Optimal data-driven sparse parameterization of diffeomorphisms for population analysis

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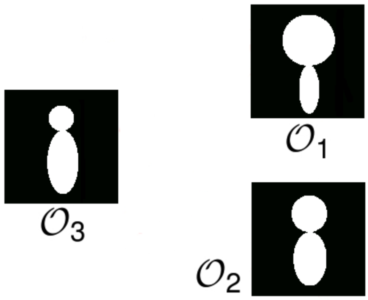
Optimal data-driven sparse parameterization of diffeomorphisms for population analysis. Inf Process Med Imaging 2011;22():123-34

September 1, 2011

Driving Application

- Given a Large collection of neuro-anatomical images of subjects with detailed Neuropsychological assessments how does one relate anatomical variation to Neuropsychological variables.
- Driving problem: The ADNI database currently has 900 subjects each with detailed Neuropsychological evaluations.
- Extract and identify patterns in brain anatomy that relate to observed clinical scores depicting cognitive abilities.

Statistics on set of images



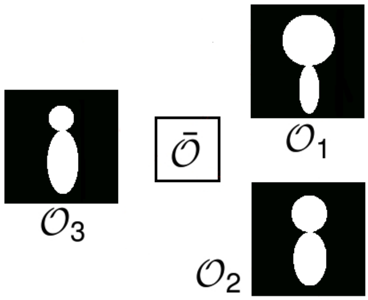
Basic Building Blocks

template image

\+ deformations

\+ *residuals*

Statistics on set of images



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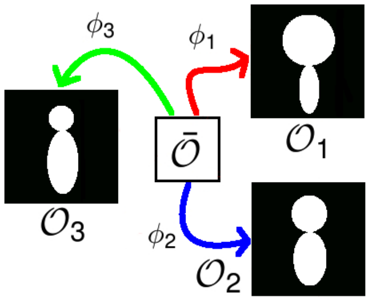

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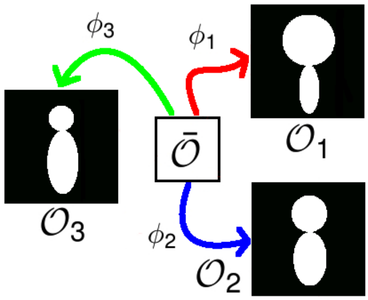

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Population Statistics

- stats on deformations: *infinite* dimension!
- too large compared to:
  - the number of samples
  - the effective number of degrees of freedom
- Need for an *adaptive* parameterization

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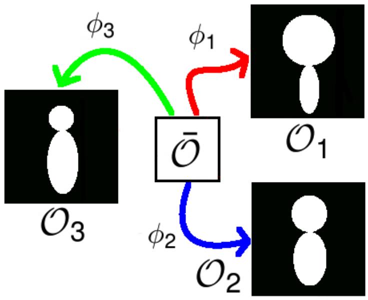

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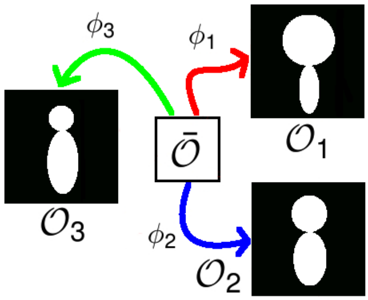

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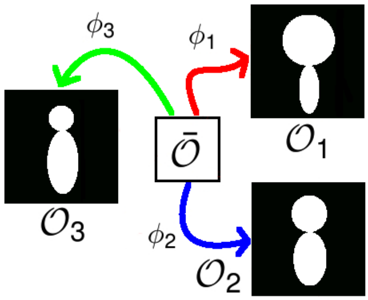

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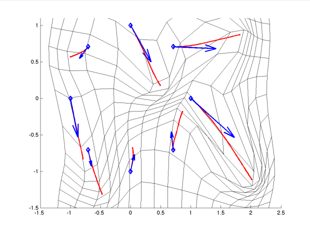
Control Points Parameterization

Enforce sparsity with a *discrete support* of the momenta:

$$\frac{dc_i(t)}{dt} = \sum_{j=1}^{N} K(c_i(t), c_j(t))\alpha_j(t)$$

$$\frac{d\alpha_i(t)}{dt} = -\sum_{j=1}^{N} \nabla K(c_i, c_j)\alpha_i(t)^T \alpha_j(t)$$



[Joshi *et al.*, T-I.P., 2000; Miller *et al.*, JMIV'06]

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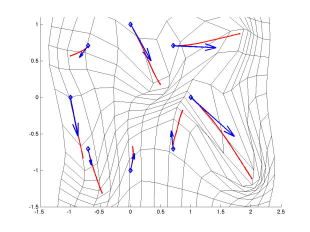

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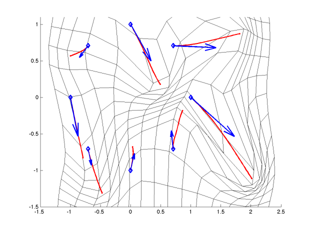


[Joshi *et al.*, T-I.P., 2000; Miller *et al.*, JMIV'06]

Not a new idea (diffeo B-spline *[Rueckert et al.]*, GRID *[Grenander et al.]*), however:

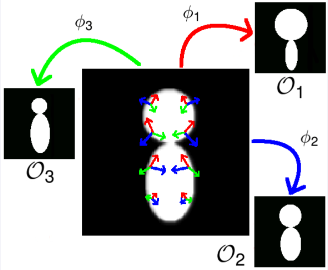
- optimal positions of the control points?
- optimal number of the control points?
- optimality for a *set* of images?

$\hookrightarrow$ Answer possible because of *explicit* dynamical system

Outline

1. Adaptive Atlas Parameterization
2. Optimization w.r.t the number of CP

Atlas estimation



Atlas =

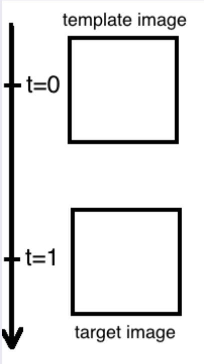
- Image I_0
- set of CP c_i
- set of momenta $\alpha_i^{(s)}$

$$E(I_0, c_i, \alpha_i^{(s)}) = \sum_{s=1}^{N_{\text{subj}}} \underbrace{\left\| I_0 \circ {\phi^{(s)}}^{-1} - I^{(s)} \right\|^2 + \text{Reg}(\phi^{(s)})}_{E^{(s)}(I_0, c_i, \alpha_i^{(s)})}$$

$$\nabla_{\alpha_i^{(s)}} E = \nabla_{\alpha_i^{(s)}} E^{(s)}, \qquad \nabla_{c_i} E = \sum_s \nabla_{c_i} E^{(s)}, \qquad \nabla_{I_0} E = \sum_s \nabla_{I_0} E^{(s)}$$

Template-to-subject registration

$$E(\underbrace{c_i, \alpha_i}) = \sum_k \underbrace{(I_0(\phi_1^{-1}(y_k)) - I(y_k))^2} + \underbrace{\mathrm{Reg}(\phi_1)}$$



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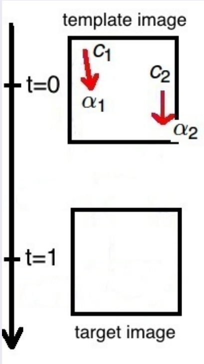
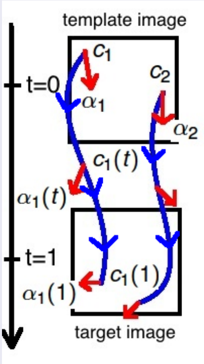
$$E(\underbrace{c_i, \alpha_i}_{S_0}) = \sum_k \underbrace{(I_0(\phi_1^{-1}(y_k)) - I(y_k))^2}_{} + \underbrace{\mathrm{Reg}(\phi_1)}_{}$$



$$\begin{cases} S_0 = \{(c_i, \alpha_i)\}_i \\ \\ \end{cases}$$

Template-to-subject registration

$$E(\underbrace{c_i, \alpha_i}_{S_0}) = \sum_k \underbrace{\left(I_0(\phi_1^{-1}(y_k)) - I(y_k)\right)^2}_{} + \underbrace{\mathrm{Reg}(\phi_1)}_{}$$



$$\begin{cases} S_0 = \{(c_i, \alpha_i)\}_i \\ \dfrac{dS(t)}{dt} = F(S(t)) \qquad S(0) = S_0 \end{cases}$$

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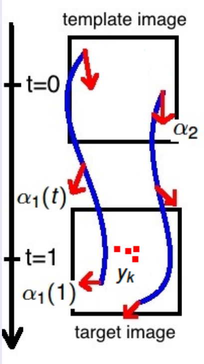

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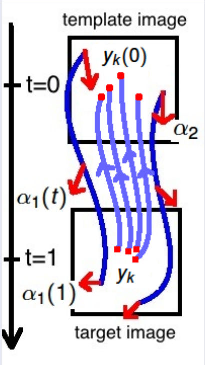
$$E(\underbrace{c_i, \alpha_i}_{S_0}) = \sum_k \underbrace{\left(I_0(\phi_1^{-1}(y_k)) - I(y_k)\right)^2}_{A(y_k(0))} + \underbrace{\mathrm{Reg}(\phi_1)}$$



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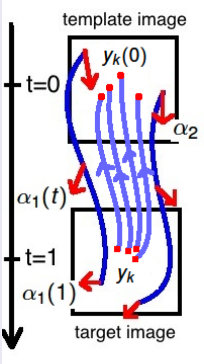

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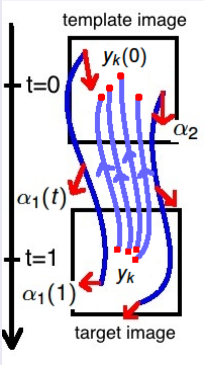

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$$\nabla_{S_0} E = d_{S_0} y(0)^T \nabla_{y(0)} A + \nabla_{S_0} L$$

$$\nabla_{y_k(0)} A = 2\,(I_0(y_k(0)) - I(y_k(1)))\,\nabla_{y_k(0)} I_0$$

- Momenta decrease image discrepancy

- Control Points attracted by image contours

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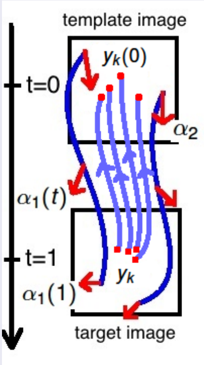

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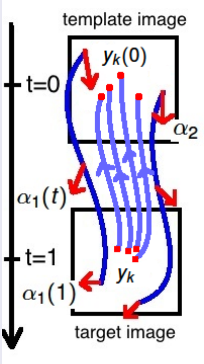

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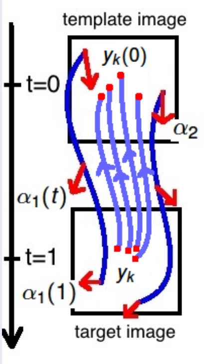

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$$\frac{d\eta(t)}{dt} = \partial_{S(t)} G^T \eta(t), \quad \eta(0) = \nabla_{y(0)} A$$

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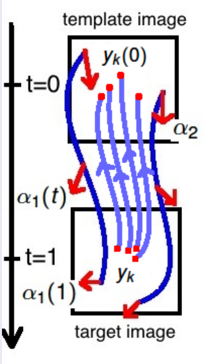

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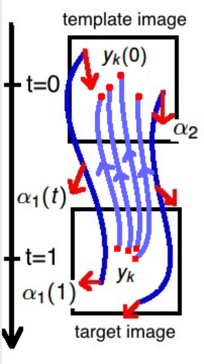

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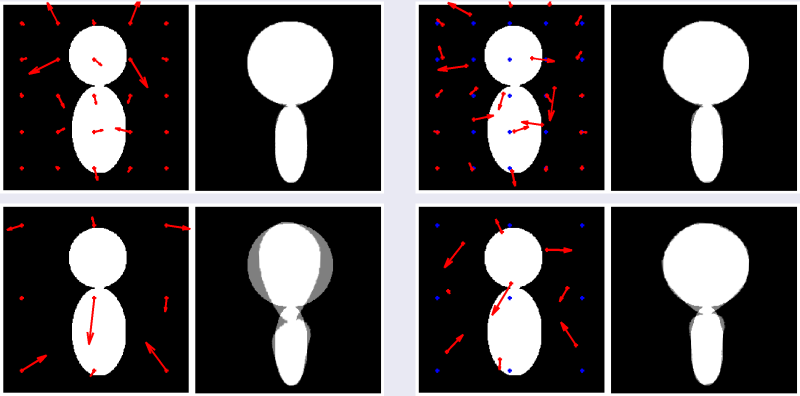
$$\boxed{\nabla_{S_0} E = \xi(0) + \nabla_{S_0} L}$$

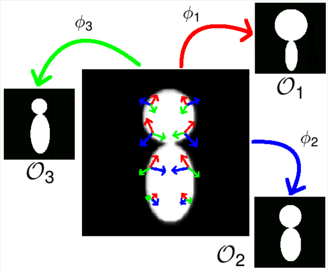
Template-to-subject registration Results

Fixed Positions | *Updated* Positions



Optimization of control points positions at NO additional cost!

Atlas estimation



Atlas =

- Image I_0
- set of CP c_i
- set of momenta $\alpha_i^{(s)}$

$$E(I_0, c_i, \alpha_i^{(s)}) = \sum_{s=1}^{N_{\text{subj}}} \underbrace{\left\| I_0 \circ {\phi^{(s)}}^{-1} - I^{(s)} \right\|^2 + \text{Reg}(\phi^{(s)})}_{E^{(s)}(I_0, c_i, \alpha_i^{(s)})}$$

$$\nabla_{\alpha_i^{(s)}} E = \nabla_{\alpha_i^{(s)}} E^{(s)}, \qquad \nabla_{c_i} E = \sum_s \nabla_{c_i} E^{(s)}, \qquad \nabla_{I_0} E = \sum_s \nabla_{I_0} E^{(s)}$$

Optimization w.r.t. the template image

Image building via linear *interpolation*:

$$\tilde{E}^{(s)}(I_0) = \left\| I_0 \circ \phi^{(s)^{-1}} - I^{(s)} \right\|^2$$
$$= \sum_k \sum_{p \in \mathcal{N}(y_k(0))} \rho_k(y_k(0)) \left(I_0(\pi_p(y_k(0)) - I_s(y_k)) \right)^2$$

gradient by *splatting* the residual $R^{(s)} = I_0 \circ \phi^{(s)^{-1}} - I^{(s)}$:

$$\nabla_{I_0} E^{(s)} = \frac{1}{2} \sum_j \left(\sum_{\{i; \exists k, \pi_k(y_i(0)) = y_j\}} \rho_k(y_i(0)) R_s(y_i) \right)$$

Optimization w.r.t. the template image

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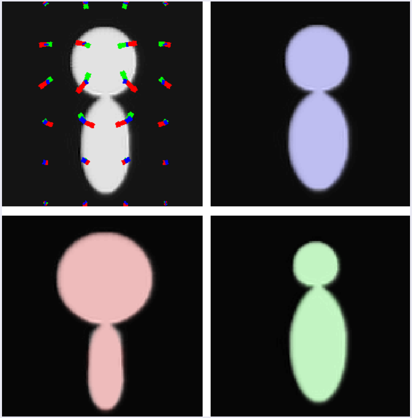
Atlas Estimation

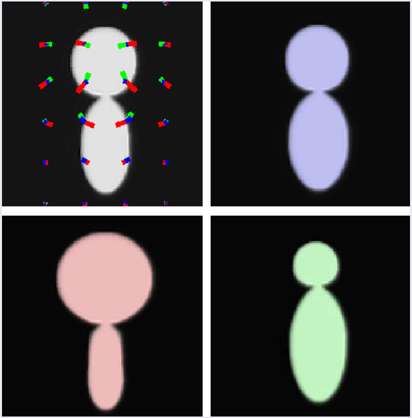


Single gradient descent:

- template image
- position of CP
- momenta

- One CP every std of the kernel (30 CPs / 128^2 pixels)
- information tends to be spread over the whole set
- needs to adjust to the actual nb of DOF of the variability

Atlas Estimation



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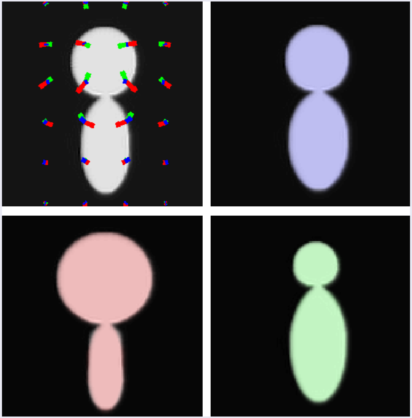

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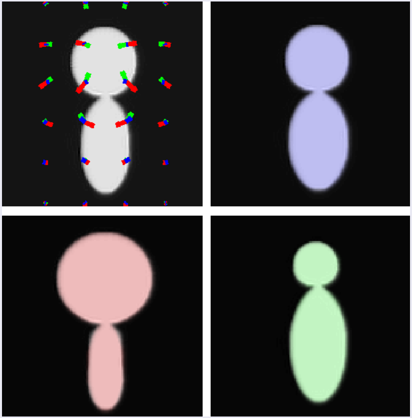

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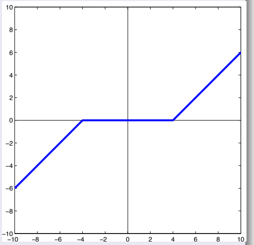
Combined L^2 and L^1 priors in the spirit of elastic net

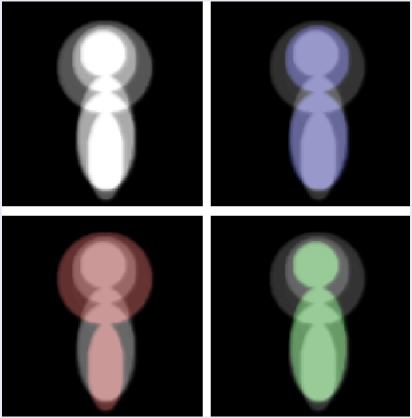
$$E(I_0, c_i, \alpha_i^{(s)}) = \sum_{s=1}^{N_{\text{subj}}} \frac{1}{2\sigma^2} \left\| I_0 \circ {\phi^{(s)}}^{-1} - I^{(s)} \right\|^2 + {\boldsymbol{\alpha}^{(s)}}^t K(\mathbf{c},\mathbf{c})\boldsymbol{\alpha}^{(s)}$$

$$+\gamma_{\text{sp}} \sum_i \left\| \alpha_i^{(s)} \right\|$$

- Minimization with F/ISTA *[Beck&Teboulle]*:
  - Update $\alpha_i^{(s)} \leftarrow \alpha_i^{(s)} - \tau \nabla_{\alpha_i^{(s)}} E^{(s)}$
  - Soft-Threshold $\alpha_i^{(s)} \leftarrow S_{\gamma_{\text{sp}}\tau}\left(\left\|\alpha_i^{(s)}\right\|\right) \frac{\alpha_i^{(s)}}{\left\|\alpha_i^{(s)}\right\|}$
  - Adapt step-size
- quadratic convergence rate (FISTA)
- adapted to 2 independent step-sizes for I_0 and $(c_i, \alpha_i^{(s)})$



Optimization with sparsity enforced

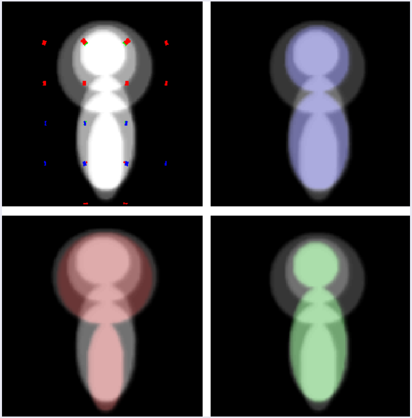


Single gradient descent:

- template image
- position of CP
- number of CP
- momenta

Image size=128^2, $\sigma_V = 25$, $\sigma^2 = 0.005$, $\gamma_{sp} = 540$

Optimization with sparsity enforced



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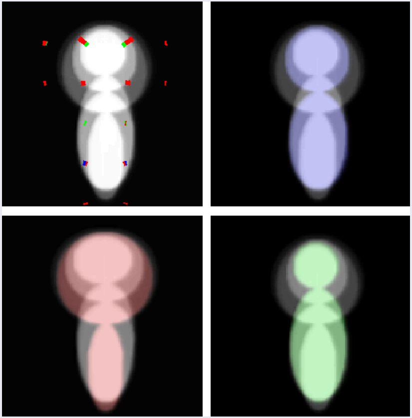

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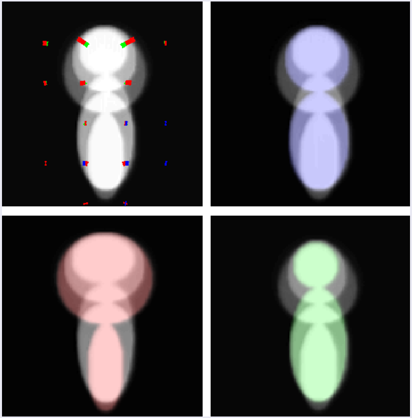

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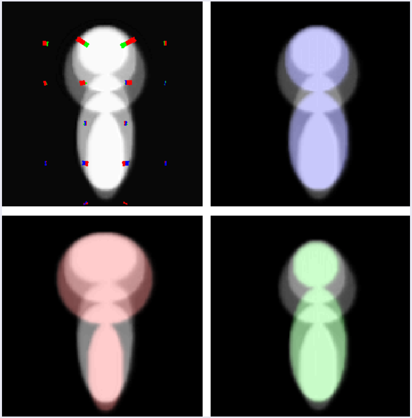

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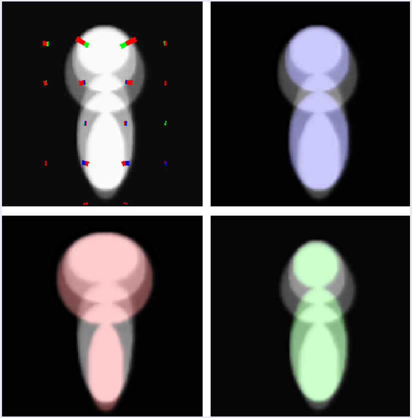

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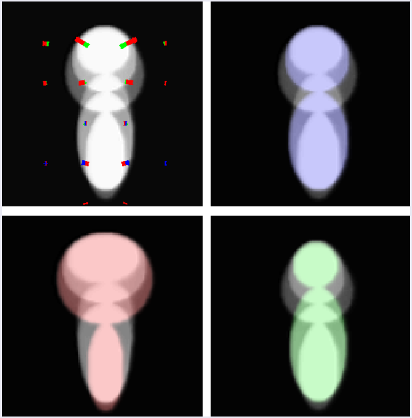

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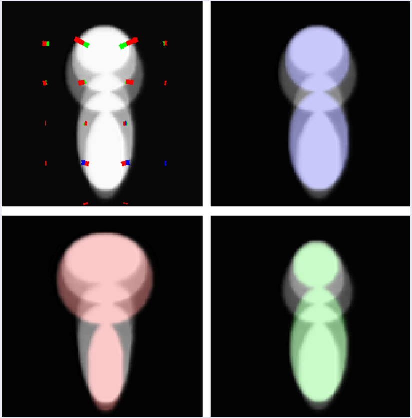

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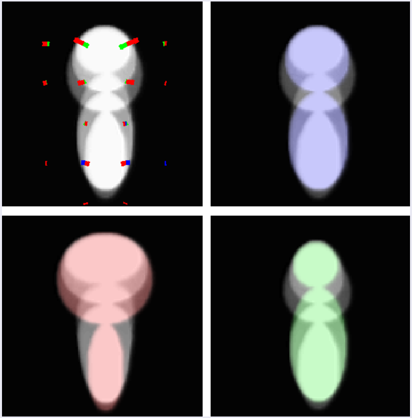

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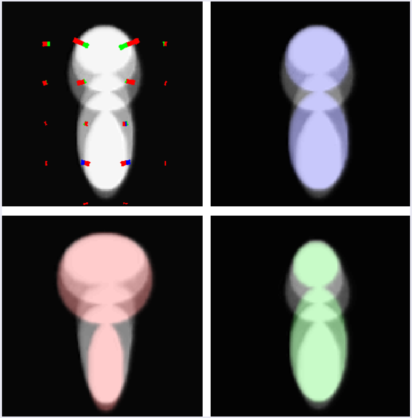

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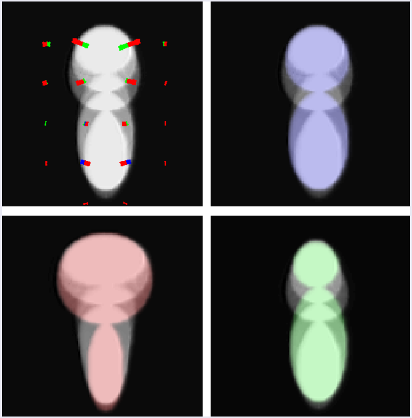

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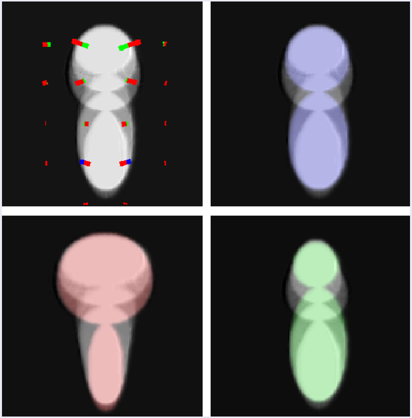

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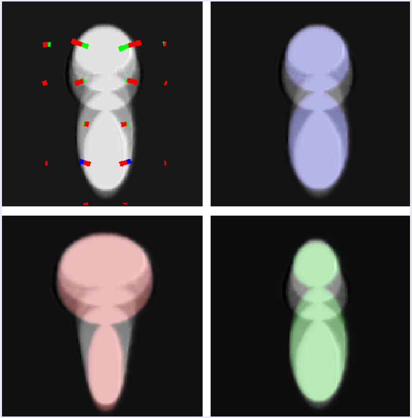

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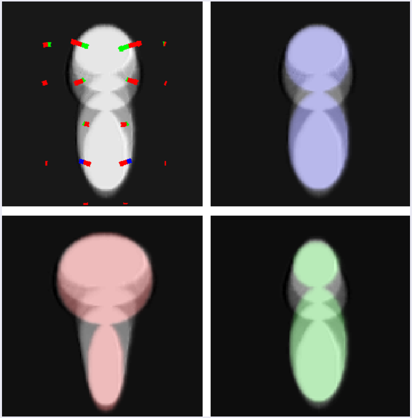

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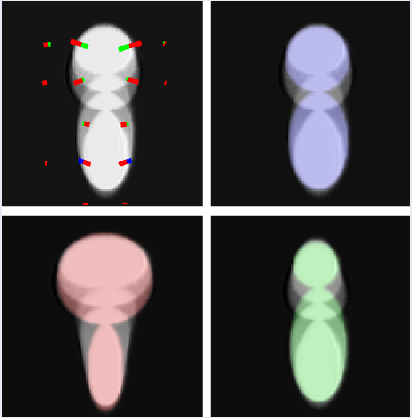

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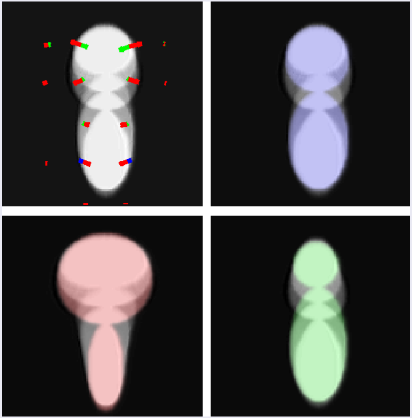

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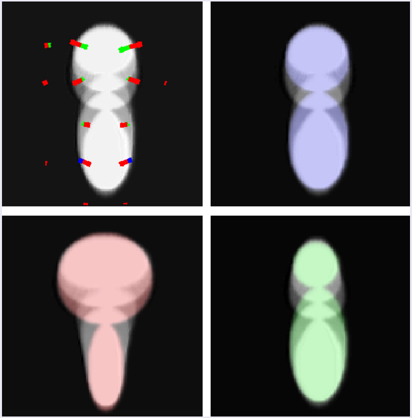

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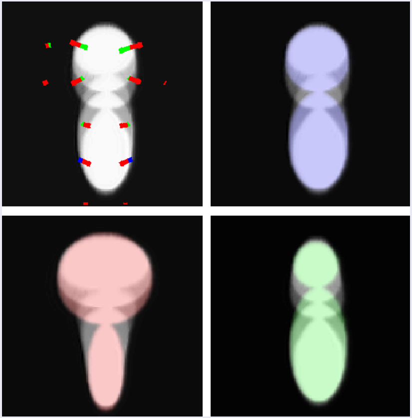

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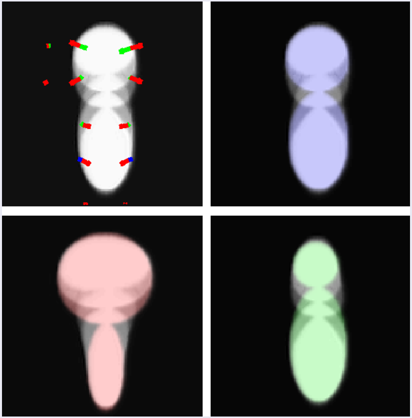

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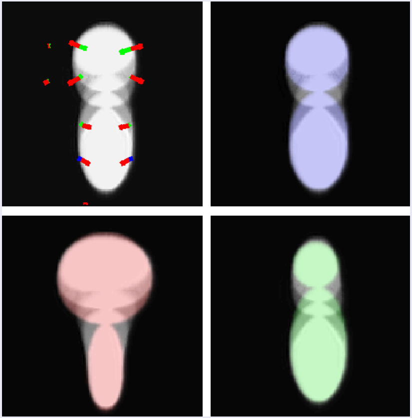

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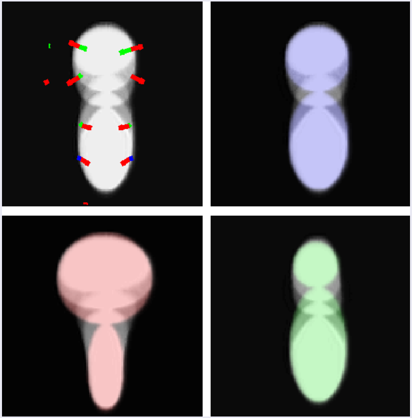

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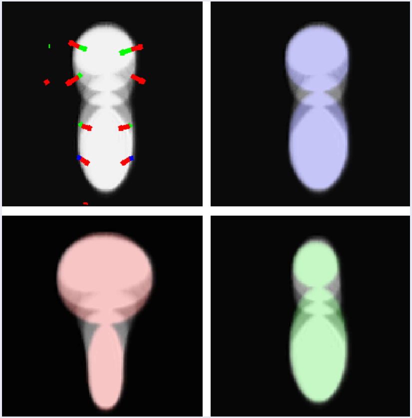

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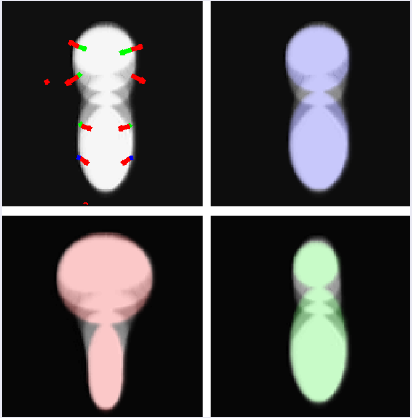

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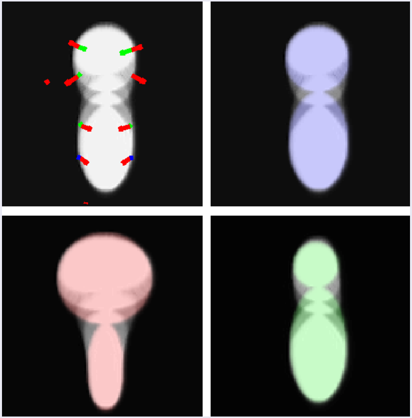

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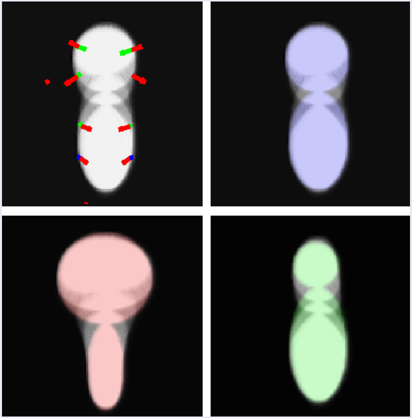

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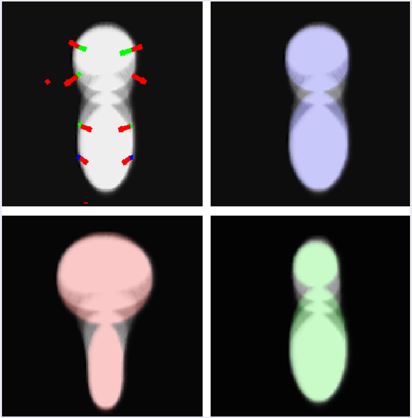

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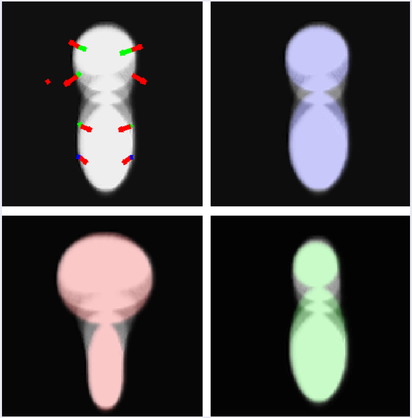

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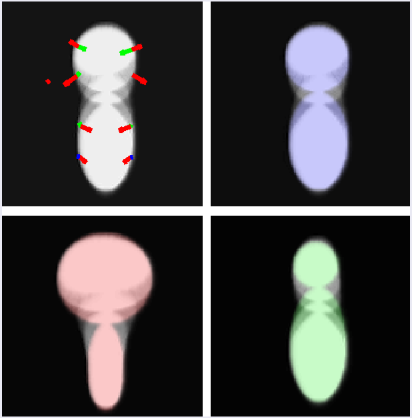

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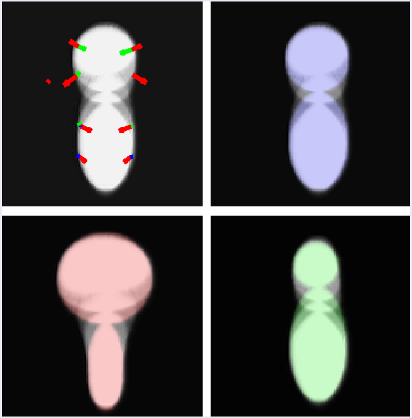

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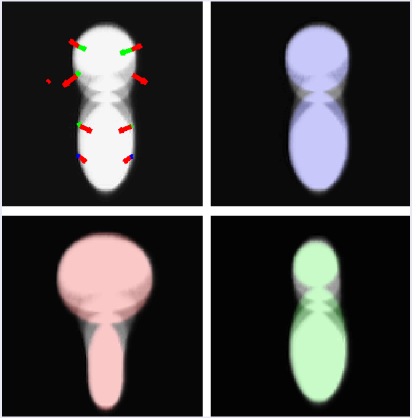

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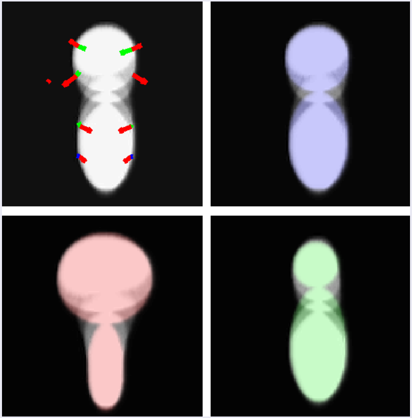

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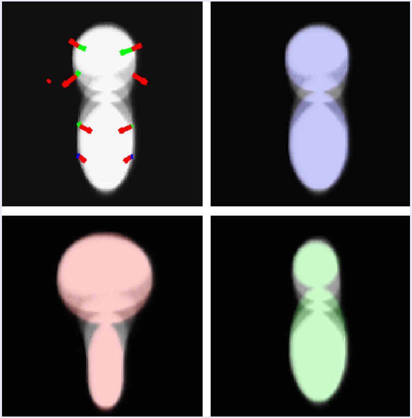

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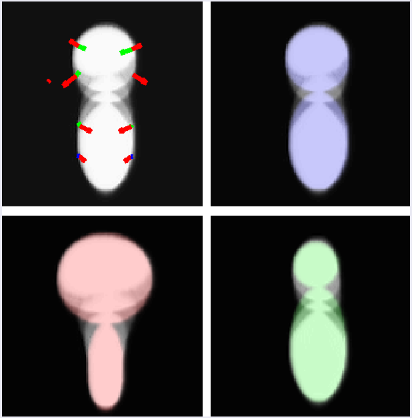

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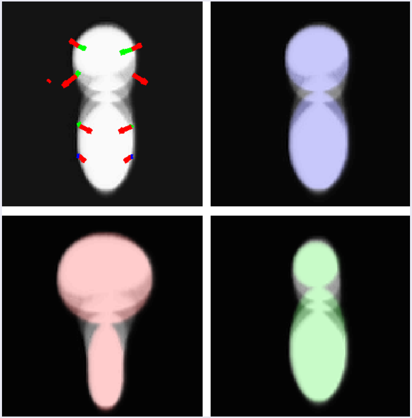

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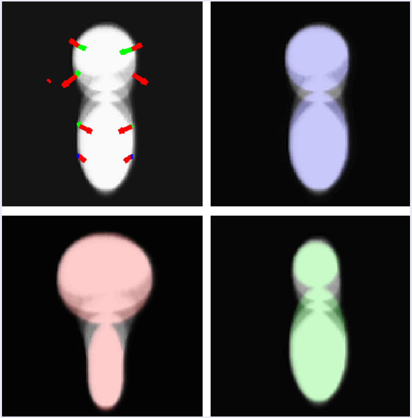

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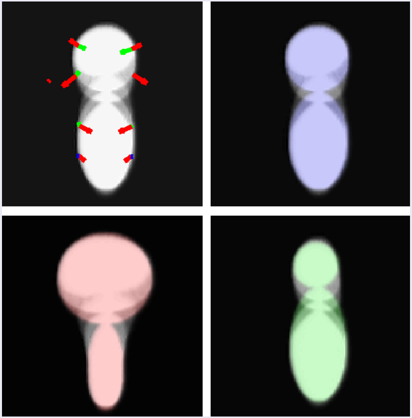

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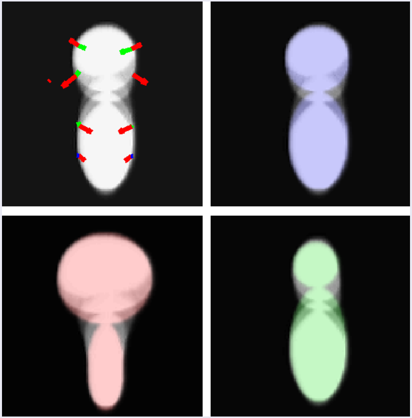

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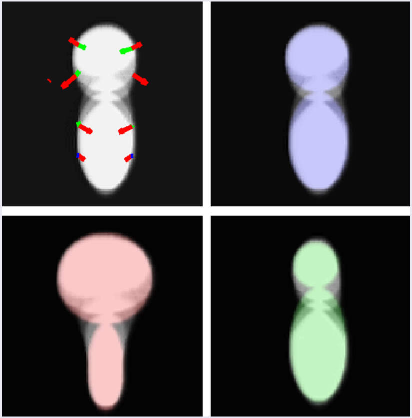

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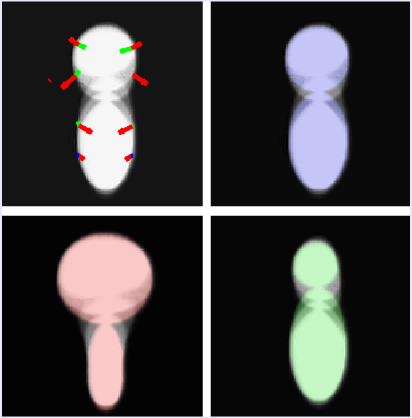

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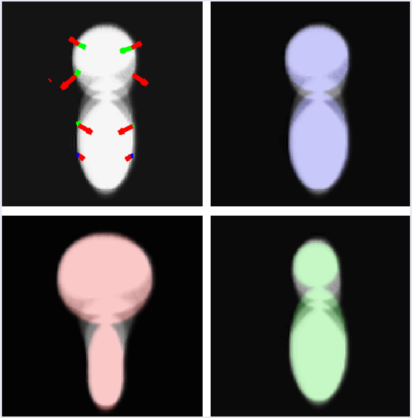

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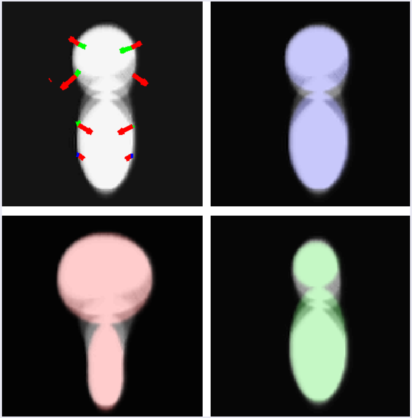

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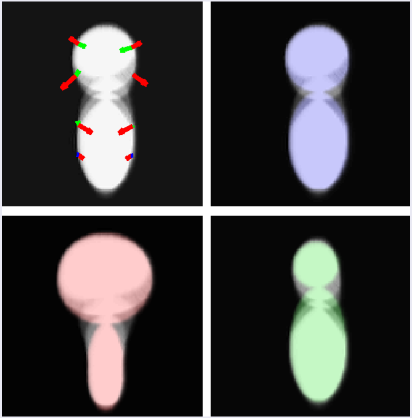

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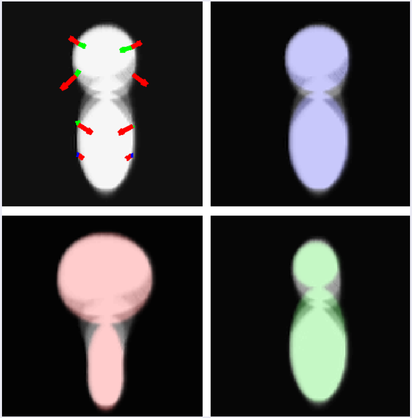

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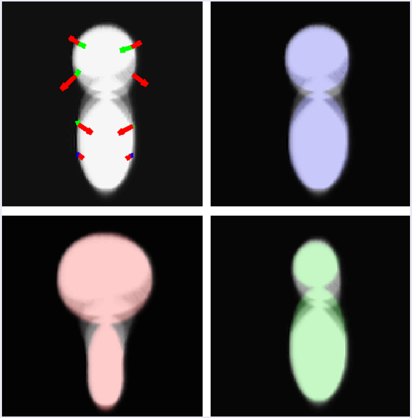

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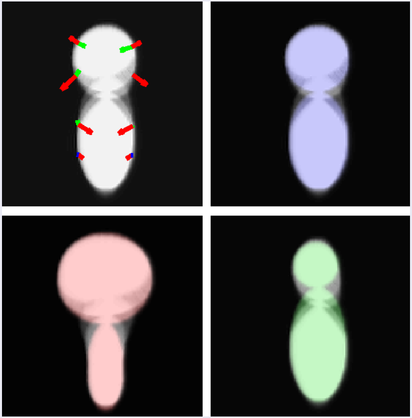

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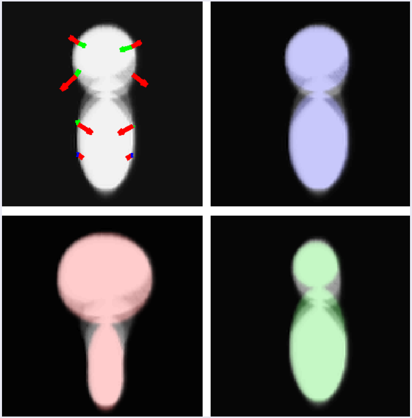
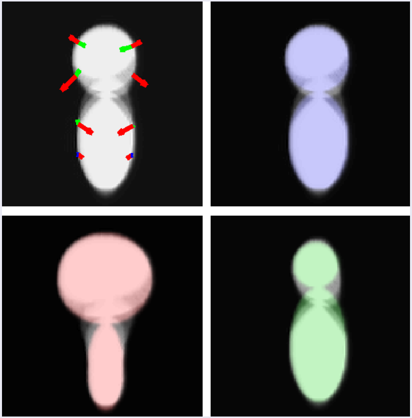


Single gradient descent:

- template image
- position of CP
- number of CP
- momenta

Image size=128^2, $\sigma_V = 25$, $\sigma^2 = 0.005$, $\gamma_{sp} = 540$

Optimization with sparsity enforced



Single gradient descent:

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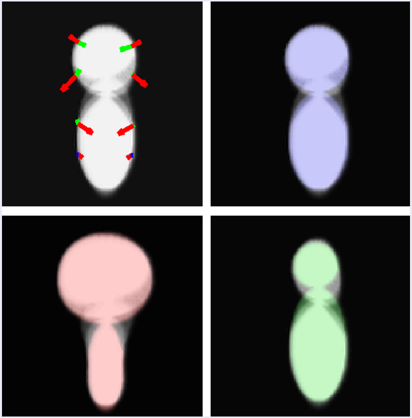
Optimization with sparsity enforced



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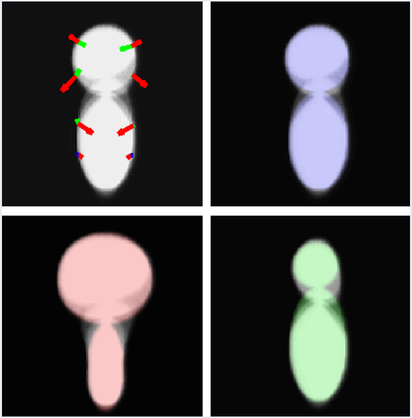

Single gradient descent:

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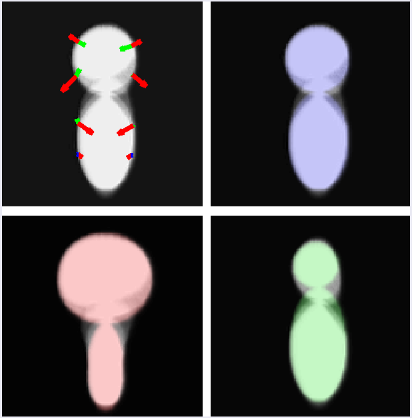

Single gradient descent:
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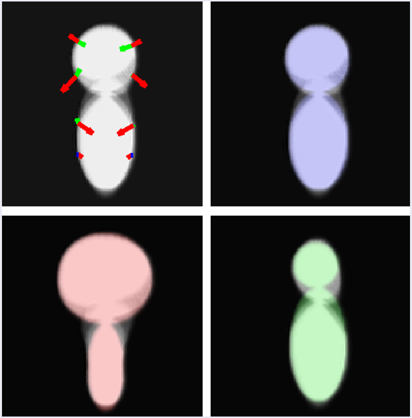

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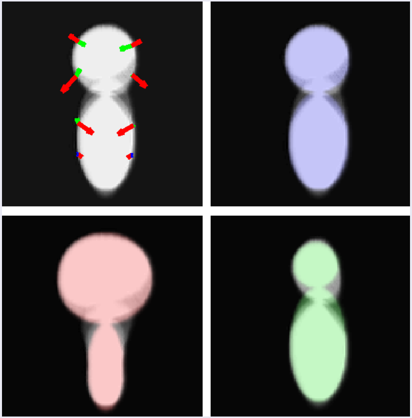

Single gradient descent:

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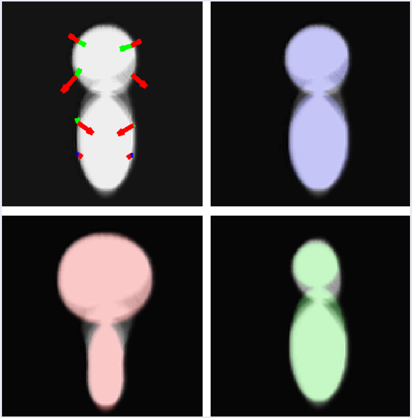

Single gradient descent:

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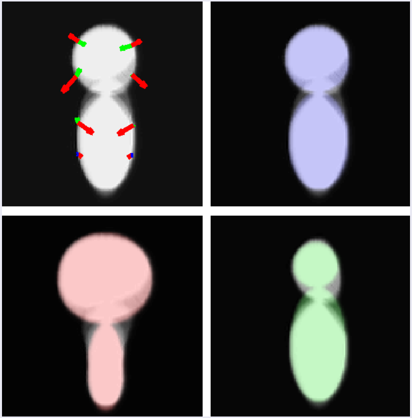

Single gradient descent:

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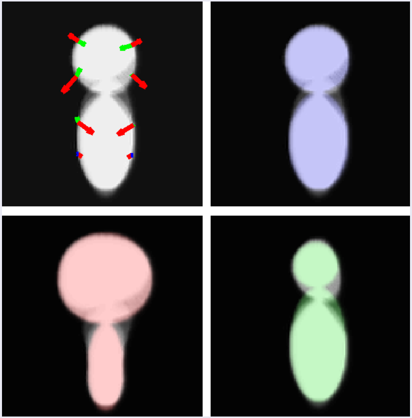

Single gradient descent:

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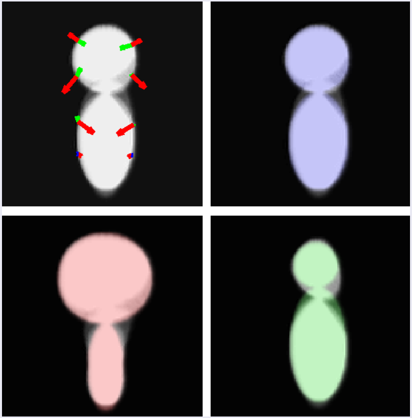

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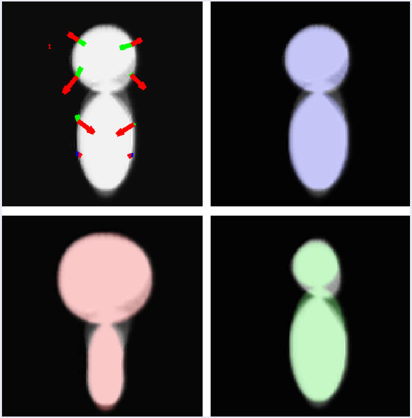

Single gradient descent:

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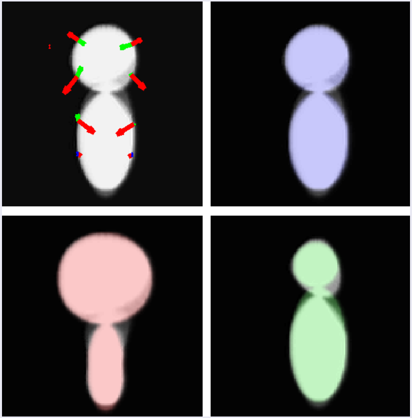

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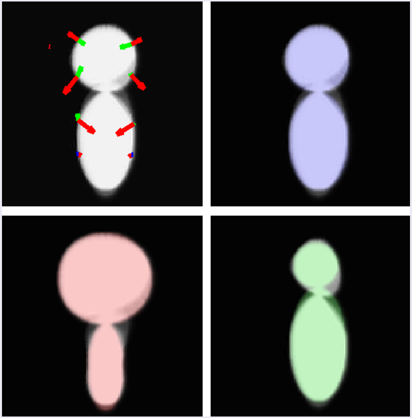

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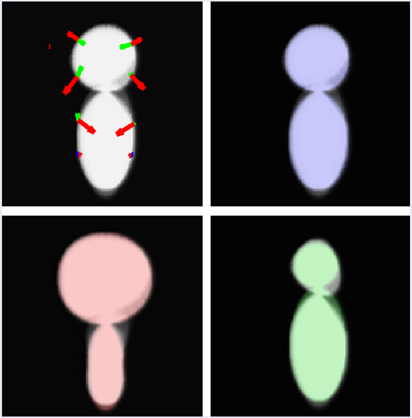

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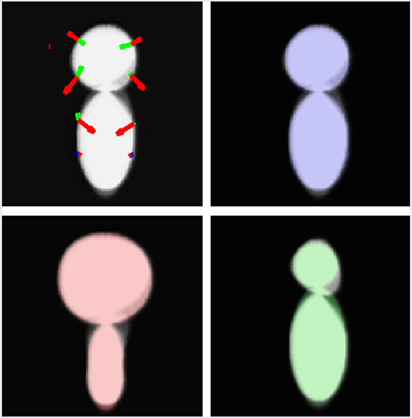

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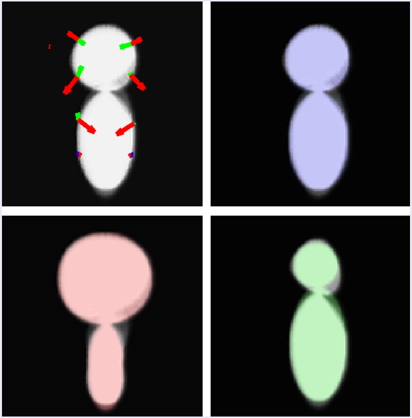

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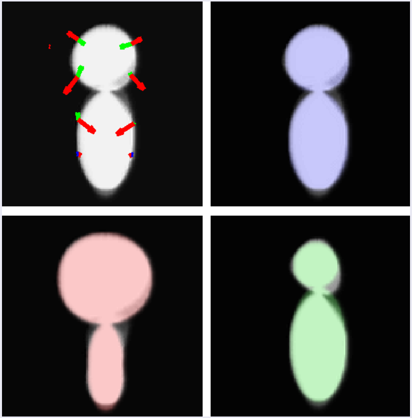

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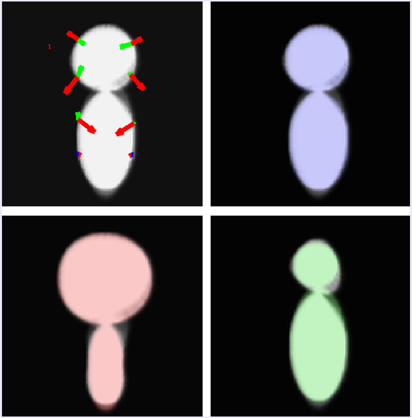

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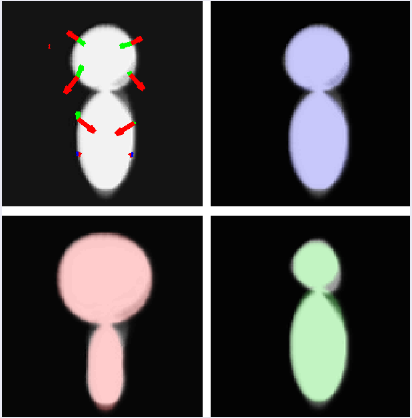

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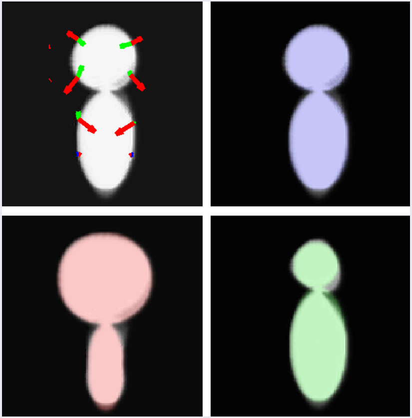

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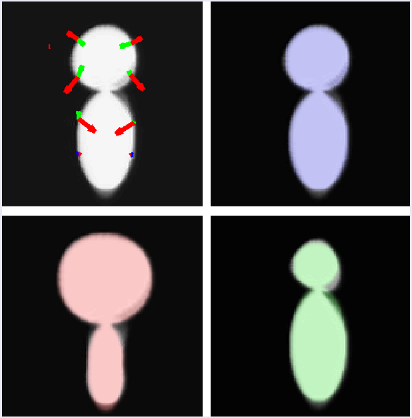

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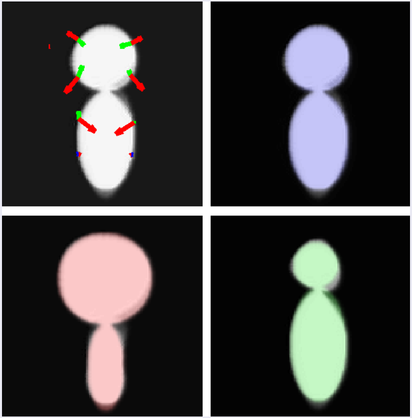

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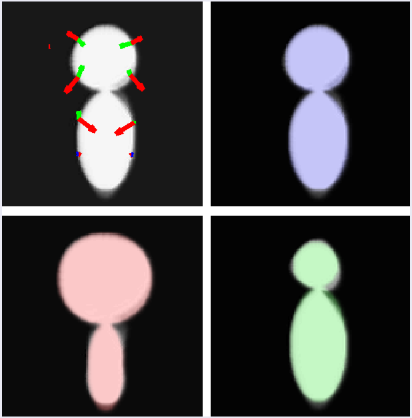

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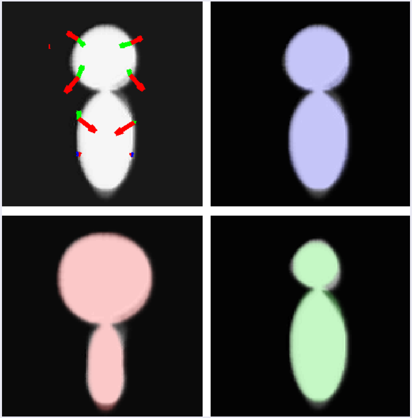

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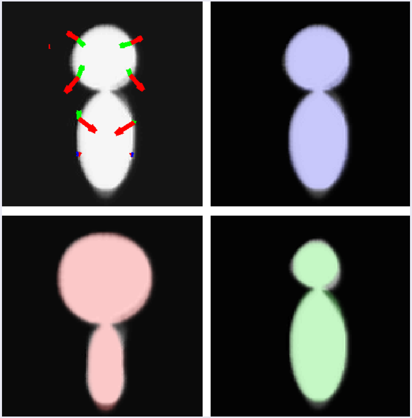

Single gradient descent:

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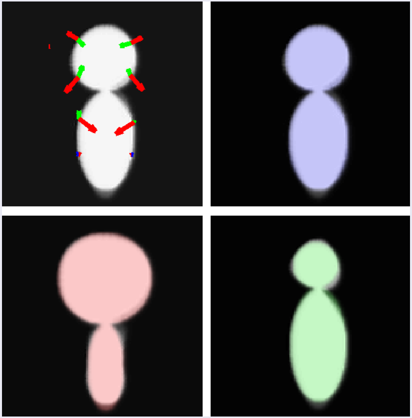

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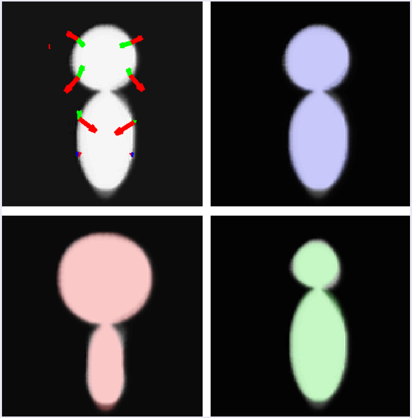

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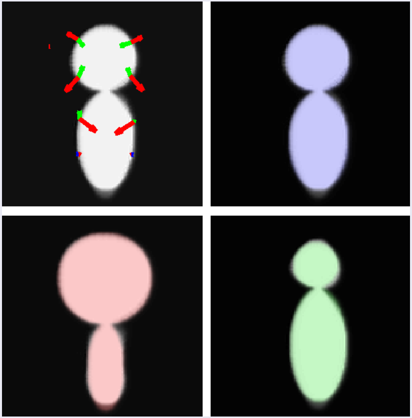

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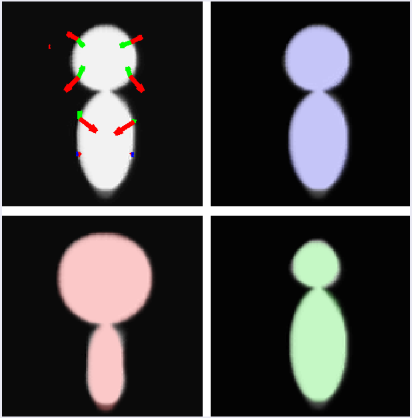

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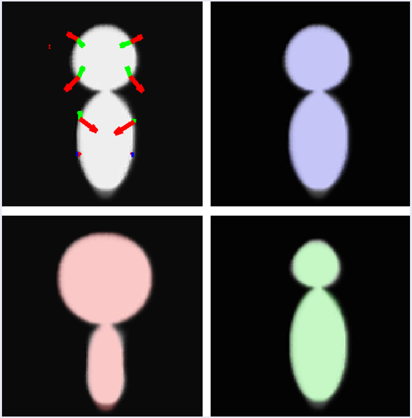

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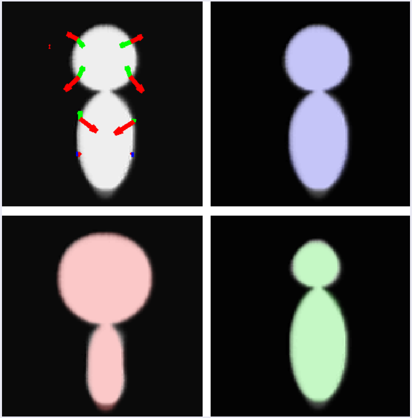

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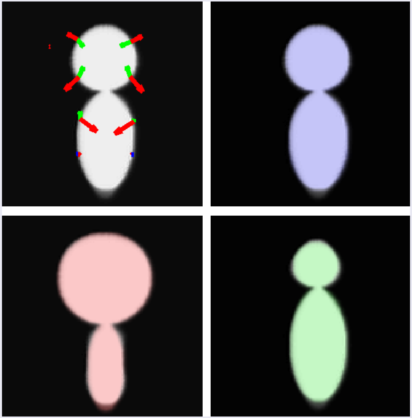

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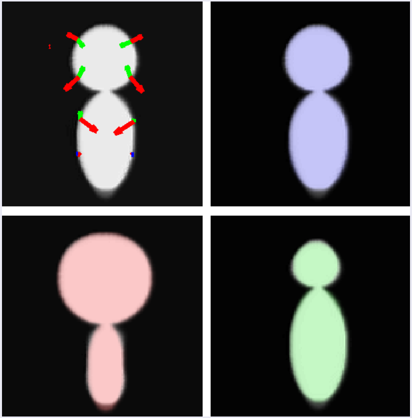

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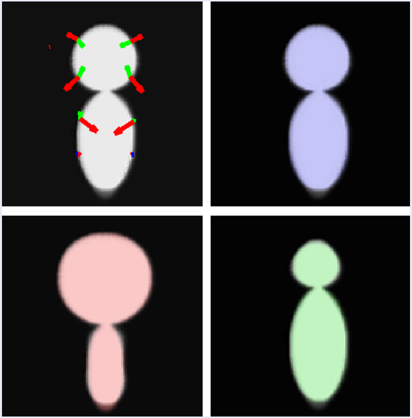

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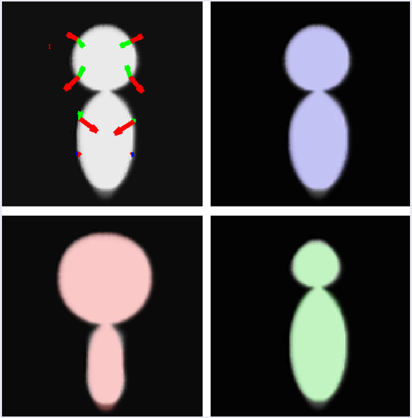

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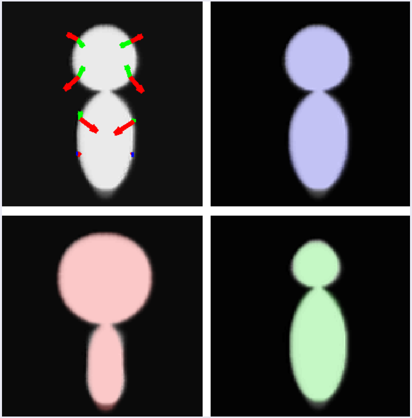

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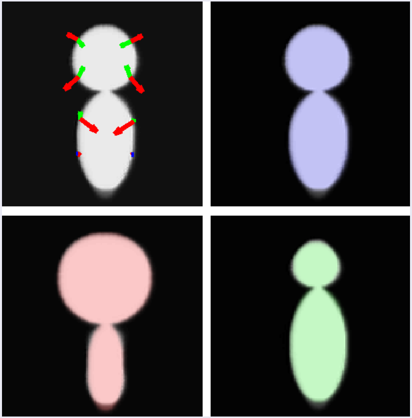

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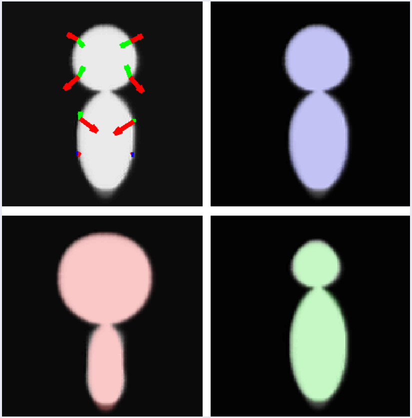

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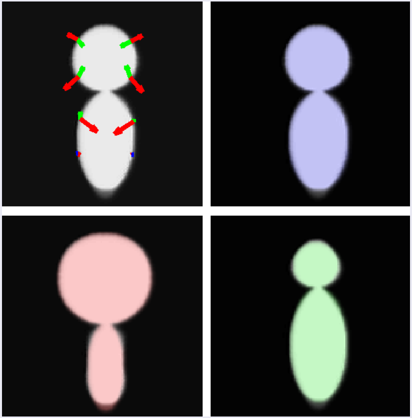

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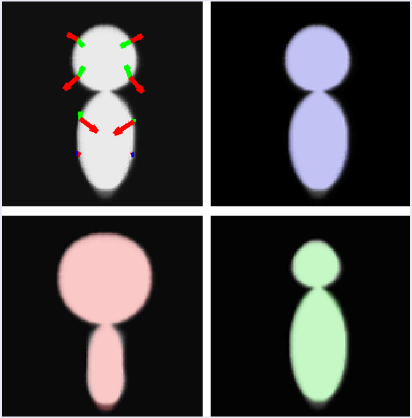

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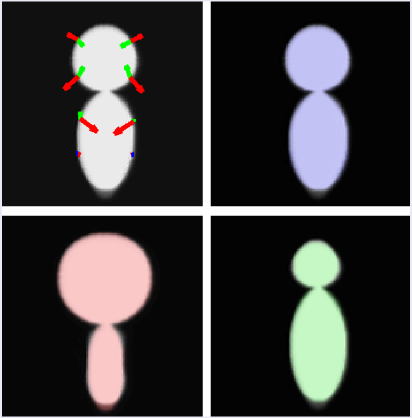

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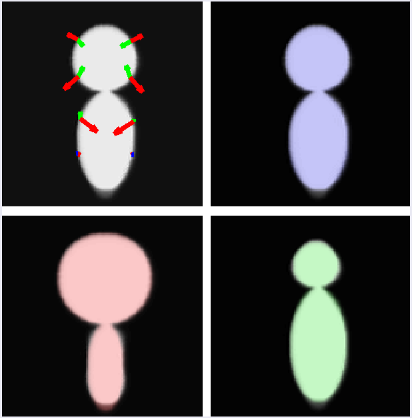

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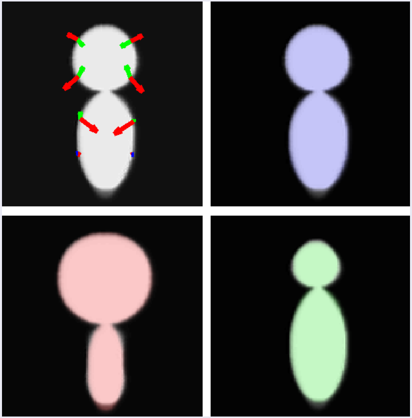

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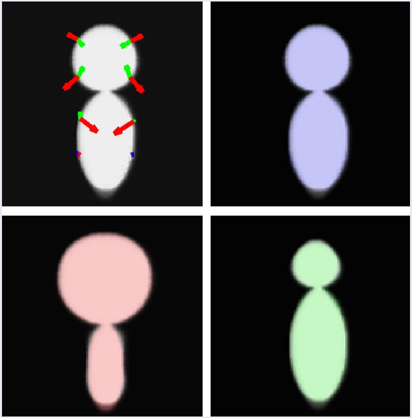

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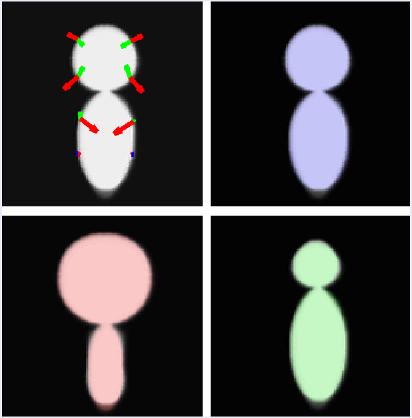

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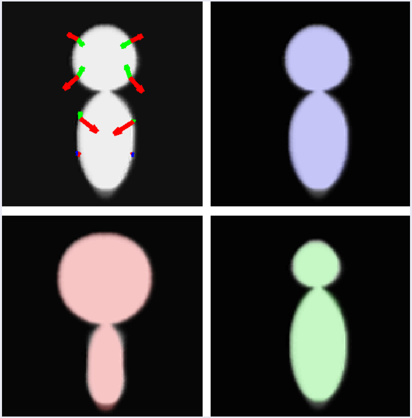

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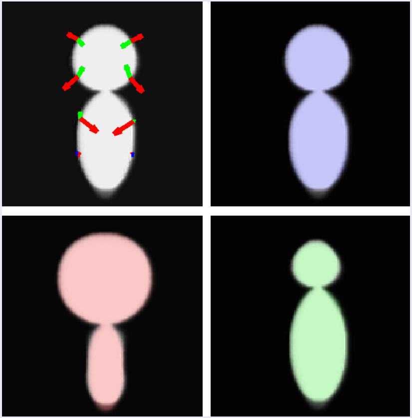

Single gradient descent:

- template image
- position of CP
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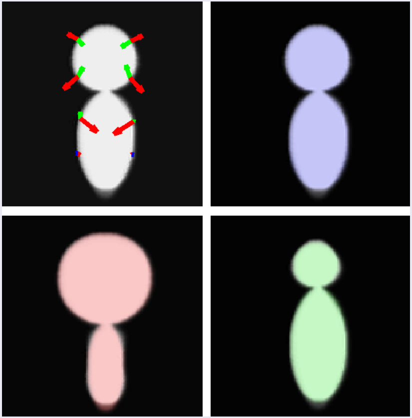

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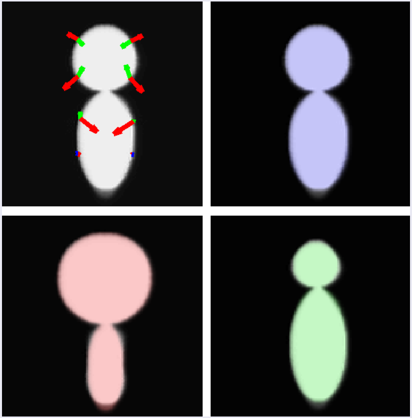

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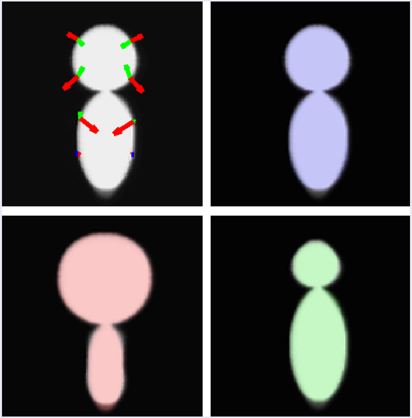

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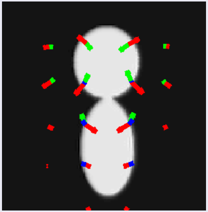
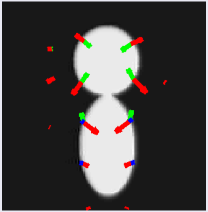
8 estimated control points!

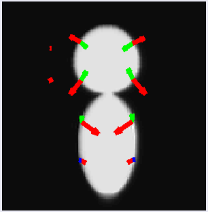
Impact of the sparsity parameter γ_{sp}



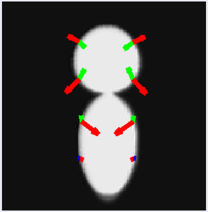
$\gamma_{sp} = 100$



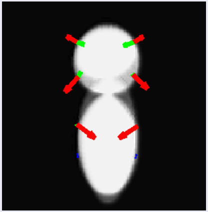
$\gamma_{sp} = 200$



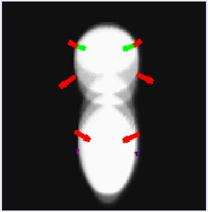
$\gamma_{sp} = 400$



$\gamma_{sp} = 500$

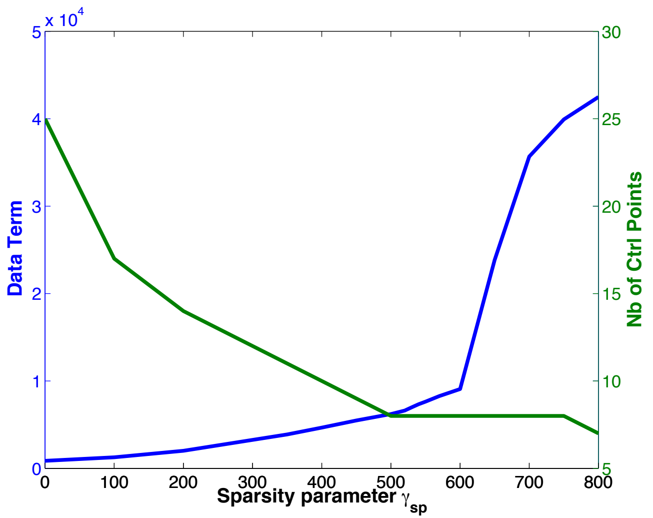


$\gamma_{sp} = 650$



$\gamma_{sp} = 700$

Impact of the sparsity parameter γ_{sp}



Results on 3D Brain images

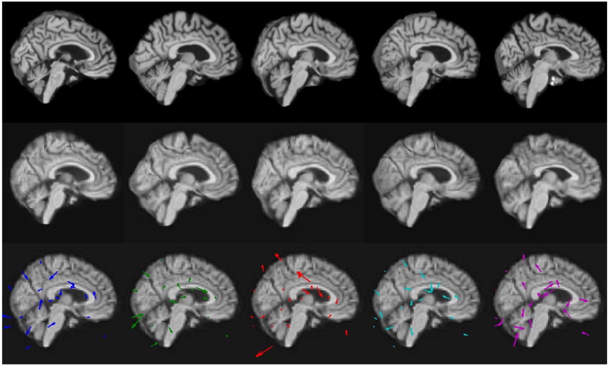


Image size=128^3, 1.25mm, $\sigma_V = 10$mm, $\sigma^2 = 0.005$, $\gamma_{sp} = 400$

923 Control Points instead of $2.1\ 10^6$

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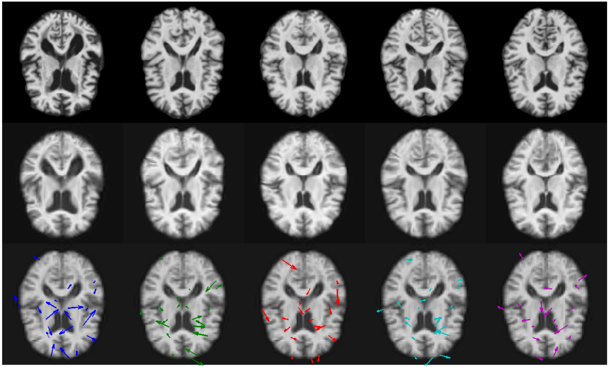

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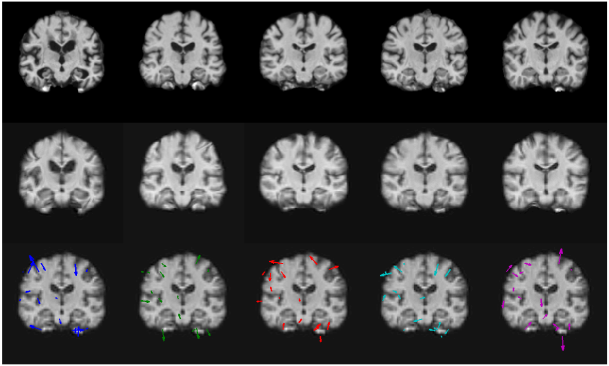

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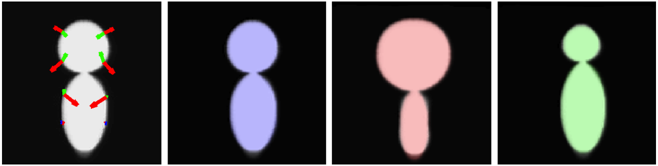
Conclusion

- Find an optimal parameterization of atlases:
  - template image
  - optimal set of CP (position and number)
  - optimal momenta -> template-to-subject registration
- Postulate: sparsity -> better statistical power
- Sparsity prior embedded into the model estimation (not as a post-processing)
- Parallel computing and GPU *[Ha et al. Best paper EGPGV 2011]*



Thanks to NIH grants: NIBIB (5R01 EB007688), NCRRR (P41 RR023953), ACE-IBIS (RO1 HD055741), and NA-MIC (U54 EB005149)

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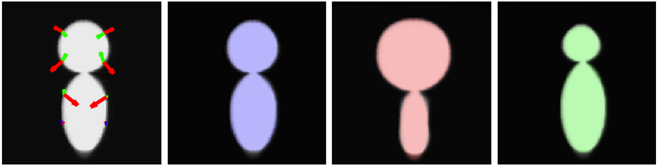

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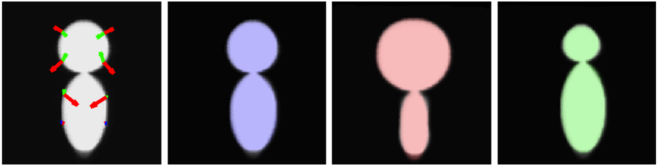

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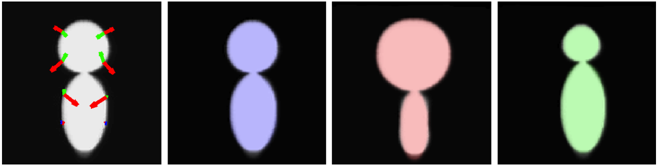

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Image LDDMM: flow of images under energy conservation law

$$\frac{dI_t}{dt} = -K\left(\alpha_t \nabla I_t\right).\nabla I_t$$

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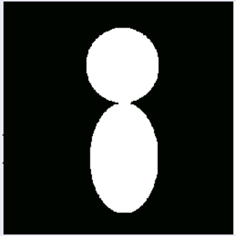

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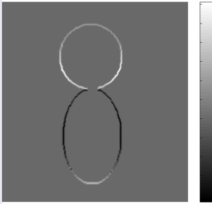

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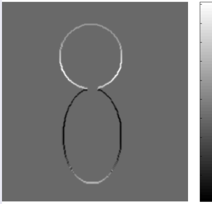

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- 1008 non-zeros pixels:
  - $1008/256^2 = 1.5\%$ of the pixels are informative!
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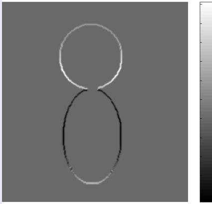

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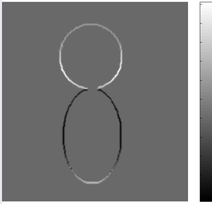

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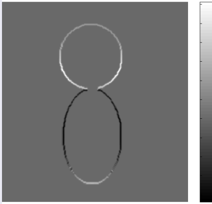

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Control Points Parameterization

Enforce sparsity with a *discrete support* of the momenta:

$$\frac{dc_i(t)}{dt} = \sum_{j=1}^{N} K(c_i(t), c_j(t))\alpha_j(t)$$

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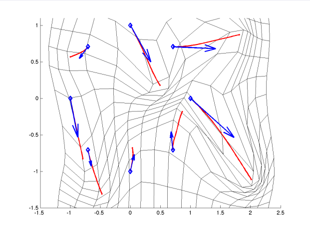

[Joshi *et al.*, T-I.P., 2000; Miller *et al.*, JMIV'06]

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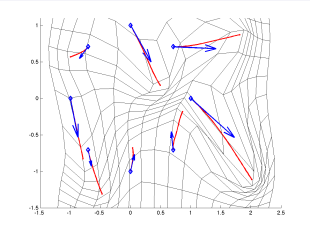

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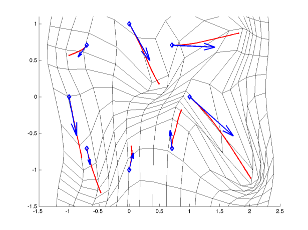

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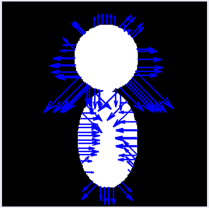
Not a new idea (see diffeo. B-spline for instance), but:

- no tangent-space parameterization
- optimal positions of the control points?
- optimal number of the control points?

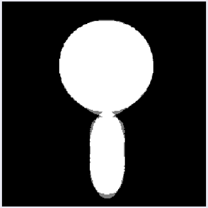
$\hookrightarrow$ Answer possible because of explicit dynamical system

Results:

Image LDDMM

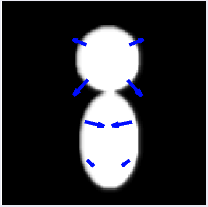


template



*s*th subject

Our solution



126 times fewer momenta for the same matching accuracy!