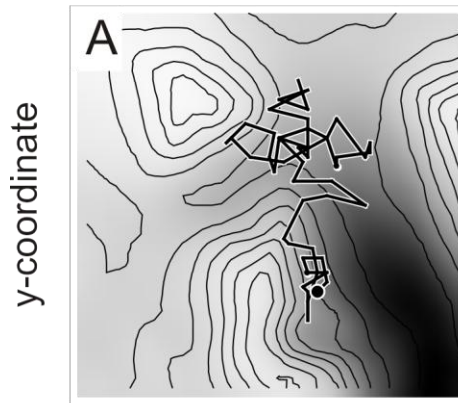
Diffusion depends on altitude

Preference for forest over rocks

Colour: probability density (the darker the higher)

Circle: initial position

Line: sample path



x-coordinate

x-coordinate

x-coordinate

Roads as corridors

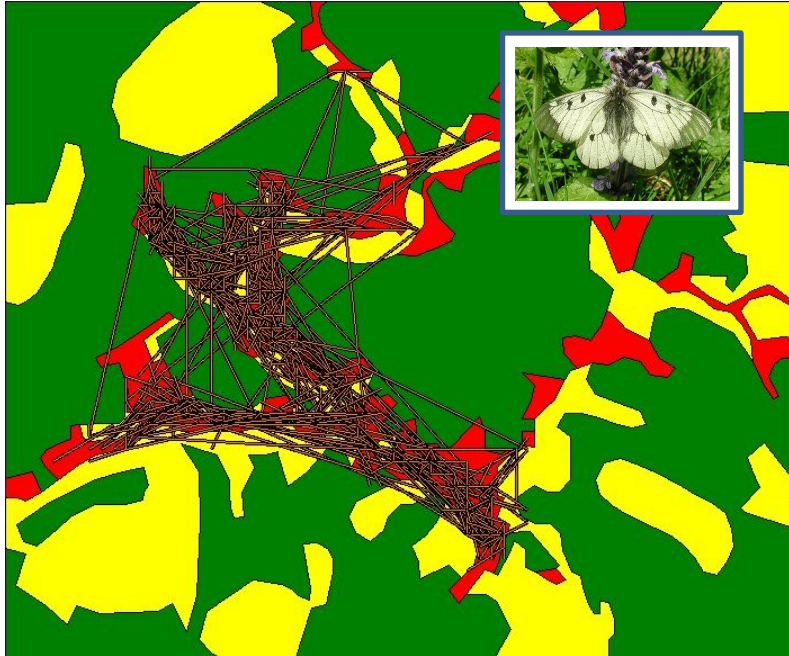
Anisotropy depends on slope

Advection downhill



# Fitting models to data

Ovaskainen 2004 (Ecology), Ovaskainen et al. 2008 (Ecology), Ovaskainen et al. 2008 (American Naturalist)



## Model with 7 parameters:

- habitat-specific diffusion coefficients (3 parameters)
- habitat selection (2 parameters, 1 normalized to one)
- mortality (1 parameter)
- capture probability (1 parameter)



Mikko Kuussaari

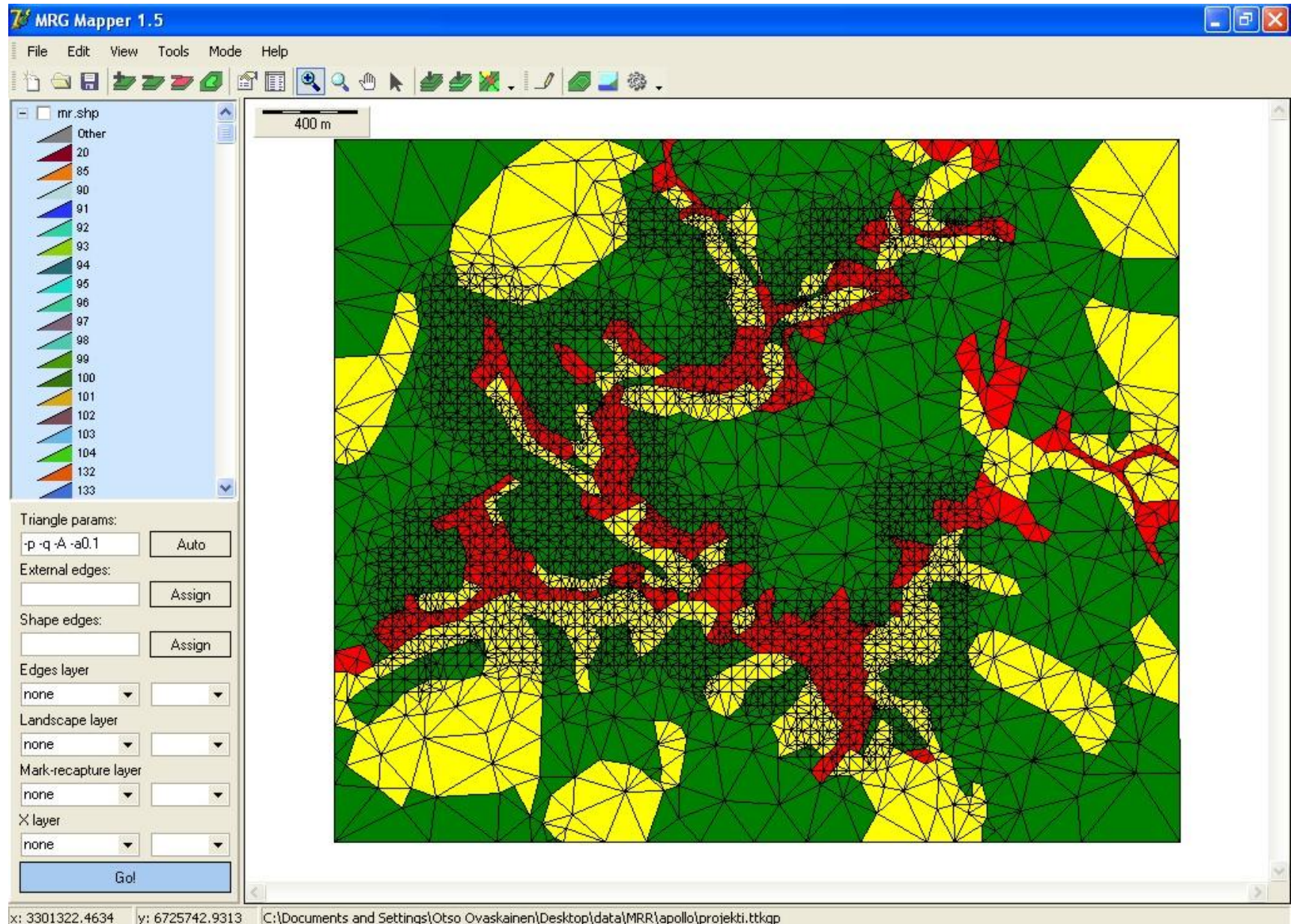


Miska Luoto



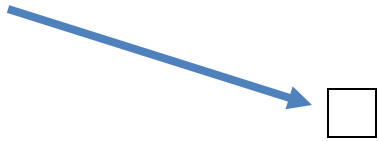
Iliri Ikonen

# Computing the likelihood with the finite element method

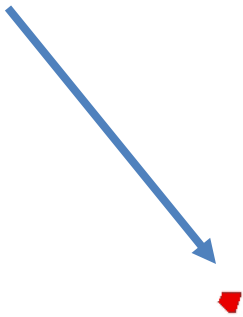


# The time-evolution of probability density

Observation  
at time  $t=5$



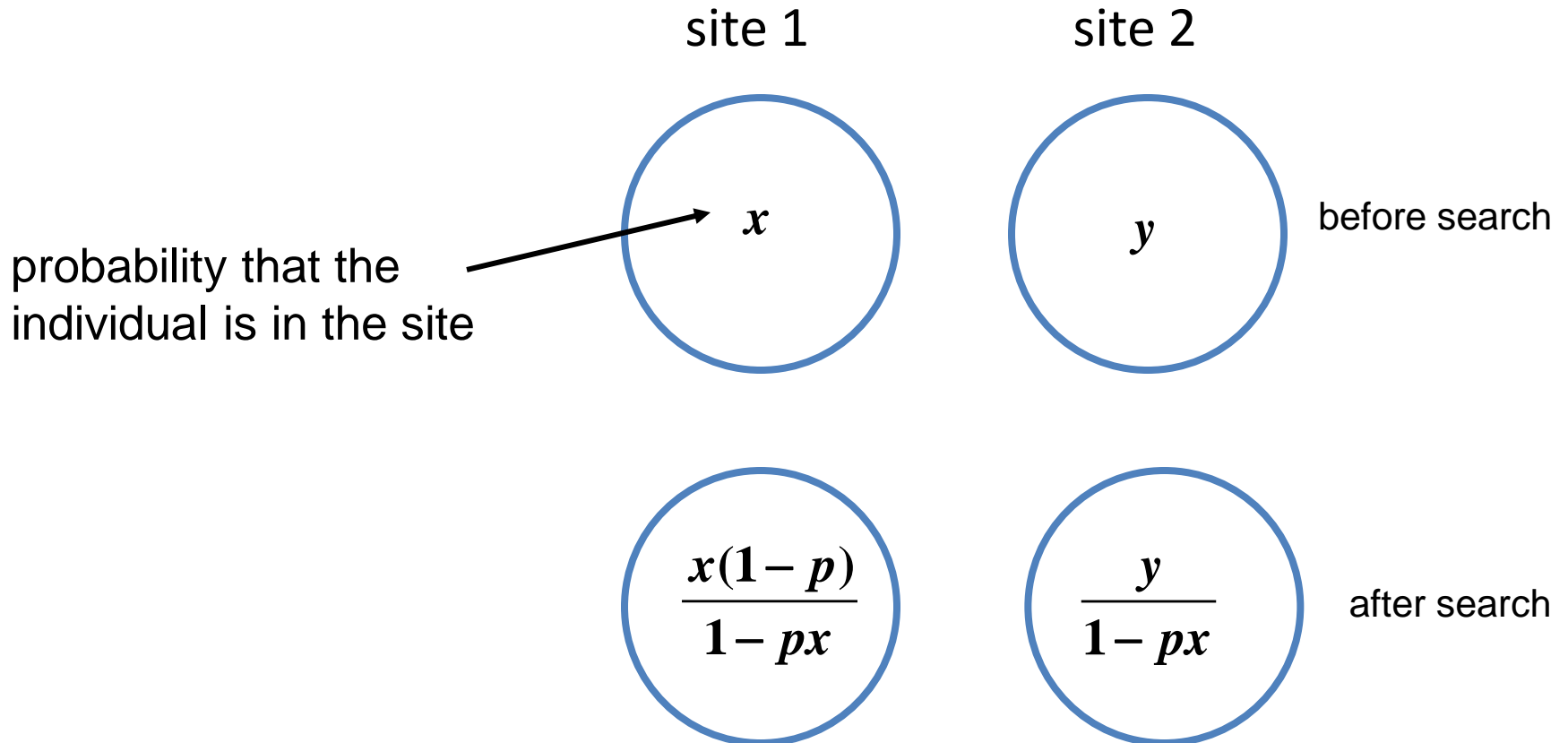
initial location



# Observation model:

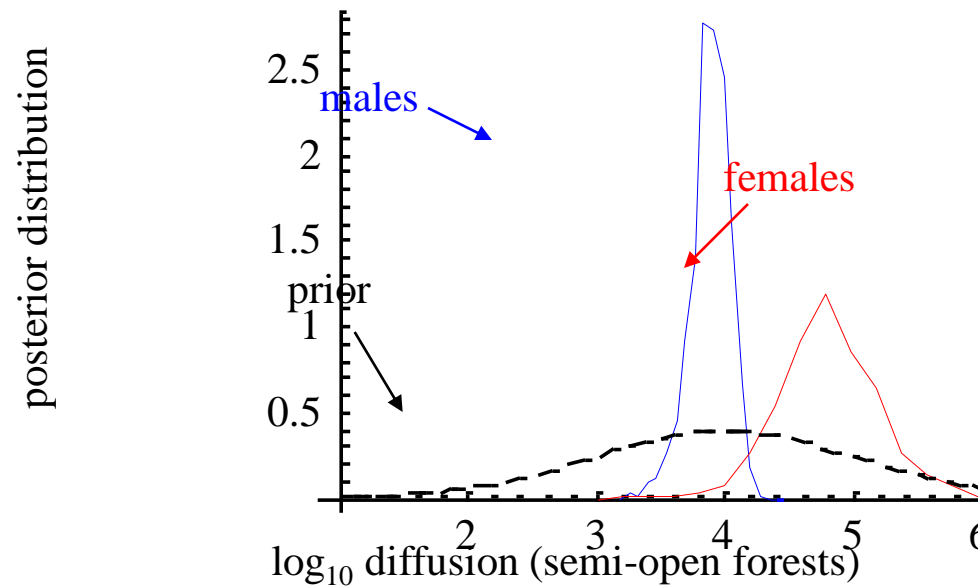
## Also not finding the individual is data

The capture probability  $p$  is the probability of observing an individual given that it actually is at the site





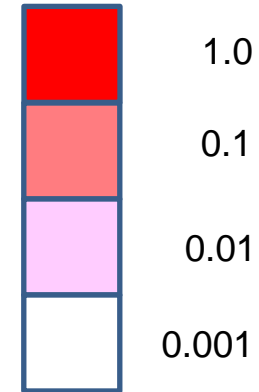
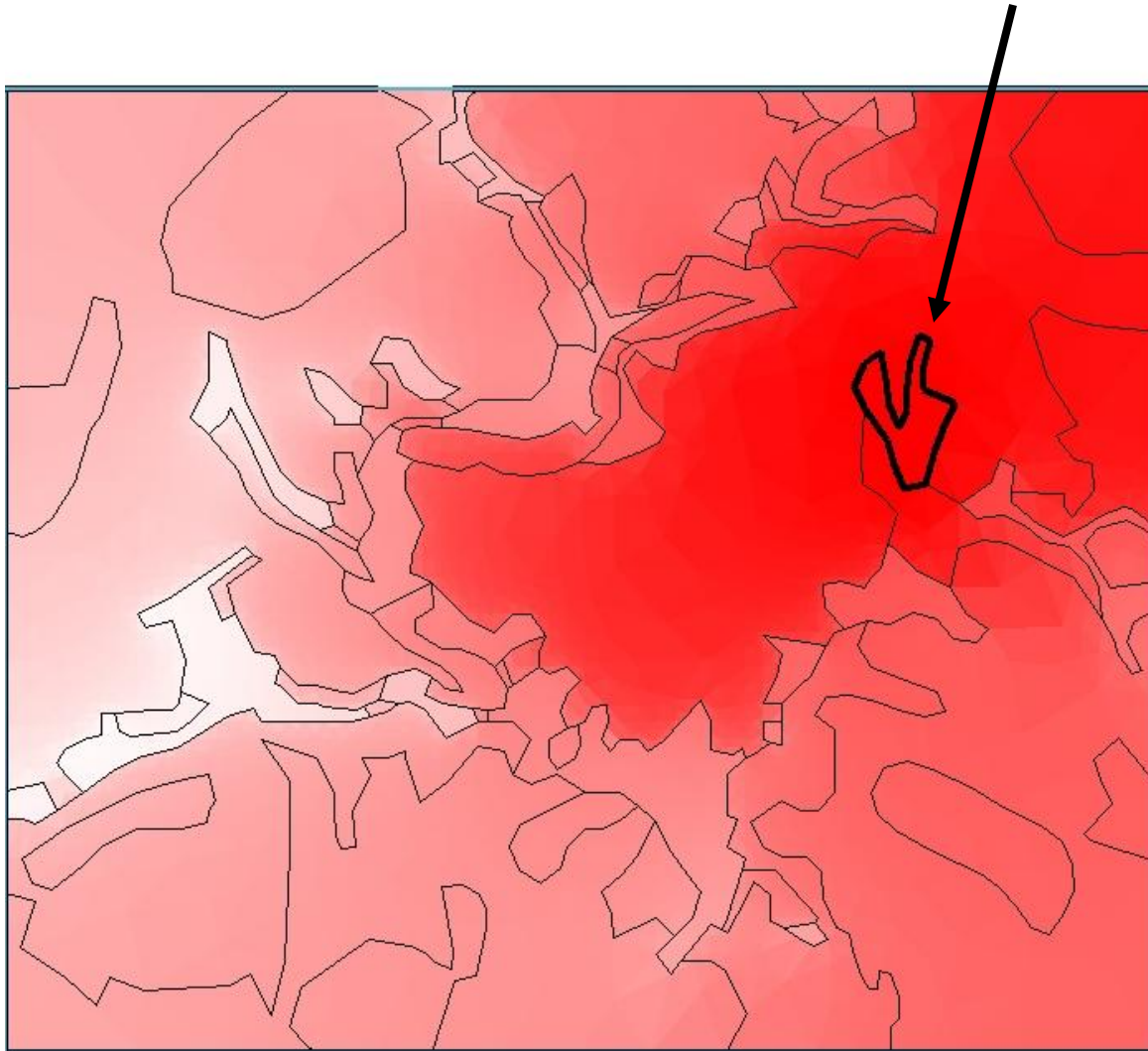
# Biological inference from parameter estimates



Females move faster than males outside the breeding habitat

# Example of model prediction

What is the probability that the butterfly ever visits this meadow?



**Theorem.** The hitting probability satisfies

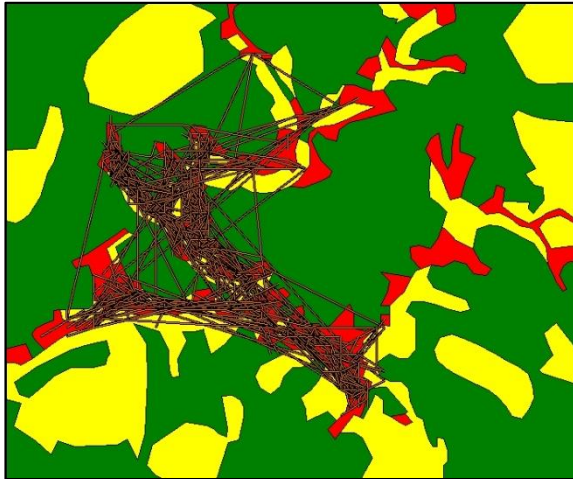
$$L^* p(x) = 0$$

with boundary condition  $B^{C_{1^*}}$

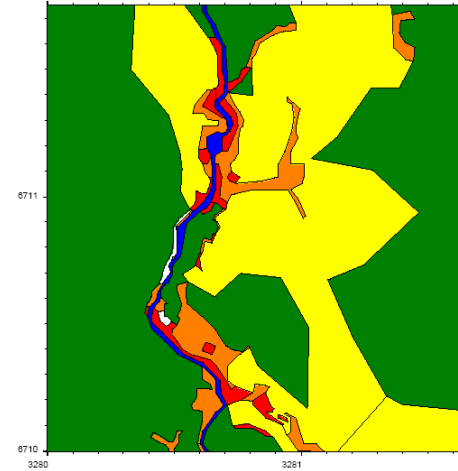
Ovaskainen & Cornell 2003  
(Journal of Applied Probability)

# Model validation

Landscape A



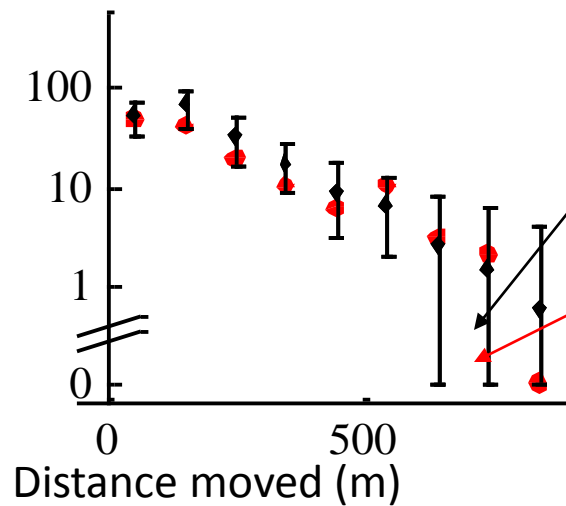
Landscape B



prediction



Number of observations



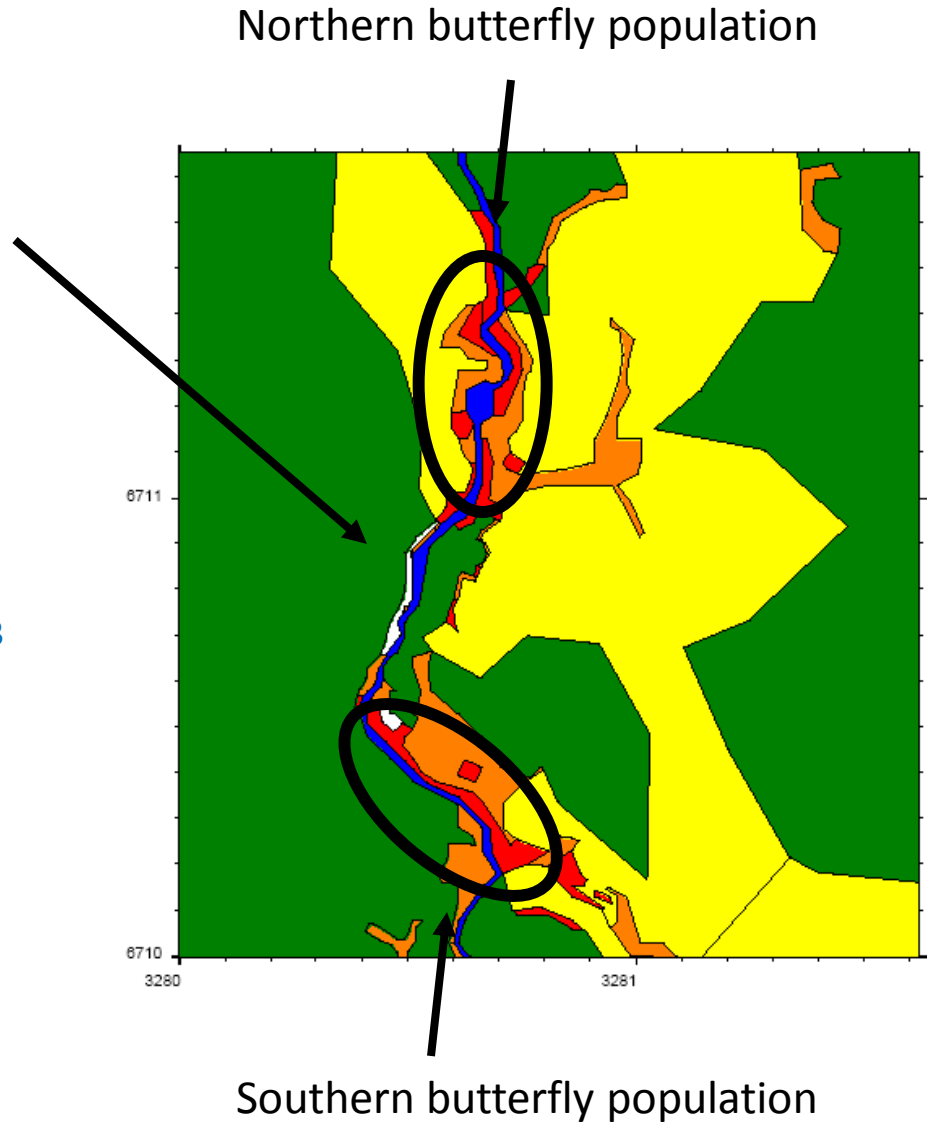
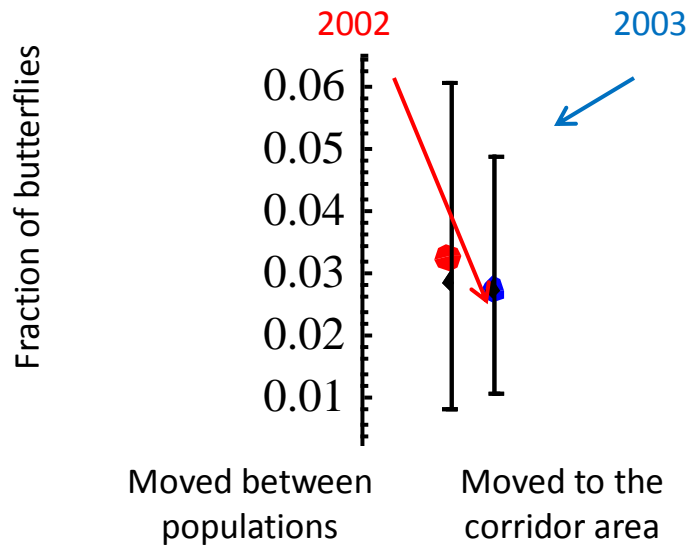
Model parameterized with data from Landscape A, prediction for Landscape B

Empirical data on Landscape B

# Effect of a movement corridor

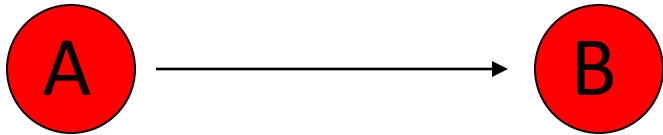


Regional environmental centre opened a movement corridor in autumn 2002



# What kind of a movement corridor would work?

Movement probability  $p_1$

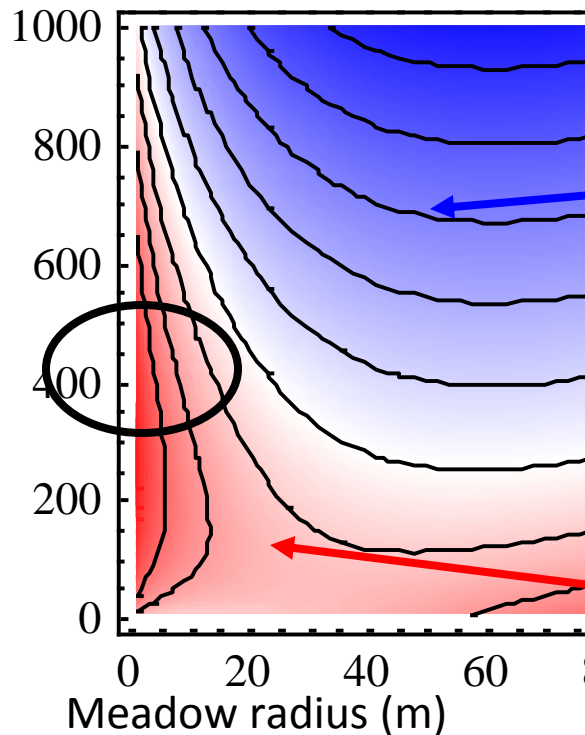


Movement probability  $p_2$



Colour: effect of the corridor ( $p_2/p_1$ )

Distance between meadows (m)



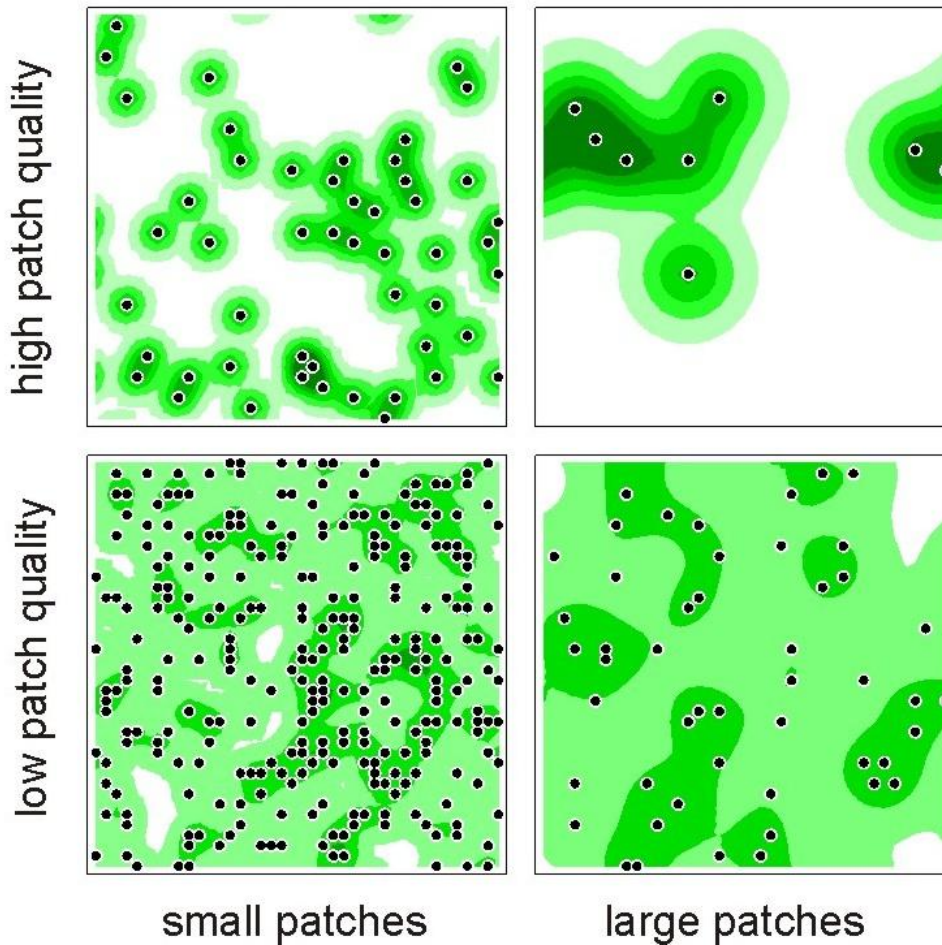
Corridor harms ( $p_2 < p_1$ )

Corridor helps ( $p_2 > p_1$ )

# Outline of the talk

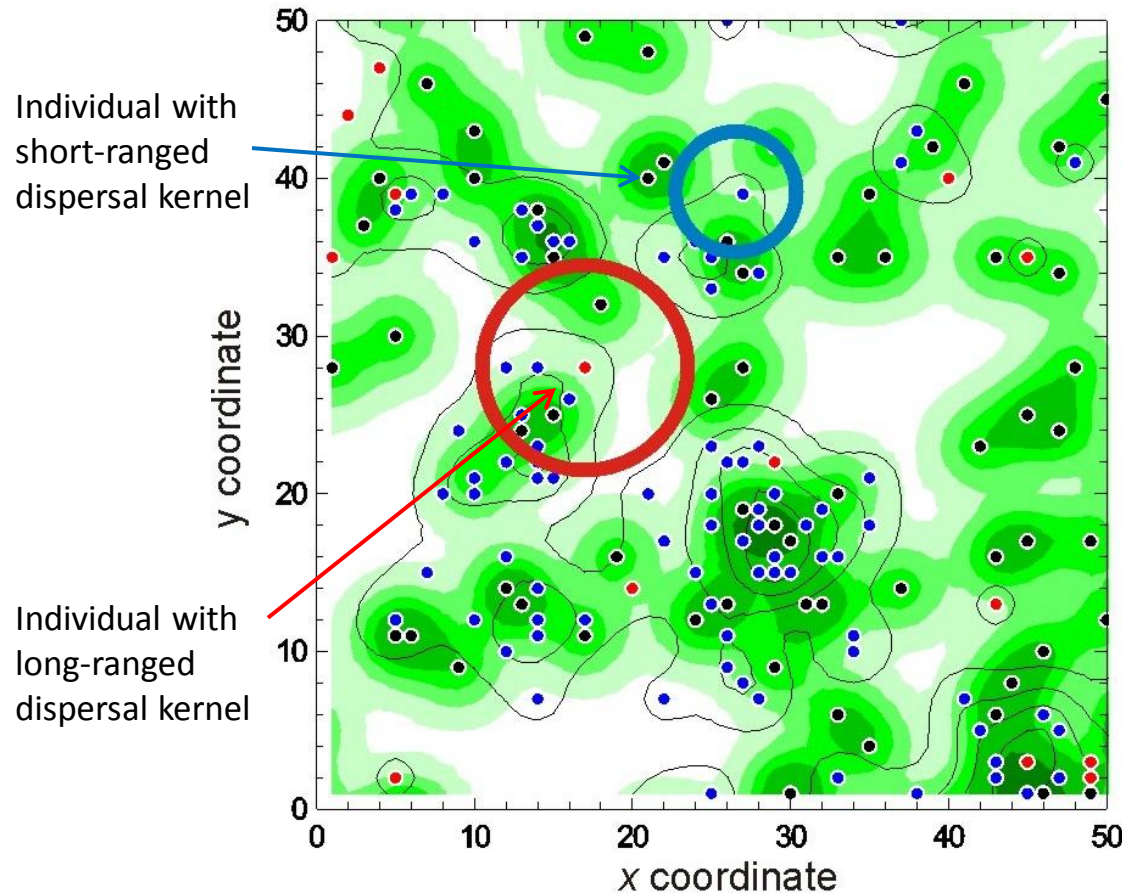
1. Animal movement in heterogeneous space
2. Evolution of dispersal in heterogeneous space

# Landscape structure controlled by patch density, size, quality and turnover



- Patches  $p$  appear at random locations at rate  $\theta\sigma$  (per unit area)
- Patches disappear at rate  $\sigma$  (per patch)
- Landscape quality
$$\omega = \Psi_p * p$$
- The kernel controls patch size (length scale) and patch quality (integral)

# Evolutionary model of dispersal



- Parameters at low density: fecundity  $f$ , establishment  $e$ , death  $d$ .
- Density-dependence affects death rate. Countour lines: local density of individuals
- Landscape quality affects fecundity

$$f = f_0 \omega$$

Mean-field model: logistic population growth:  $\frac{dN}{dt} = rN(1 - N / K)$



The model is a marked point process...

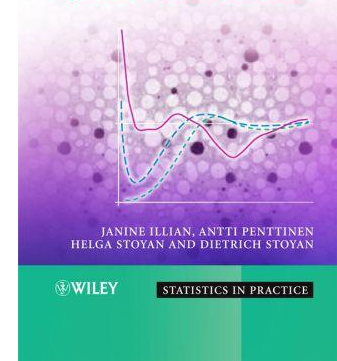


Prof. Yuri Kondratiev

...or a Markov evolution  
in the space of finite  
configurations...

...and you can write it down as a spatial  
moment equation

Statistical Analysis  
and Modelling of  
Spatial Point Patterns

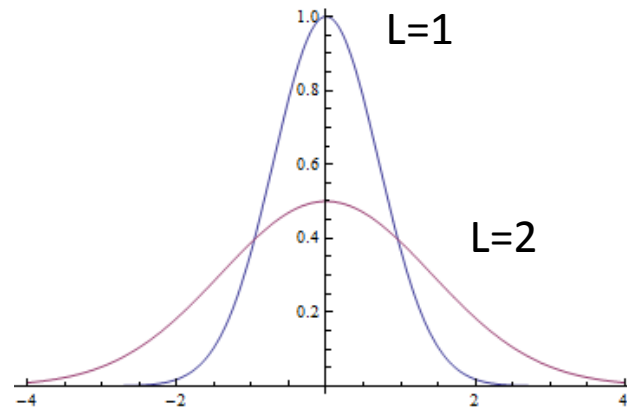


Prof. Ben Bolker

# Model analysis by moment closures or perturbation expansions

Approaching mean-field limit with length scale parameter  $L$

$$K(x) = \frac{k(x/L)}{L^d}$$

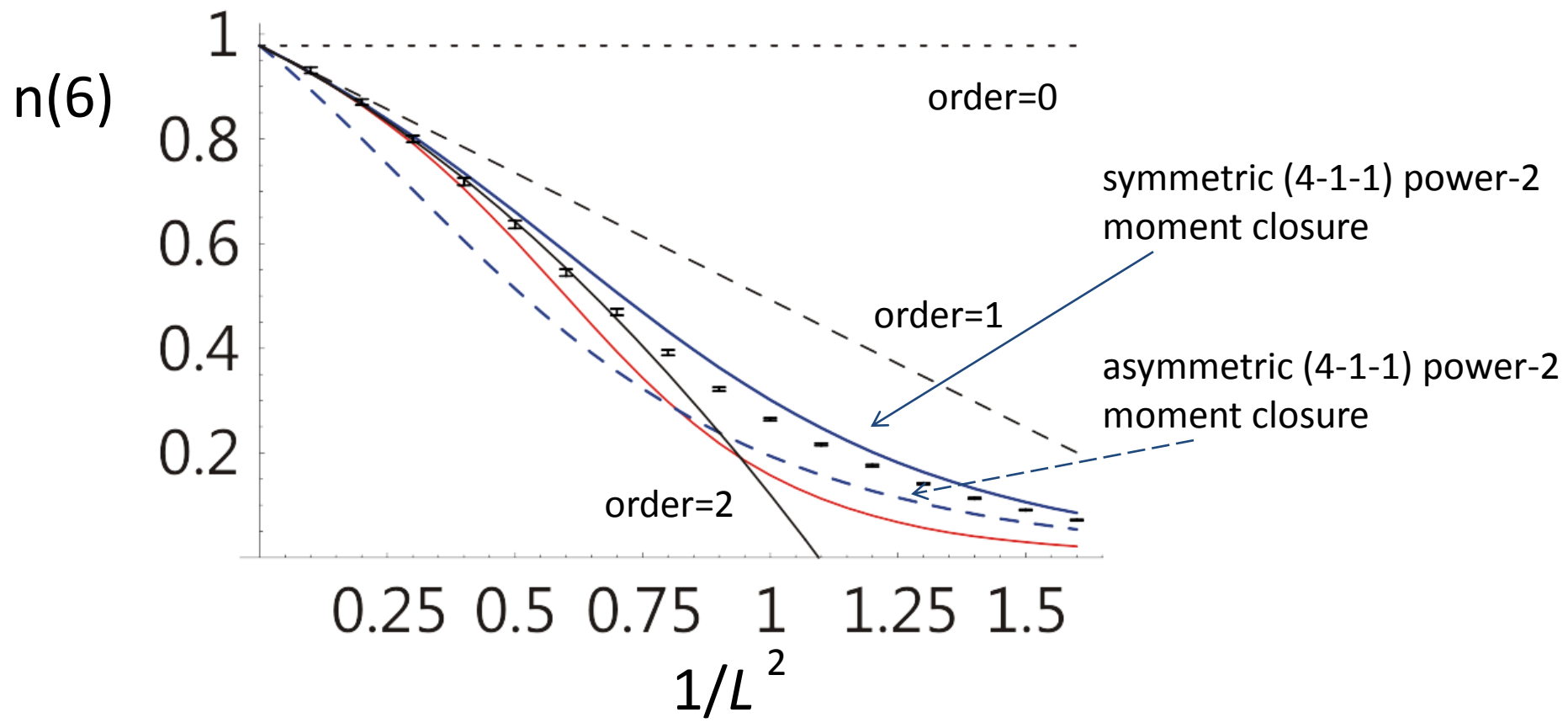


Expansion of density:  $\overline{n(t)} = n_0(t) + \frac{n_1(t)}{L^d} + \frac{n_2(t)}{L^{2d}} + \dots$

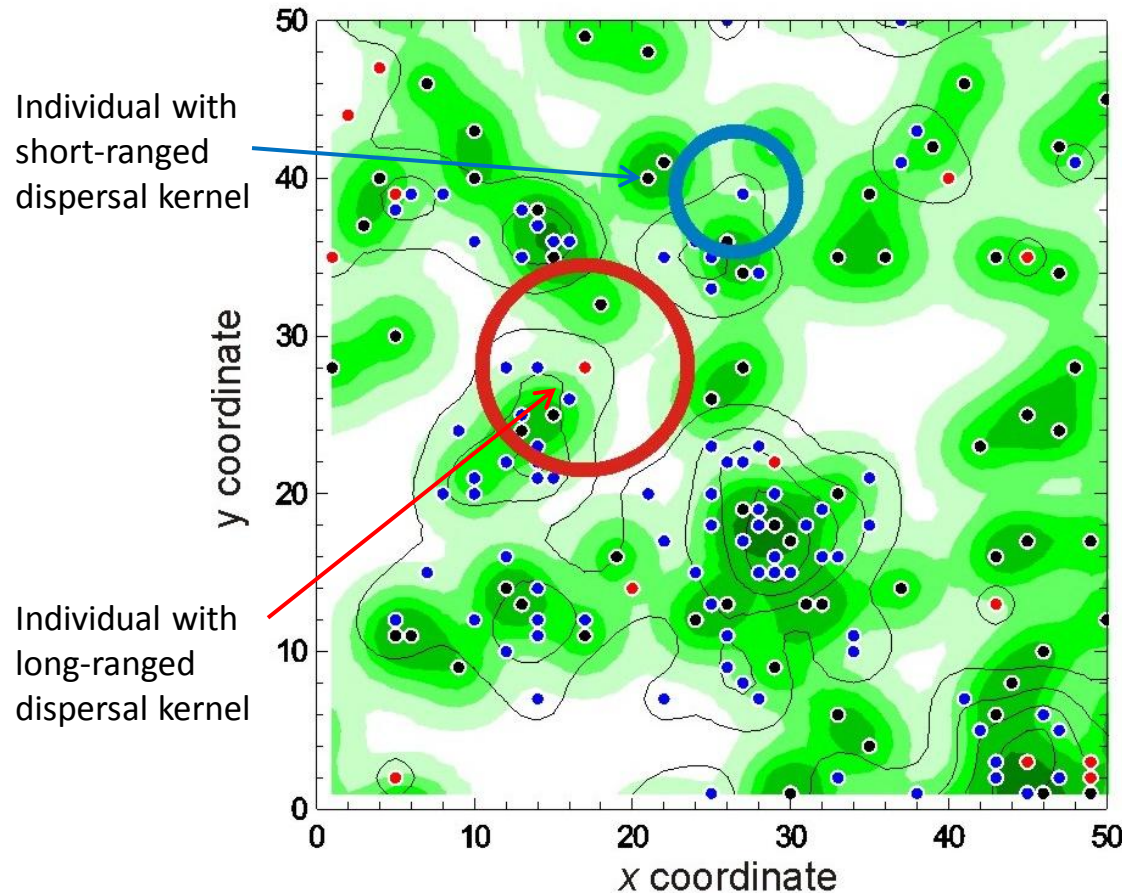


Ovaskainen & Cornell, PNAS (2006)

The expansion is asymptotically exact, but some moment closures may work better for small  $L$



# Evolutionary model of dispersal



- Parameters at low density: fecundity  $f$ , establishment  $e$ , death  $d$ .
- Density-dependence affects death rate. Countour lines: local density of individuals
- Landscape quality affects fecundity

$$f = f_0 \omega$$

Mean-field model: logistic population growth:  $\frac{dN}{dt} = rN(1 - N / K)$

# Two ways of modeling evolution

## Evolutionary Stable Strategy (ESS)

Assume a monomorphic population, see if a mutant can invade (adaptive dynamics, separation of ecological and evolutionary time-scales)

## Evolutionary Stable Frequency Distribution (ESFD)

Assume a polymorphic population persisting in a balance between mutation and selection (coupled ecological and evolutionary dynamics)



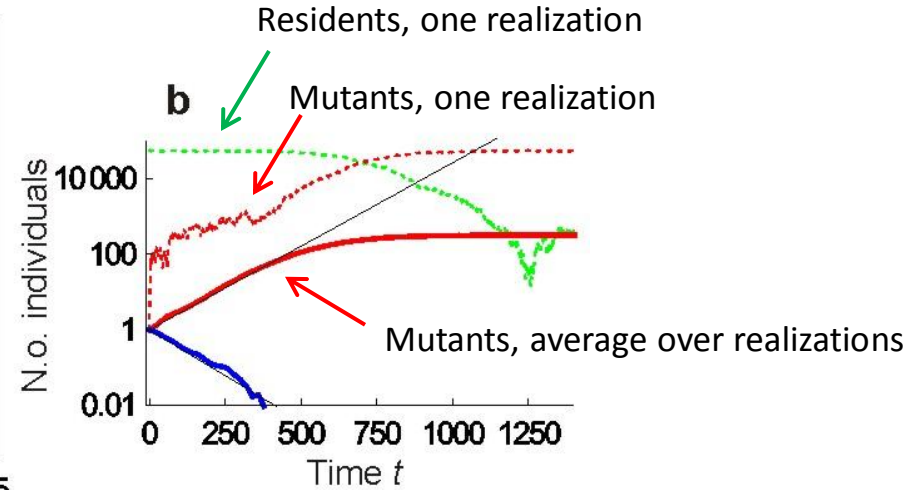
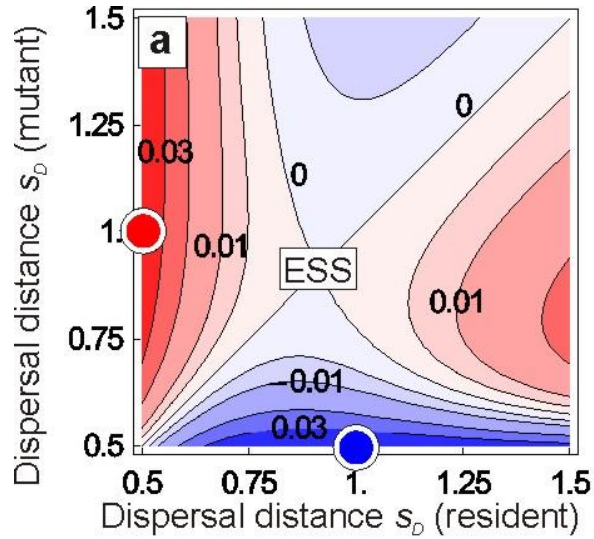
North, Cornell and Ovaskainen, *Evolution* (2011)

North et al., *Evolution* (2010)

# Two ways of modeling evolution

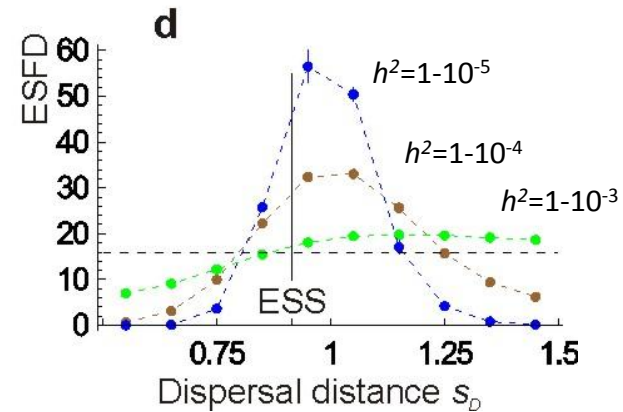
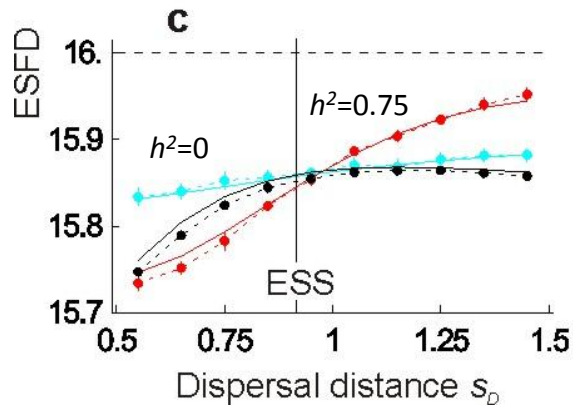
## ESS

Eigenvalue  
perturbation  
expansion,  
pairwise  
invasibility plots



## ESFD

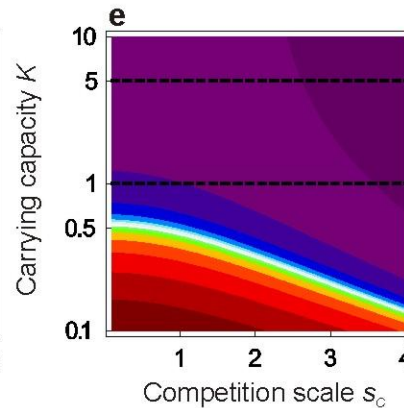
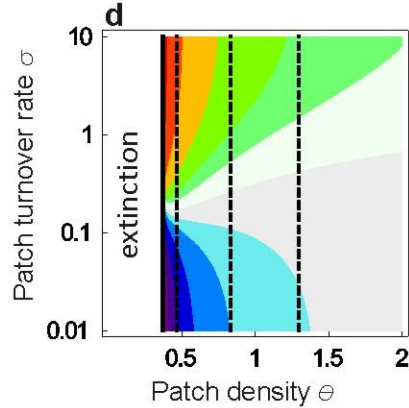
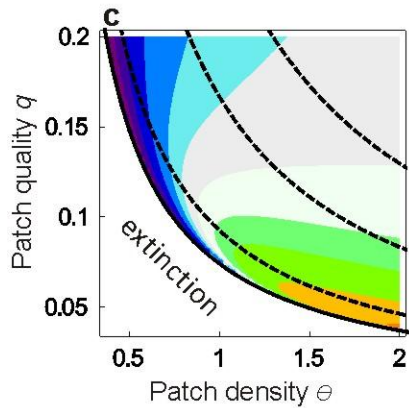
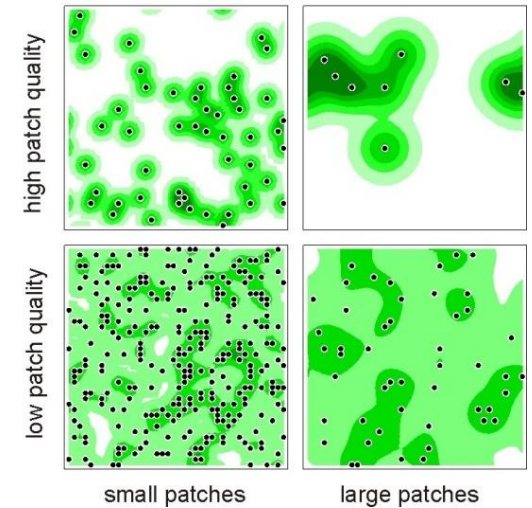
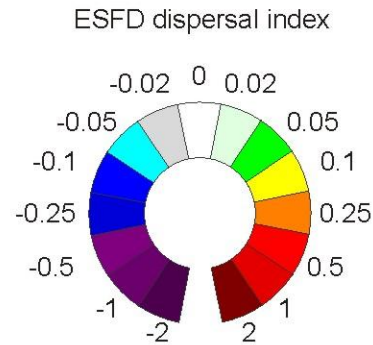
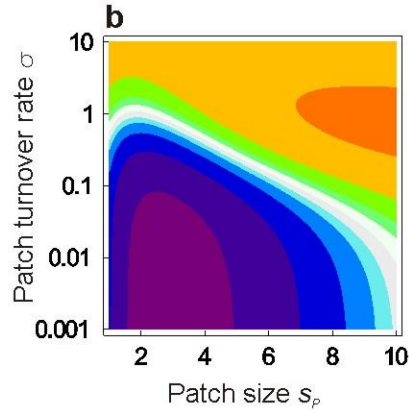
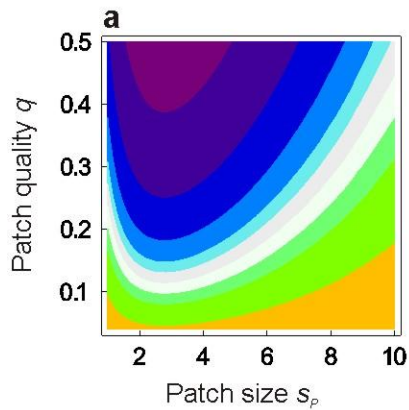
Density  
perturbation  
expansion



# Life-history & landscape structure influencing dispersal evolution

STATIC LANDSCAPES, TOTAL AMOUNT OF HABITAT CONSTANT

DYNAMIC LANDSCAPES, TOTAL AMOUNT OF HABITAT CONSTANT

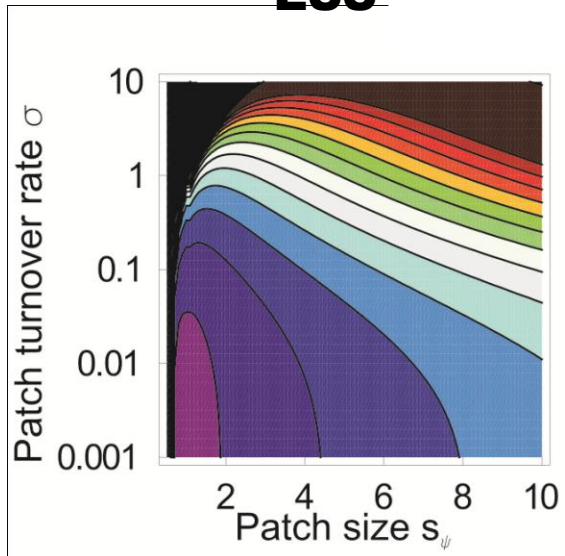


STATIC LANDSCAPES UNDERGOING HABITAT LOSS

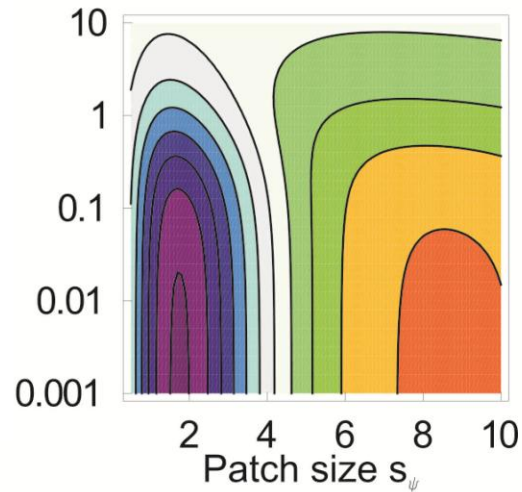
DYNAMIC LANDSCAPES UNDERGOING HABITAT LOSS

# Sometimes the coupling between ecological and evolutionary dynamics makes a qualitative difference

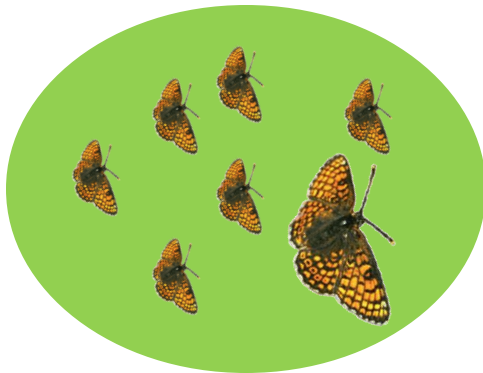
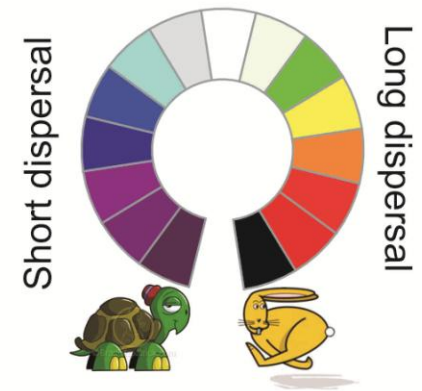
**ESS**



**ESFS**



Dispersal evolution





# Summary

- State-space models include a process model and an observation model, allowing one to bring biological knowledge into statistical inference, and estimate parameters that are not observed directly
- Diffusion-advection models provide a flexible framework for incorporating environmental heterogeneity into movement models
- Marked point processes can be used to study ecological and evolutionary dynamics in heterogeneous space. The model assumptions are formulated at the level of individuals, and emerging population level patterns can be studied using simulations or various approximations.

# References (2008-)

1. Ovaskainen, O. Smith, A. D., Osborne, J. L., Reynolds, R. D., Carreck, N. L., Martin, A. P., Niitepöld, K. and Hanski, I. 2008. Tracking butterfly movements with harmonic radar reveals an effect of population age on movement distance. *PNAS* **105**, 19090-19095.
2. Cornell, S. J. and Ovaskainen, O. 2008. Exact asymptotic analysis for metapopulation dynamics on correlated dynamic landscapes. *Theoretical Population Biology*, **74**, 209-225.
3. Ovaskainen, O., Luoto, M., Ikonen, I., Rekola, H., Meyke, E. and Kuussaari, M. 2008. An empirical test of a diffusion model: predicting clouded apollo movements in a novel environment. *American Naturalist* **171**, 610-619.
4. Ovaskainen, O., Rekola, H., Meyke, E. and Arjas, E. 2008. Bayesian methods for analyzing movements in heterogeneous landscapes from mark-recapture data. *Ecology* **89**, 542-554.
5. Ovaskainen, O. 2008. Analytical and numerical tools for diffusion based movement models. *Theoretical Population Biology* **73**, 198-211.
6. Patterson, T. A., Thomas, L., Wilcox, C., Ovaskainen, O. and Matthiopoulos, J. 2008. State-space models of individual animal movement. *Trends in Ecology and Evolution* **23**, 87-94.
7. Ovaskainen, O., Cano Arias, J. M. and Merilä, J. 2008. A Bayesian framework for comparative quantitative genetics. *Proceedings of the Royal Society B: Biological Sciences* **275**, 669-678.
8. Gripenberg, S., Ovaskainen, O., Morriën, E. and Roslin, T. 2008. Spatial population structure of a specialist leaf-mining moth. *Journal of Animal Ecology* **77**, 757-767.
9. Haag-Liautaud, C., Pederson, J., Ovaskainen, O. and Keller, L. 2008. Breeding system and reproductive skew in a highly polygynous ant population. *Insectes Sociaux* **55**, 347-354.
10. Sundell, J., Trebatická, L., Oksanen, T., Ovaskainen, O., Haapakoski, M., and Ylönen, H. 2008. Predation on two vole species by a shared predator: antipredatory response and prey preference. *Population Ecology* **50**, 257-266.
11. O'Hara, R. B., Cano Arias, J. M., Ovaskainen, O., Teplitsky, C. and Alho, J. 2008. Bayesian approaches in evolutionary quantitative genetics. *Journal of Evolutionary Biology*, **21**, 949-957.
12. Arellano, L., León-Cortés, J. L., and Ovaskainen, O. 2008. Patterns of abundance and movement in relation to landscape structure: a study of a common scarab (*Canthon cyanellus cyanellus*) in Southern Mexico. *Landscape Ecology*, **23**, 69-78.
13. Hottola, J., Ovaskainen, O. and Hanski, I. 2009. A unified measure of the number, volume and diversity of dead trees and the response of fungal communities. *Journal of Ecology* **97**, 1320-1328.
14. Zheng, C., Weisser, W.W., Härril, S.A. and Ovaskainen, O. 2009. Hierarchical metapopulation dynamics of two aphid species on a shared host plant. *American Naturalist* **174**, 331-341.
15. Roslin, T., Avomaa, T., Leonard, M., Luoto, M. and Ovaskainen, O. 2009. Some like it hot: microclimatic variation affects the abundance and movements of a critically endangered dung beetle. *Insect Conservation and Diversity* **2**, 232-241.
16. Niitepöld, K., Smith, A. D., Osborne, J. L., Reynolds, D. R., Carreck, N. L., Martin, A. P., Marden, J. H., Ovaskainen, O. and Hanski, I. 2009. Flight metabolic rate and *Pgi* genotype influence butterfly dispersal rate in the field. *Ecology* **90**, 2223-2232.
17. Zheng, C., Pennanen, J. and Ovaskainen, O. 2009. Modelling dispersal with diffusion and habitat selection: analytical results for highly fragmented landscapes. *Ecological Modelling* **220**, 1495-1505.
18. Zheng, C., Ovaskainen, O. and Hanski, I. 2009. Modelling single nucleotide effects in the *Pgi* gene on dispersal in the Glanville fritillary butterfly: coupling of ecological and evolutionary dynamics. *Philosophical Transactions of the Royal Society of London Series B-Biological Sciences* **364**, 1519-1532.
19. Tack, A. J. M., Ovaskainen, O., Harrison, P. J. and Roslin, T. 2009. Competition as a structuring force in leaf miner communities. *Oikos* **118**, 809-818.
20. Ovaskainen, O., Nokso-Koivisto, J., Hottola, J., Rajala, T., Pennanen, T., Ali-Kovero, H., Miettinen, O., Oinonen, P., Auvinen, P., Paulin, L., Larsson, K.-H. and Mäkipää, R. 2010. Identifying wood-inhabiting fungi with 454 sequencing - what is the probability that BLAST gives the correct species? *Fungal Ecology* **3**, 274-283.
21. Tack, A. J. M., Ovaskainen, O., Pulkkinen, P. and Roslin, T. 2010. Spatial location dominates over host plant genotype in structuring an herbivore community. *Ecology* **91**, 2660-2672.
22. Ovaskainen, O., Hottola, J. and Siitonen, J. 2010. Modeling species co-occurrence by multivariate logistic regression generates new hypotheses on fungal interactions. *Ecology* **91**, 2514-2521.
23. Sebastián-Conzález, E., Sánchez-Zapata, J. A., Botella, F. and Ovaskainen, O. 2010. Testing the heterospecific attraction hypothesis with time-series data on species co-occurrence. *Proceedings of the Royal Society B: Biological Sciences* **277**, 2983-2990.
24. Ovaskainen, O. and Meerson, B. 2010. Stochastic models of population extinction. *Trends in Ecology & Evolution* **25**, 643-652.
25. North, A., Pennanen, J., Ovaskainen, O. and Laine, A.-L. 2011. Local adaptation in a changing world: the roles of gene-flow, mutation, and sexual reproduction. *Evolution* **65**, 79-89.
26. Hanski, I., Mononen, T. and Ovaskainen, O. 2011. Eco-evolutionary metapopulation dynamics and the spatial scale of adaptation. *American Naturalist* **177**, 29-43.
27. Gurarie, E., Suutarinen, J., Kojala, I. and Ovaskainen, O. 2011. Summer movements, predation and habitat use of wolves in human modified boreal forests. *Oecologia* **165**, 891-903.
28. Wang, R., Ovaskainen, O., Cao, Y., Chen, H., Zhou, Y., Xu, C., and Hanski, I. 2011. Dispersal in the Glanville fritillary butterfly in fragmented versus continuous landscapes: comparison between three methods. *Ecological entomology* **36**, 251-260.
29. Ovaskainen, O. and Soinen, J. 2011. Making more out of sparse data: hierarchical modeling of species communities. *Ecology* **92**, 289-295.
30. North, A., Cornell, S. J. and Ovaskainen, O. 2011. Evolutionary responses of dispersal distance to landscape structure and habitat loss. *Evolution* **65**, 1739-1751.
31. Entling, M., Stämpfli, K. and Ovaskainen, O. Increased propensity for aerial dispersal in disturbed habitats due to intraspecific variation and species turnover. *Oikos*, in press.
32. Sarhan, A., Ovaskainen, O. and Hanski, I. Size and genetic composition of the colonizing propagules in a butterfly metapopulation. *Oikos*, in press.
33. Gurarie, E. and Ovaskainen, O. Characteristic spatial and temporal scales unify models of animal movement. *American Naturalist*, in press.
34. Harrison, P. J., Hanski, I. and Ovaskainen, O. Bayesian state-space modeling of metapopulation dynamics in the Glanville fritillary butterfly. *Ecological Monographs*, in press.