Weighted ℓ_1 Minimization: Stability, robustness, and some implications

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Collaboration

Joint work with:

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- Rayan Saab
- Özgür Yılmaz

Outline

Part 1: Introduction and Overview

Part 2: Stability and Robustness of Weighted ℓ_1 Minimization

Part 3: Experimental Results and Stylized Applications

Part 3: Some implications of the weighted ℓ_1 result

Motivation

- We want to recover a k-sparse signal $x \in \mathbb{R}^N$.
- Given $n \ll N$ linear and noisy measurements $y = Ax + e$.
- If A has the RIP with $\delta_{2k} < \sqrt{2} - 1$ or $\delta_{(a+1)k} < \frac{a-1}{a+1}, a > 1$,
- Suppose k, n and N are such that ℓ_1-minimization fails to recover x, and we have prior information on the support of x.
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Definition: Restricted Isometry Property (RIP)

The RIP constant δ_k is defined as the smallest constant such that $\forall x \in \Sigma_k^N$

$$(1 - \delta_k)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1 + \delta_k)\|x\|_2^2,$$

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Constrained ℓ_1-minimization

- $\min_{u \in \mathbb{R}^N} \|u\|_1 \quad \text{subject to } \|Au - y\|_2 \leq \|e\|_2, \qquad k \lesssim n/\log(N/n)$
- $\|x^* - x\|_2 \leq C_0\|e\|_2^2 + C_1 k^{-1/2}\|x - x_k\|_1$

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Failed recovery and prior information

- Eg. when $k > \hat{k} \approx n / \log(N/n)$
- Eg. indices 1, 3, and 6 are non-zero.

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Signals with Prior Information

- In many applications, it is possible to draw an estimate of the support of the signal, for example:
  - Natural images have large DCT coefficients that are localized in the low frequency subbands.
  - Video sequences are temporally correlated, resulting in a shared subset of their support.
  - Other signals such as seismic data, ...

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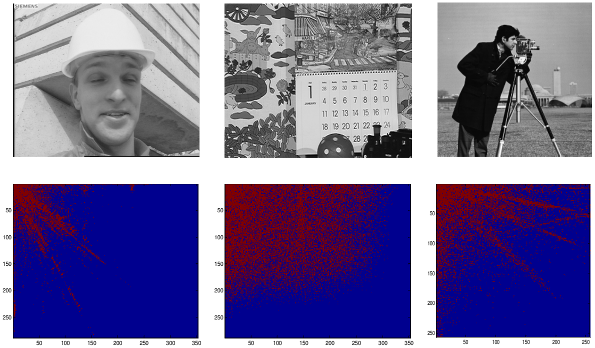

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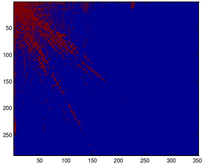

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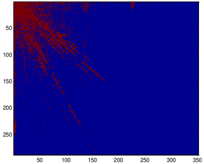

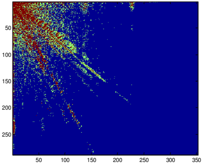

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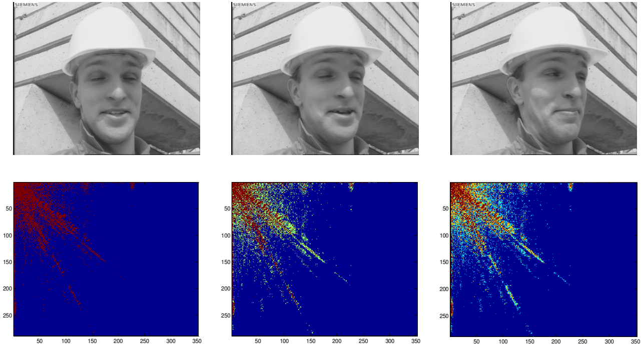

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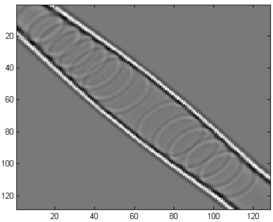

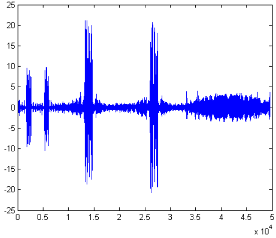

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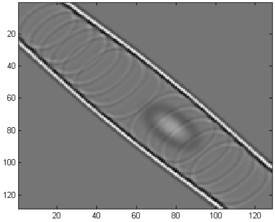

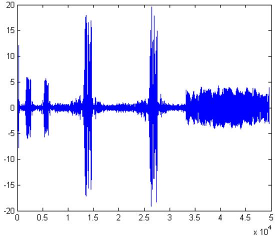

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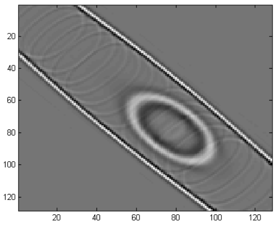

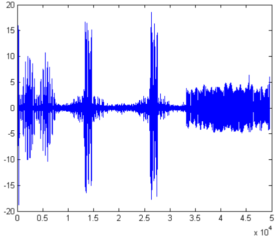

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- But, the ℓ_1 minimization formulation is non-adaptive, i.e., aside from sparsity, no prior information on x is used in the recovery.

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Problem Setup

- Suppose that x is a k-sparse signal supported on an unknown set T_0.
- Let $\widetilde{T}$ be a known support estimate that is partially accurate.
- We want to:
  1. Recover x by incorporating $\widetilde{T}$ in the recovery algorithm.
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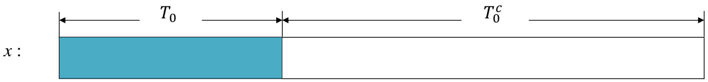
- Our approach: weighted ℓ_1 minimization.



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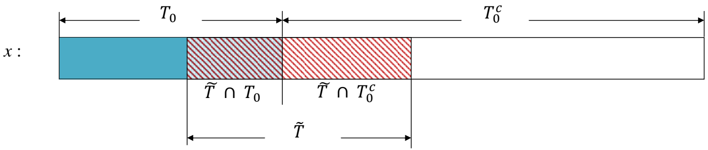

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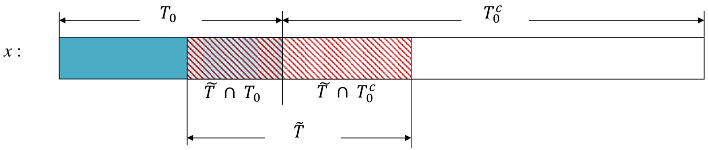

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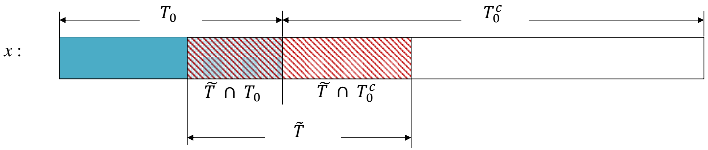

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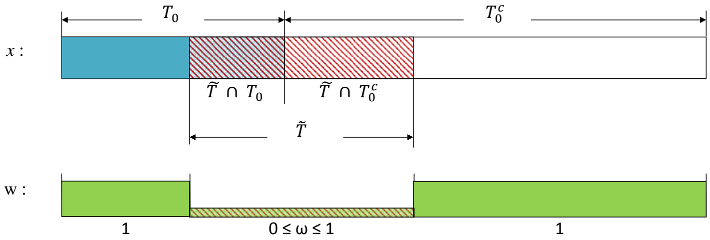
Given a set of measurements y, solve

$$\min_x \ \|x\|_{1,\mathrm{w}} \text{ subject to } \|Ax-y\|_2 \leq \epsilon \quad \text{with} \quad \mathrm{w}_i = \begin{cases} 1, & i \in \widetilde{T}^c, \\ \omega, & i \in \widetilde{T}. \end{cases}$$

where $0 \leq \omega \leq 1$ and $\|x\|_{1,\mathrm{w}} := \sum_i \mathrm{w}_i |x_i|$, $\|e\|_2^2 \leq \epsilon$.



Contributions

- We adopt weighted ℓ_1 minimization and derive stability and robustness guarantees for the recovery of a signal x with partial support estimate $\widetilde{T}$.
- We show that if at least 50% of $\widetilde{T}$ is accurate, then weighted ℓ_1 minimization guarantees recovery with
  - weaker RIP conditions
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- We demonstrate through extensive experiments that assigning weights $0 < \omega < 1$ on $\widetilde{T}$ results in the best reconstruction performance, especially if x is compressible.

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Related Work

- Borries et al. '07: empirically demonstrate that x is recoverable with s fewer measurements by setting $\omega = 0$ on a known subset of the support of size s.
- Khajehnejad et al. '09: find a class of signals x, defined by a probabilistic model on sparsity and by the weight vector, that can be recovered with high probability using weighted ℓ_1 minimization.
- Vaswani et al. '10: propose weighted ℓ_1 minimization with zero weights and find weaker sufficient recovery conditions in the noise-free case.
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Find the vector x from a set of measurements y using the support estimate $\widetilde{T}$ by solving

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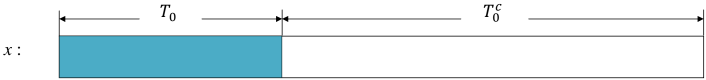
where $0 \leq \omega \leq 1$ and $\|x\|_{1,\mathrm{w}} := \sum_i \mathrm{w}_i |x_i|$.

Stability and Robustness

- Let x be in $\mathbb{R}^N$ and let x_k be its best k-term approximation, supported on T_0.
- Let $|\widetilde{T}| = \rho k$ and define $\alpha = \frac{|\widetilde{T} \cap T_0|}{|\widetilde{T}|}$, and $0 \leq \omega \leq 1$.



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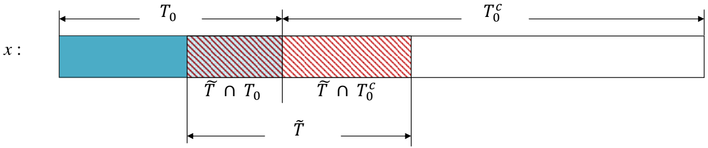

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Theorem (Main Result)

Suppose there exists an $a \in \frac{1}{k}\mathbb{Z}$, with $a \geq (1-\alpha)\rho$, $a > 1$, and that A satisfies

$$\delta_{ak} + a\gamma\delta_{(a+1)k} < a\gamma - 1.$$

Then the solution x^* to the weighted ℓ_1 problem obeys

$$\|x^* - x\|_2 \leq C_0'\epsilon + C_1' k^{-1/2}\left(\omega\|x_{T_0^c}\|_1 + (1-\omega)\|x_{\widetilde{T}^c \cap T_0^c}\|_1\right).$$

- $\gamma = \frac{1}{\left(\omega + (1-\omega)\sqrt{1+\rho-2\alpha\rho}\right)^2}$

Sufficient Recovery Condition

It is sufficient to have:

- $\delta_{(a+1)k} < \hat{\delta}^{(\omega)} := \frac{a-\left(\omega+(1-\omega)\sqrt{1+\rho-2\alpha\rho}\right)^2}{a+\left(\omega+(1-\omega)\sqrt{1+\rho-2\alpha\rho}\right)^2}$
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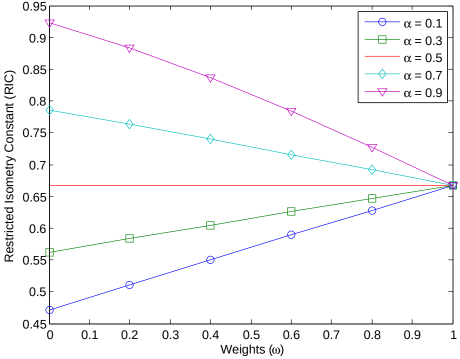

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- Take for example: $\hat{\delta}^{(1)} = 0.6667$, and $\omega = 0.5$, $\rho = 1$,
  - if $\alpha = 0.7$, then $\hat{\delta}^{(\omega)} = 0.7279$.
  - if $\alpha = 0.3$, then $\hat{\delta}^{(\omega)} = 0.6151$.

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Error Bound Constants

Measurement noise constant C_0':

- $C_0' = \dfrac{2\left(1+\left(\omega+(1-\omega)\sqrt{1+\rho-2\alpha\rho}\right)/\sqrt{a}\right)}{\sqrt{1-\delta_{(a+1)k}}-\frac{\omega+(1-\omega)\sqrt{1+\rho-2\alpha\rho}}{\sqrt{a}}\sqrt{1+\delta_{ak}}}$

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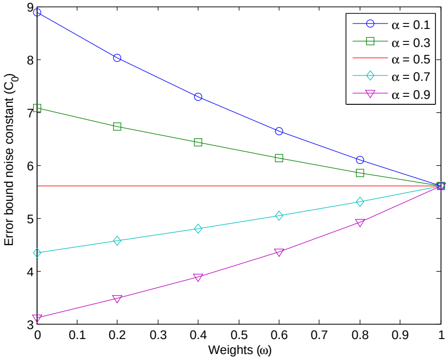

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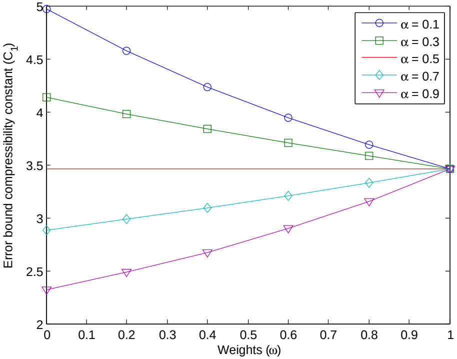

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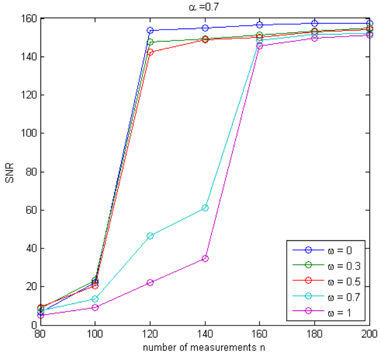

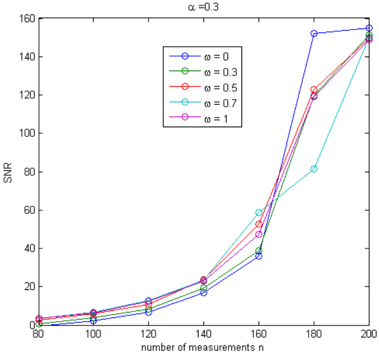

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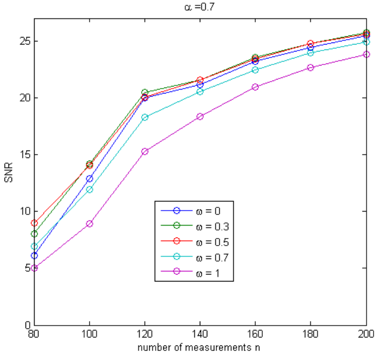

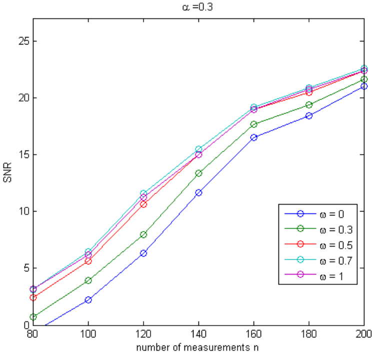

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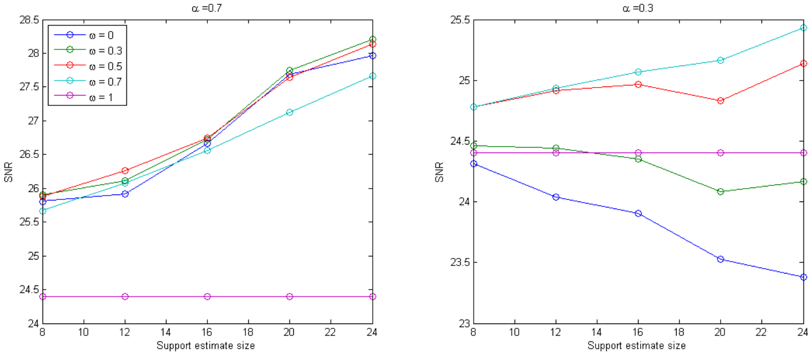

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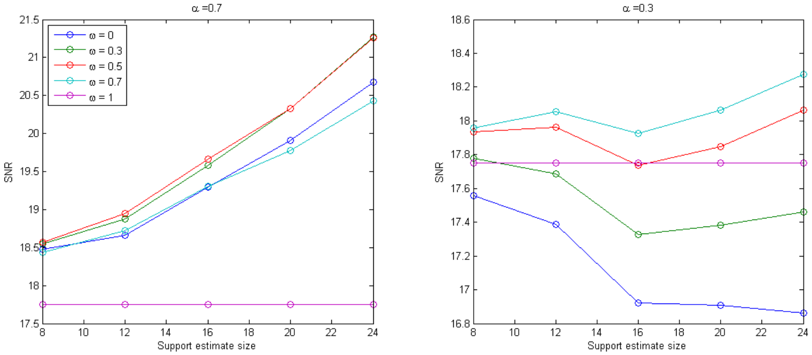

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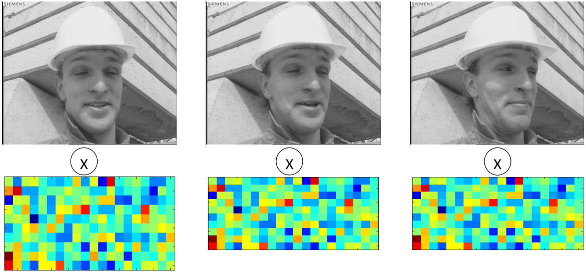

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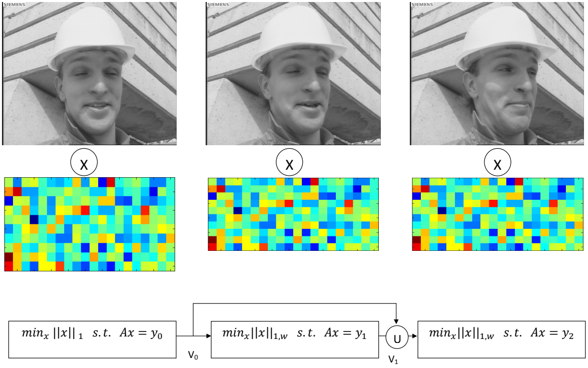
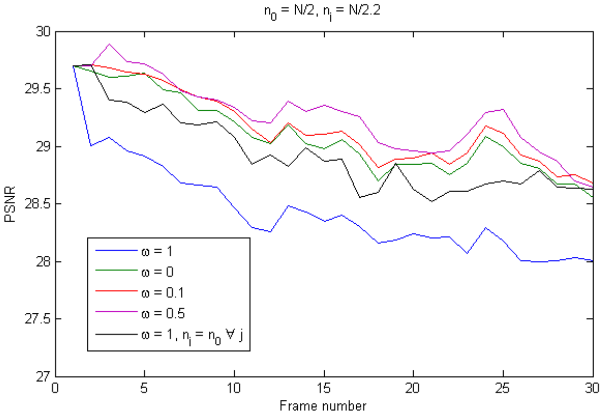


Video Compressed Sensing Results

- $n_0 = N/2$, $n_j = N/2.2$ for $j = 1, 2, \ldots$



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- Weighted ℓ_1 minimization can recover less sparse signals than standard ℓ_1 when enough prior information is available.
- We showed that the recovery is stable and robust.
- We also showed that if at least 50% of the support estimate is accurate, then the recovery is guaranteed with weaker RIP conditions and smaller error bounds.
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Let $x \in \mathbb{R}^N$ be k-sparse and suppose the measurement matrix A is such that ℓ_1 minimization cannot recover x.

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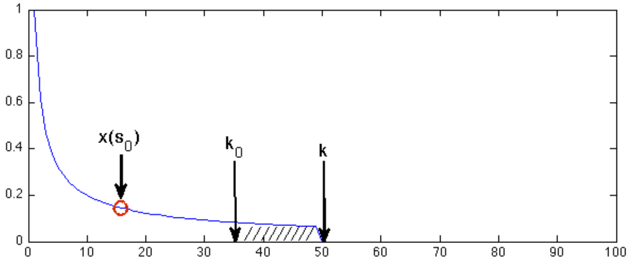

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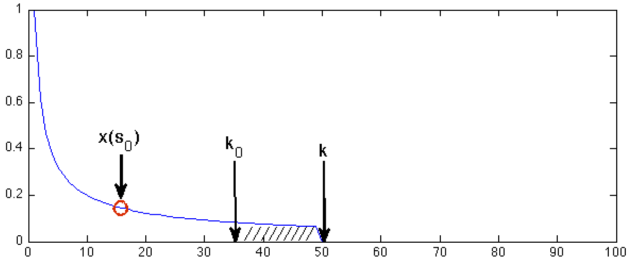

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Let $x \in \mathbb{R}^N$ be k-sparse and suppose the measurement matrix A is such that ℓ_1 minimization cannot recover x.

- If $\alpha > 0.5$ and $\omega < 1$, then $s_1 \geq s_0$.
- Assuming x decays according to weak ℓ_p, the above condition requires $p \geq 3$!
- More conditions on signal decay are required to ensure $s_1 > s_0$.
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Iterative weighted ℓ_1 algorithm (work in progress)

1. Solve an initial ℓ_1 minimization problem to obtain a support estimate.
2. Solve weighted ℓ_1 minimization with weight equal to 0.5 on the previous support estimate.
3. Obtain a new support estimate.
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Iterative weighted ℓ_1 algorithm (work in progress)

Input $b = Ax$

Output $x^{(t)}$

Initialize $\hat{p} = 0.99$, $\hat{k} = n\log(N/n)/2$, $\omega_1 = 0.5$, $\omega_2 = 0$,
$T_1 = \emptyset$, $T_2 = \emptyset$, $\Omega = \emptyset$,
$l = 0$, $t = 0$, $s^{(0)} = 0$, $x^{(0)} = 0$

while $\|x^{(t)} - x^{(t-1)}\|_2 \leq \mathrm{Tol}\|x^{t-1}\|_2$ **do**

  $t = t + 1$

  $W = \mathbf{1}$

  $\Omega = \mathrm{supp}(x^{(t-1)}|_{s^{(t-1)}})$

  $T_2 = T_1 \cap \Omega$

  $W_{T_1} = \omega_1$, $W_{T_2} = \omega_2$

  $x^{(t)} = \arg\min_u \|u\|_{1,W}$ s.t. $Au = b$

  $l = \min_\Lambda |\Lambda|$ s.t. $\|x_\Lambda^{(t)}\|_2 \geq \hat{p}\|x^{(t)}\|_2$

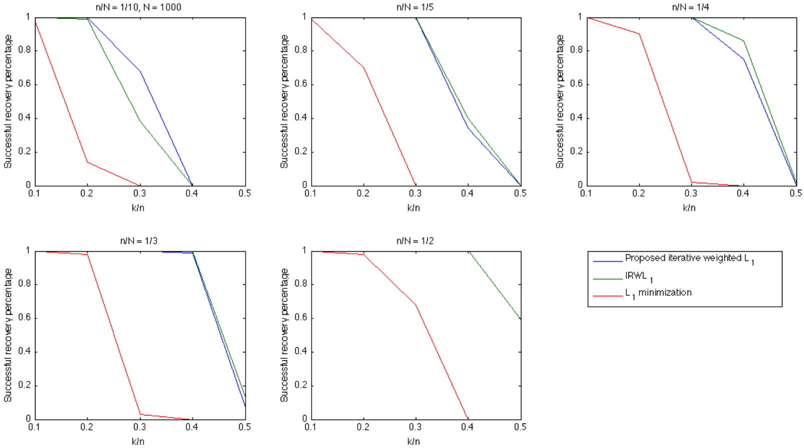
  $s^{(t)} = \min\{l, \hat{k}\}$

  $T_1 = \mathrm{supp}(x^{(t)}|_{s^{(t)}})$

end while

Iterative weighted ℓ_1 algorithm (work in progress)

N = 1000



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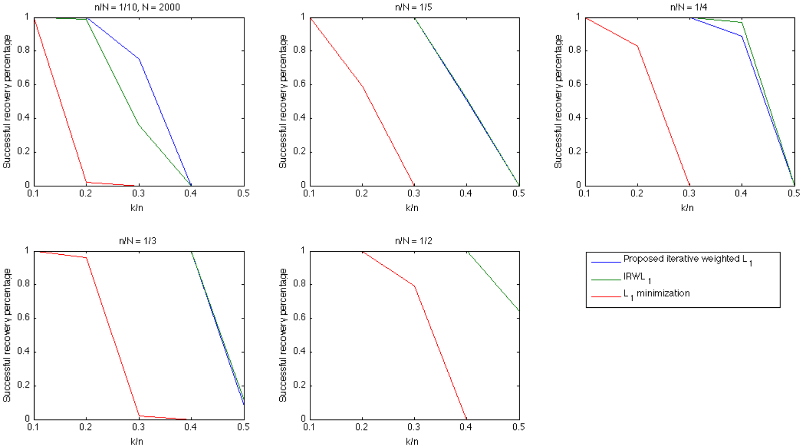

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Thank you!

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