COORDINATED MATHEMATICAL MODELING OF INTERNAL WAVES

N.J Balmforth (UBC),
G.I. Ogilvie (U. Cambridge),
T. Peacock (MIT),
B.R. Sutherland (U. Alberta)

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This report presents a review of the material covered at the workshop on ”Coordinated Mathematical Modeling of Internal Waves”. The scope of the workshop was very broad, covering internal wave dynamics that arises in geophysical and astrophysical contexts. Five plenary lectures were given on the topics of oceanic, atmospheric and astrophysical internal waves. The presenters of these five lectures herein provide an overview of the state-of-play of research in each of these fields, and furthermore summarize the major outstanding issues and questions that were raised at the workshop.

1 Astrophysical Internal Waves I (by J. Goodman)

If internal waves are defined as periodic fluid motions restored by buoyancy and coriolis forces, then the internal waves observed in astronomical bodies are mainly global modes of oscillation rather than traveling waves such as those observed in the Earth's atmosphere and oceans. This is largely a selection effect, since at astronomical distances only waves that modulate the net light output from the nearer face of a star can be directly detected. Small-scale traveling waves are probably excited by instabilities, turbulence, and sometimes astronomical tides, and such waves may be important for mixing and momentum transport. But the absence of *in situ* measurements makes them difficult to constrain. This review concentrates on directly observed or potentially observable modes/waves; the subject of tidally excited internal waves—dear to my own heart—has been taken up by G. Ogilvie and others at this meeting.

A *g-mode* is the usual astronomical term for a global oscillation supported mainly by buoyancy due to stable stratification of entropy or composition. The radiatively diffusive core of the Sun, which encompasses 70% of its radius and more than 97% of its mass, is stratified, and g-modes surely exist there, but none have yet been securely detected [7]. Their eigenfunctions are evanescent in the outer 30% of the Sun, which is convective and therefore unstratified. The predicted velocity amplitude at the photosphere (visible surface) is $\lesssim 1\,\mathrm{mm\,s^{-1}}$ if the g-modes are excited by the convective turbulence, as the p-modes are. The latter are basically sound waves; $\sim 10^6$ p-modes are seen with typical amplitudes $\sim 10\ \mathrm{cm\,s^{-1}}$ and periods 3-6 min, and these have been used extensively to probe the Sun's internal structure [15].

A possible example of the indirect influence of astrophysical internal waves is the suggestion that g-mode coupling explains why the core rotates synchronously with the convection zone, as is inferred from p-mode rotational splittings, even though the Sun has gradually lost angular momentum to the solar wind over its lifetime [60]. As is often the case with indirect effects of internal waves in astrophysics, however, there are competing candidates for the coupling mechanism.

Many stars pulsate at amplitudes much larger than would be expected from turbulent forcing. Some of the frequencies are compatible with g-modes. These include subclasses of main-sequence B stars (i.e., surface temperatures 10^4-$10^{4.5}$ K) [20]. The excitation mechanisms are thermal: modulation of the radiative or convective heat flux (luminosity) of the star produces mechanical work in a manner somewhat analogous to—-but thermodynamically much less efficient than—a Carnot engine [17]. That this does not occur in all stars is due to requirements on the thermal timescale at those depths where modulation is possible, which translate to requirements on the surface temperature. It is worth noting that whereas the real parts of the mode frequencies are straightforwardly calculable by linear theory,[1] calculation of growth rates, stochastic forcing, and nonlinear saturation are challenging and more vulnerable to uncertainties in the input physics: e.g. radiative opacities, turbulent viscosities, etc.

By far the best observed and perhaps best understood g-mode pulsators are not main-sequence stars but white dwarfs. These are "dead" stars supported against their own gravity by electron degeneracy rather than thermal pressure and composed mainly of thermonuclear ash (helium through magnesium) rather than hydrogen. The residual heat supports, in addition to the observable luminosity of these objects, a slight thermal stratification of the outermost layers; additional stratification occurs deeper at the interfaces among helium, carbon, oxygen (etc.) zones due to the slight differences in nuclear mass per (pressure-ionized) electron rather than the molecular weight *per se*. In particular, the DAVs (a.k.a. ZZ Ceti stars) are white-dwarf pulsators with surface temperatures in the range $11,000 - 12,000\,\mathrm{K}$. Due to the compactness of white dwarfs ($R \sim 10^3$-$10^4\,\mathrm{km}$, $\bar{\rho} \gtrsim 10^6\,\mathrm{g\,cm^{-3}}$), the g-mode periods are 10^2-10^3 seconds—as compared to hours to days for main-sequence pulsators—enabling useful time series to be obtained relatively quickly. Precise measurements of mode frequencies diagnose the internal structure of the white dwarfs. The intrinsic linewidths are so small and the mode lifetimes so long in some cases that the gradual change in frequencies due to cooling—on timescales of order $10^9\,\mathrm{yr}$—is directly detected ([63] and references therein).

Excitation of DAV g-modes is understood to occur by a thermal instability in which the surface convection zone is crucial to modulating the heat flux, even though it contains only $\sim 10^{-14}$ of the stellar mass [13, 30]. As with any linear instability, it is necessary to address nonlinear saturation. This is less well understood than excitation (and much less well than the linear eigenfrequencies), but for the smaller-amplitude pulsators with many active modes, there are quantitative reasons to believe that saturation occurs by three-mode couplings, and in particular by parametric instabilities [65].[2]

This review includes a brief discussion of *r-modes* in neutron stars. The maximum observed rate of rotation of neutron stars, $\approx 700\,\mathrm{Hz}$, is less than the "break-up" rate where centrifugal force balances gravity (thought to be $\approx 1\,\mathrm{kHz}$); it is speculated this is due to loss of angular momentum by gravitational radiation [11], which requires the star to be slightly nonaxisymmetric. This may occur by linear instability of r-modes, which can be spontaneously excited by emission of gravitational waves at high rotation rates if the viscosity of the neutron star is sufficiently small [6]. This is another example of a somewhat speculative indirect effect of internal waves. There is, however, hope that the gravitational waves may be directly detected in the not too distant future [62].

Unlike g-modes, r-modes are restored by Coriolis rather than buoyancy forces. They are a special case of the more general class of rotationally supported internal waves, namely inertial oscillations. r-modes are distinguished by their long wavelengths and simple dispersion relation; in fact they are approximately polynomial in Cartesian coordinates. Quadrupolar modes varying longitudinally $\propto \exp(2i\phi)$ have angular frequency $\approx 4\Omega/3$ in an inertial frame, where Ω is the rotational frequency of the star, but $\approx -2\Omega/3$ in the rotating frame. This makes them modes of negative energy and angular momentum, so that they can be excited by emission of positive-energy gravitational waves; since the emitted power is proportional to the square of the wave amplitude (and to the sixth power of frequency), this produces linear instability. Here too saturation may occur via a network of nonlinear three-mode couplings [14, 54]. However, it appears that a steady balance between parametric growth and viscous dissipation of the daughter modes is not possible, so that growth and saturation of the primary mode—and its potentially observable gravitational waves—may undergo limit cycles.

2 Astrophysical Internal Waves II (by G.I. Ogilvie)

Internal waves play an important role in astrophysics, in the context of tidal interactions between stars and planets. In comparison with terrestrial studies, the astrophysical approach takes a broad and often simplistic

[1] This is true at least for nonrotating, spherical stars; the basic equations are summarized in the accompanying presentation. Even linear theory can be conceptually challenging with rotation, however, as witnessed by the talks given at this meeting by B. Dinstrans, G. Ogilvie, J. Papaloizou, M. Rieutord, & Y. Wu.

[2] There may be close parallels here to three-mode couplings of oceanic internal waves, discussed at this conference by J. MacKinnon and N. Balmforth, among others.

view, because we must deal with a vast range of systems and parameters, and have very few observational data, usually of a highly indirect nature.

Tidal interactions can have a significant effect on the orbital and spin evolution of binary stars over astronomical timescales if the orbital period is less than ten days or so [68]. They have also affected the Earth–Moon system and the satellites of other planets in the solar system [49]. Interest has been rekindled in this subject through the ongoing discovery of many extrasolar planets that orbit very close to their host stars [69].

Typically, tidal interactions lead to a synchronization and alignment of the spin of the bodies with their orbital motion, together with a circularization of the orbit. These dissipative processes are accompanied by heating, which can have dramatic consequences, as in the case of Jupiter's closest moon, Io. In systems of extreme mass ratio, such as planets orbiting stars, the large body usually cannot achieve synchronization, and the tidal exchange of angular momentum leads instead to orbital migration, which is inward if the large body spins more slowly than the orbit. This process limits the lifetime of planets found in close orbits around stars.

A general mathematical formalism can be constructed for problems of tidal forcing, in which the tidal potential experienced by a body is expanded in solid spherical harmonics and in a Fourier series in time. For orbits of significant eccentricity, a broad spectrum of forcing frequencies is present [64]. At least in linear theory, our aim is to calculate the potential Love number, which is a dimensionless measure of the response of the body to periodic forcing; it depends on the degree and order of the spherical harmonic that is applied, and also on the tidal frequency. The Love number measures the external gravitational potential perturbation generated by the deformed body, which is the only means by which energy and angular momentum can be exchanged with the companion. It is a complex response function, and its imaginary part $\mathrm{Im}(k)$, which determines the part of the response that is out of phase with the forcing, governs the energy and angular momentum exchanges.

One possible viewpoint is that an astrophysical body supports a spectrum of discrete global oscillation modes, which might form a complete set of orthogonal functions under certain conditions. These modes would typically be computed for an ideal fluid, and their damping rates due to non-adiabatic effects or viscosity would be estimated by perturbative methods. Each mode can then be expected to respond to periodic forcing in the same way as a damped harmonic oscillator, and the overall response function of the body would contain a succession of Lorentzian peaks corresponding to the various modes with the appropriate natural frequencies and damping constants, and weighted according to their spatial overlap with the tidal potential.

There are at least two important ways in which this viewpoint is questionable. First, the relevant low-frequency oscillation modes in convective regions of stars and giant planets are thought to be inertial waves, which do not generally form discrete oscillation modes in an ideal fluid unless they propagate within simple containers such as a full sphere. If the inertial waves are confined to a spherical shell, for example because of the presence of a dense planetary core or stellar radiative zone, then after multiple reflections they exhibit a complicated behavior that depends strongly on the tidal frequency [47, 52]. Singularities associated with the critical latitude and with wave attractors have been found to be important, and connections can be made with problems studied in the Earth's ocean and in laboratory experiments. The tidal response is much more complicated than a succession of Lorentzian peaks, but on the other hand $\mathrm{Im}(k)$ may achieve a viscosity-independent asymptotic regime in restricted intervals. Recent work by several participants at the workshop has revealed the importance of inertial waves for tidal dissipation in astrophysical bodies as well as their remarkable complexity [10, 32, 36, 48, 53, 66]. Broadly speaking, this work implies that global modes are relevant when the inertial waves propagate in a full sphere, while singularities dominate the response when the core exceeds a certain size.

Second, when internal waves are involved, global oscillation modes may not be established because the waves can break. This is a problem especially for internal gravity waves that are excited in stellar radiative zones. Since the tidal frequency is usually much smaller than the Brunt–Väisälä frequency, the gravity waves can have a very short radial wavelength and propagate slowly. They are especially susceptible to breaking as they approach the surface of a star (for stars more massive than the Sun) or the center (for solar-type stars). Wave breaking can prevent the resonant excitation of global modes and leads instead to efficient tidal dissipation over a broad range of frequencies [29, 31]. This is an example of a situation in which the nonlinear behaviour is much simpler, in broad terms, than the linear behaviour. It is also an area in which terrestrial studies can provide valuable information for astrophysicists.

3 Atmospheric Internal Waves (by D.C. Fritts)

Internal Gravity Waves (IGWs) play enormous roles in the dynamics, structure, and variability of Earth's atmosphere extending from the surface well into the thermosphere ($\sim$500 km and above). Their importance derives from their many sources, their efficient transport of energy and momentum to higher altitudes, and increases in their amplitudes and effects accompanying rapid density decreases with altitude (see [22], for a recent review). The dominant sources for smaller-scale IGWs include topography, convection, and wind shears. Wavelengths arising from these sources range from $\sim$10 to 100's of km in the horizontal and $\sim$1 to 100 km or more in the vertical, with the larger scales more prevalent at higher altitudes. Unbalanced jet stream flows, solar energy inputs in the auroral zones, and body forces accompanying IGW dissipation processes at higher altitudes yield larger-scale IGWs which have influences at lower or higher altitudes depending on their phase speeds.

The IGWs having the largest influences on atmospheric circulation and structure, and the weather and climate processes driving our forecasting needs, are the subset accounting for the dominant transport of energy and momentum from source regions to higher altitudes. These are the IGWs having the largest amplitudes, vertical group velocities, and energy and momentum fluxes at each altitude. IGWs excited at larger amplitudes and smaller scales account for the dominant fluxes and play the major roles in the troposphere and stratosphere. Because atmospheric density decreases by $\sim 10^6$ and $\sim 10^{11}$ from Earth's surface to $\sim$100 and $\sim$300 km (for mean solar conditions), respectively, the dominant IGWs in the mesosphere and thermosphere have larger scales and amplitudes than at lower altitudes by $\sim$1 to 2 decades.

IGW amplitudes increase strongly with increasing altitude because conservative IGW motions maintain a constant pseudo-momentum flux, $F = < u'_h w'(1 - f^2/\omega^2) >$ as they propagate, where $\rho_0(z) \sim e^{-z/H}$ is mean density, $H \sim 7$ km at lower altitudes, u'_h and w' are the IGW horizontal and vertical perturbation velocities in the plane of propagation, f and ω are the inertial and IGW intrinsic frequencies, and primes and angle brackets denote perturbations and a suitable spatial or temporal average, respectively. This implies IGW amplitudes that vary with density or altitude as $(u'_h, w', \rho'/\rho_0) \sim \rho_0^{-1/2}(z) \sim e^{z/2H}$. However, increasing amplitudes cause IGWs to be increasingly susceptible to various non-conservative instability processes which constrain IGW amplitudes, induce various interaction and instability dynamics, and drive IGW energy and momentum deposition. Larger-scale effects of these dynamics include: 1) systematic changes in the mean circulation and thermal structure throughout the atmosphere, 2) generation of secondary IGWs at higher altitudes, 3) modulation of, and by, tidal and planetary wave motions and mapping of these structures to much higher altitudes, and 4) apparently strong influences of these neutral dynamics on plasma dynamics and instabilities throughout the ionosphere. Smaller-scale effects include: 5) turbulence and mixing throughout the atmosphere with intensities and influences that increase with altitude into the thermosphere, and 6) an approximately "universal" IGW spectrum in wavenumber and frequency remote from IGW sources.

Stability theory provides valuable guidance on the occurrence, character, and time scales of the instability dynamics influencing IGWs (e.g., [1, 43, 57]), while numerical modeling provides insights into the instability and turbulence dynamics and mixing in idealized environments (e.g., [23, 24]). Despite many advances in theoretical, modeling, and observational studies, however, current parameterizations of these dynamics, and their influences on the large-scale circulation and structure of the atmosphere, are recognized to have major deficiencies due to an incomplete understanding of these dynamics at present (e.g., [40, 56]).

Solar tides are also important components of the atmospheric motion field at higher altitudes. They can be viewed as large-scale IGWs forced by solar thermal absorption and modified by Earth's curvature and rotation. Thermal absorption in the troposphere excites deep tropical convection exhibiting maxima over Africa, the Amazon basin, and the western Pacific (except during El Niño) that induces both migrating (sun synchronous) and non-migrating modes (having eastward and westward phase speeds different than the sun) at harmonics of a solar day. Solar UV absorption by ozone in the stratosphere yields additional tidal forcing at larger vertical scales. The result of these thermal sources and interactions among the various tidal modes and planetary waves is a rich superposition of tidal harmonics exhibiting complex but systematic phase structures and winds of 50 to 100 ms^{-1} or larger extending from $\sim$80 km well into the thermosphere. Like IGWs at smaller spatial scales, the tides contribute to energy and momentum transport over considerable depths. Major tidal roles include the modulation of IGW propagation and transport to higher altitudes extending well into the thermosphere and tidal influences on plasma dynamics and instabilities in the ionosphere.

Despite significant progress to date in many areas, there remain major unknowns spanning the spectrum of linear, quasi-linear, and nonlinear dynamics of IGWs in Earth's atmosphere. While linear theory provides a reasonable, though qualitative, description of the dominant IGW sources and propagation in variable environments, it often fails to describe the details. This is surely due, in part, to the lack of complete characterization of the spatial and temporal scales of the IGW sources and the environments in which they arise and through which they propagate. Observations cannot describe the airflow over terrain at high resolution and thus are unable to describe the effects of boundary layer dynamics, separation, or temporal modulation. Similarly, convection is poorly defined in space and time (in observations and models) relative to the scales of the most intense updrafts (and strongest IGW sources). Hence even an accurate statistical description of IGW responses to convection is beyond our present capabilities. Other IGW sources, such as jet streams, wind shears, or body forces, are even less well characterized.

Fine structure in the wind and/or temperature fields through which IGWs propagate almost certainly influences their propagation and tendency for instability throughout the atmosphere. Yet we have limited abilities to characterize these influences at present, despite indications that such superpositions of spatial scales may dramatically influence the tendency toward, and character of, instabilities influencing IGW amplitudes and transport. Likewise, quasi-linear influences (e.g., IGW-induced mean flows) are well documented at larger scales throughout the atmosphere, but we know little about transient or localized body forcing or their influences on IGW propagation, interaction, and instability dynamics.

By far the largest current unknowns concerning IGWs, however, are the nonlinear dynamics and spectral transfers accompanying wave-wave interactions and IGW instability and turbulence generation. While valuable insights have come from theory, laboratory studies, and atmospheric observations, the parameter space for these dynamics is enormous, and studies to date have only provided a few enticing glimpses of the likely diversity. These dynamics and their effects depend in detail on both the dominant properties of these flows (i.e., IGW and environmental parameters) as well as the fine-structure flow that may or may not be observable, but which may have significant influences on the flow evolution. Key questions include: 1) when are linear or quasi-linear dynamics sufficient to describe IGW effects, 2) when are nonlinear effects essential to account for observed IGW character, 3) which dynamics determine the IGW spectrum with altitude (and under what conditions), and 4) what dynamics are critical to parameterize these IGW effects in our numerical weather prediction, climate, and general circulation models?

4 Oceanic Internal Waves I (by J.A. MacKinnon)

Internal gravity waves are ubiquitous in the stratified ocean, and play an important role in both local dynamics and ecology and the Earth's climate as a whole. Oceanic internal waves are Boussinesq and often have low enough amplitudes that a linear dispersion relation accurately describes their polarization and propagation characteristics (see, e.g. [28]). Vertical wavelengths range from the full ocean depth (km) to tens of meters. Wave frequencies are bounded at the low end by the local (latitude-varying) inertial frequency and at the high end by the local buoyancy frequency. Oceanic internal waves tend to be spectrally red in both frequency and wavenumber, with variance dominated by low-vertical-mode, low-frequency waves [25].

Though there are many motivations to study internal waves in the ocean (heaving of density surfaces affects everything from sound propagation to light limitation for phytoplankton), most research is inspired by the large role internal waves in diapycnal mixing. Away from surface and bottom boundary layers the magnitude and geography of diapycnal mixing in the ocean interior is largely set by the dynamics of breaking internal gravity waves. Over the last two decades it has become clear that wave breaking, and the resultant turbulent mixing, are strong inhomogeneous in both space and time. The patterns are driven by details of internal wave generation, propagation, interaction, and dissipation. In turn, the patchiness of diapycnal mixing has significant consequences for both regional and global flow patterns. Current generation climate models include little if any of these patterns or the internal wave dynamics that produce them [33, 38, 55]. Climate models that do not appropriately represent the turbulent fluxes of heat, momentum, and CO_2 across critical interfaces will not accurately represent the ocean's role in present or future climate.

Open questions remain for every stage of the internal wave life cycle. Energy is input into the internal wave field primarily by the tides and wind [67]. Internal tides are generated where the barotropic tide rubs over rough topography. Near the generation site, internal tides often take beam-like form, with the detailed structure dependent on tidal strength and shape of the topography [8, 16, 26]. Some of the resultant baroclinic energy dissipates locally, producing a global map of mixing hotspots that mirrors internal tide generation sites [59]. However, most of the energy radiates away in the form of low (vertical) mode waves [58]. Where this low-mode energy dissipates is still very much up in the air - contenders include scattering over deep topography [39], breaking on the continental slope [46], nonlinear interactions with the ambient internal wave field, including the special case of parametric subharmonic instability [5, 34, 45, 44], or interactions with mesoscale features [50, 51].

Near-inertial internal waves start with surface wind forcing of near-inertial motions in the mixed layer [3]. Beta-plane and eddy-interactions change the horizontal wavenumber so this variance can move equatorward and into the pycnocline, turning purely inertial motions into near-inertial waves that can propagate [18, 19]. Subsequent interactions within the internal wave field and with topography likely determine their role in turbulence production but these pieces of the puzzle are not well-understood. Local dissipation of higher-mode near-inertial waves plays a large role in turbulent fluxes of heat, dissolved gases, and nutrients in the stratified transition layer just beneath the mixed layer. As with internal tides, higher-mode waves are likely to be generated and dissipated locally, while low-mode waves escape to propagate thousands of km across ocean basins [4].

5 Oceanic Internal Waves II (By G.N. Ivey)

In the coastal ocean environment, the combination of finite depth and often complex coastal bathymetry means the role of boundaries becomes all important in the internal wave dynamics. The interaction of internal waves with the boundaries often promotes turbulent mixing, of central importance not only to local coastal ocean dynamics but also to basin-scale dynamics. The coastal region is also of particular importance to industry such as the offshore oil and gas industry, fisheries and the ecological functioning of the region.

Internal waves at density interfaces can grow from an initial small amplitude η_0 to form large amplitude highly non-linear internal wave trains. The final state of these evolving internal waves is dependent upon the two parameters η_0/H and h/H, where h is the upper layer depth and H the total depth [35]. In extreme cases, the induced interfacial shear can be so strong that mixing occurs, but the occurrence of mixing is very dependent upon both the strength of the shear as well as how long the shear is locally sustained [9]. Rather than in the interior, internal waves at density interfaces are most easily broken down when shoaling over sloping bottoms where, depending on relative magnitudes of the bottom slope and wave slope, from zero up to a maximum of 25% of wave energy can be converted to increased PE [2, 12].

In continuously stratified environments, a feature of both observational and numerical modeling work, particularly in the coastal ocean, is the crossing of obliquely propagating internal wave beams. Resonant interactions and turbulent can occur at these intersection regions and, while this has been demonstrated in the laboratory [61], it has not been observed in the field but could well be important in coastal regions such as Monterey Bay or the South China Sea with complex and energetic internal wave fields. Internal wave breakdown is clearly more dramatic and active near boundaries and especially due to wave reflection at critical slopes where energy conversions can again be up to 25% efficient [37]. While the process is well known, the sensitivity of the process to topographic shape and near boundary ambient flows is less understood.

In general more is known about internal wave reflection than generation. The major generation mechanisms are from turbulence in the surface mixing layer and particularly tidally forced flow over bottom topography which can generate both modal and beam-like responses [21]. Field measurements suggest highly dynamic mean flow fields and intense turbulent mixing in boundary regions and it remains unclear how this impacts the effectiveness of wave generation, particularly in the beam case [27, 42]

Some implications for field scale numerical models are clearly the need for three-dimensional effects, but there remains challenges over when (or where in domain) non-hydrostatic models are needed, and how to deal with the resulting computational constraints for field scale applications. What spatial resolution is needed, especially near boundaries, to describe the topographic shape, horizontal excursion length and (especially) in vertical? It is not clear how these factors influence internal wave beam width as it leaves the bottom. Intimately linked to this is the need for simple but dynamic parameterization of turbulence [41] in the vicinity of topography where waves may overturn and break.

6 Concluding Q & A

In a concluding session, the following questions were posed and answered by the participants. This is a brief synopsis of this session.

6.1 What is the spectrum of internal waves produced by turbulence or penetrative convection?

There appears to be some organization/weak coupling with a preferred excitation frequency $\omega \sim 0.6N - 0.8N$ (Sutherland). Some evidence for support of this in ocean data (MacKinnon). Basically a complicated broad spectrum in stars (Rogers). BV frequency varies significantly with location in stars (Goodman). Interaction sites between waves set the spectrum; after that it is just propagation (Sutherland).

6.2 Is geometric focusing relevant in a domain with a large aspect ratio?

Focusing was first studied in a thin shell case around the equator. Important when waves only get trapped in an equatorial region (Rieutord).

6.3 ”Universal” wave spectrum - what causes it?

In atmosphere it is instability processes (Fritts). Saturation phenomena due to wave-wave interactions in the ocean. Reproduced by numerical models. GM spectrum established since 1970’s and proven if energy enters in $M2$ and $K1$. Not necessarily triad interactions (StLaurent). Lots of tests/foundation came from near Woods Hole. There have been ocean observations elsewhere that disagree (Alford). Ocean expressed in terms of modes. Atmosphere has very different processes (Sutherland). Maybe some small-scale processes are the same in the ocean and atmosphere (MacKinnon). Don’t need breaking to achieve saturation (StLaurent). Energy dissipation timescales are different in the ocean (50 days) and atmosphere (10 days). Scales in the ocean different to atmosphere. Ocean is limited to 10km vertical scales whereas atmosphere can go upto 100km (general comments).

6.4 Why isn’t momentum deposition important in the ocean?

Not necessarily true. In Antarctic Circumpolar Current it may be that momentum deposition by lee waves is important. Also in equatorial undercurrent (MacKinnon). Momentum deposition in the ocean appears not to have been measured (Sommeria,StLaurent).

6.5 Why perform lab experiments?

Basically there are still processes that are challenging for numerics. Can see unexpected things in the lab. Numerical models are for finite periods of time; if you are interested in long time behavior then need experimental validation (general comments).

6.6 Is there an atmospheric tide?

Yes. Created by heating and cooling. Also gravitational tides. This is very important in the ionosphere (Fritts).

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