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Effective couplings for brane & bundle moduli

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arXiv:0912.3265 [hep-th] with Peter Mayr, Johannes Walcher

arXiv:0909.1842 [hep-th], **work in progress** with Murad Alim, Michael Hecht, Peter Mayr, Adrian Mertens, Masoud Soroush

arXiv:0904.4674 [hep-th], **arXiv:0808.0761** [hep-th] with Masoud Soroush

Introduction & Motivation

✓ Effective couplings

- Ingredients: Fluxes, D-branes, O-planes, bundles, F-theory scenarios
- Mirror symmetry + other dualities (e.g. F-theory/type II dualities, F-theory/heterotic dualities):
  - Compute couplings of N=1 low energy effective theories
- N=1 effective superpotential/Kähler potentials for type II/heterotic/F-theory compactifications
- Computation of non-perturbative corrections (D-instantons, space-time instantons) via dualities

✓ Topological strings, mirror symmetry & invariants

- Moduli spaces of classical geometries are identified with moduli space of quantum geometries
- Dualities to compute the partition function of the topological A-model
  - Extract (integer) invariants (GW Invariants, OV invariants, . . .)
  - Topological disk partition function for branes in compact Calabi-Yau geometries
  - 4-fold GW invariants related to 3-fold OV invariants

Topological strings & Mirror Symmetry

✓ Mirror symmetry

	Topological B-model		Topological A-model
Bulk geometry (closed sector):	Calabi-Yau Y C.S. moduli space const. maps	closed string ⟷ mirror map	Calabi-Yau X Kähler moduli space holomorphic maps
	Classical geometry		Quantum geometry

Genus 0 partition function/holomorphic prepotential

Brane geometry (open sector):	Calabi-Yau Y + hol. vect. bundle (sheave)	open-/closed string ⟷ mirror map	Calabi-Yau X + spec. Lagr. submanifold with flat connection
	Classical obstructions		Quantum obstructions

Disk partition function/holomorphic superpotential

✓ <u>Dualities:</u> Interplay with effective superpotentials from F- & Het. theory

Outline

1. B-model for divisors in Calabi-Yau threefolds
2. Disk invariants via mirror symmetry
3. Dualities to F-theory & heterotic strings
4. Effective couplings
5. Conclusions

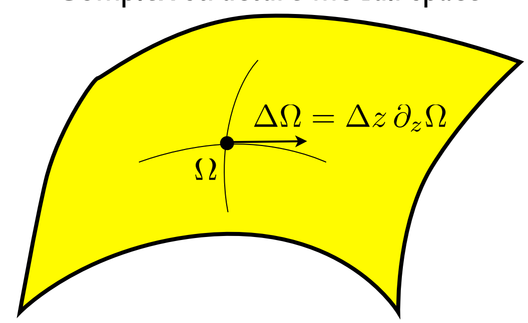
Closed-string mirror symmetry

✓ Complex structure moduli space of the B-model

- Variation of the holomorphic three form

$$(3,0)_Y \xrightarrow{\partial_z} (2,1)_Y \xrightarrow{\partial_z} (1,2)_Y \xrightarrow{\partial_z} (0,3)_Y$$

$$\mathcal{L}^{\rm PF}(z,\partial_z)\Omega(z) \sim 0$$



- Period integrals & N=2 special geometry

$$X(z) = \int_A \Omega(z)\,, \qquad \mathcal{F}_X(z) = \frac{\partial\mathcal{F}}{\partial X} = \int_B \Omega(z)$$

(A,B) symplectic basis of $H_3(Y)$

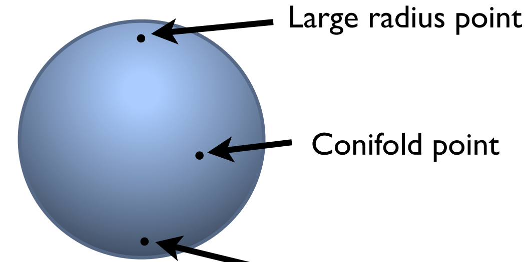
$$\mathcal{L}^{\rm PF}(z,\partial_z)X(z) = 0\,, \qquad \mathcal{L}^{\rm PF}(z,\partial_z)\mathcal{F}_X(z) = 0$$

✓ Quantum Kähler moduli space of the A-model

- Mirror map & flat periods

$$(X(z),\mathcal{F}_X(z)) = (1\,, t(z)\,, \mathcal{F}_t(z)\,, 2\mathcal{F}(z) - t(z)\mathcal{F}_t(z))$$

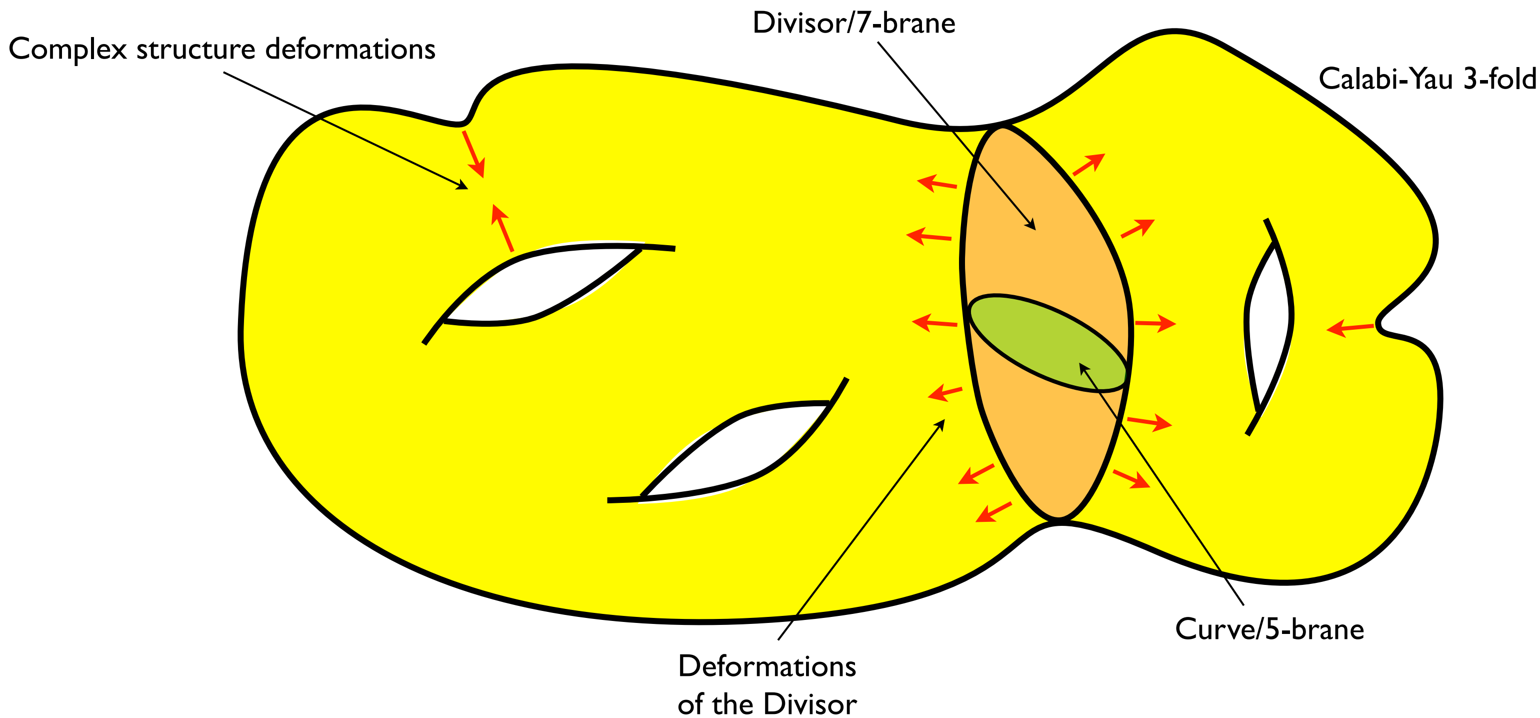
$$\mathcal{F}(z) \xrightarrow{z(t)} \mathcal{F}(t) = \mathcal{F}(z(t))$$



Open-string mirror symmetry

Alim, Hecht, Mayr, Mertens, Soroush, HJ; Aganagic, Beem

✓ B-model deformations: C.S. of the Calabi-Yau + (holomorphic) cycles

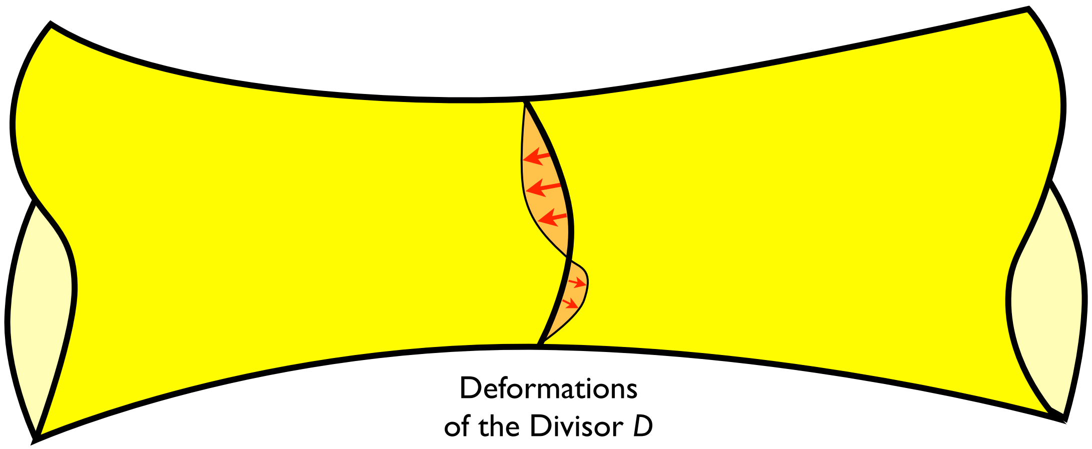


✓ Generalizations

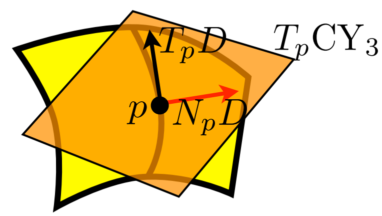
- Intersecting divisors/branes
- Complex of line bundles/coherent sheaves & matrix factorizations

Intuition for the brane deformations

Mayr, Lerche, Warner; Louis, HJ



✓ Holomorphic deformations



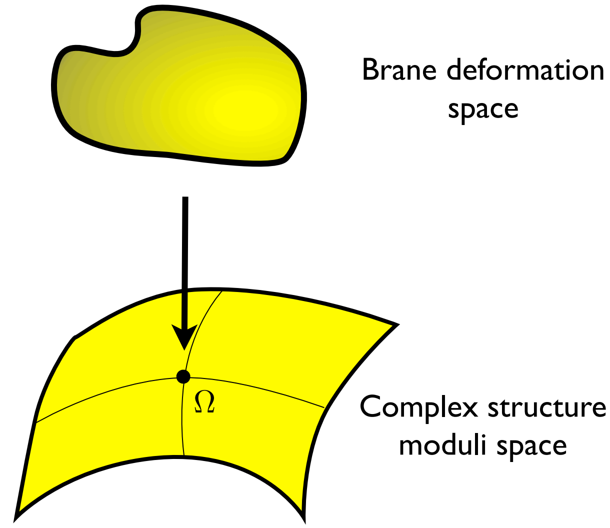
$$T_p \mathrm{CY}_3 \simeq T_p D \oplus N_p D \;, \qquad p \in D$$

$$\zeta \in H^0(D, ND) \;\longleftrightarrow\; \omega_\zeta = \Omega_{ijk}\, \zeta^i \, dz^j \wedge dz^k \in H^{2,0}(D)$$

- Normal bundle sections correspond to infinitesimal deformations
- Holomorphic deformations depend on the complex structure of the ambient Calabi-Yau 3-fold

Open-closed deformation space

Mayr, Lerche, Warner; Soroush, HJ



✓ Variation of mixed Hodge structure

$$\begin{array}{ccccccc} & & (2,0)_D & \xrightarrow{\delta_z} & (1,1)_D & \xrightarrow{\delta_z} & (0,2)_D \\ & \overset{\delta_u}{\nearrow} & & \overset{\delta_u}{\nearrow} & & \overset{\delta_u}{\nearrow} & \\ (3,0)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (2,1)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (1,2)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (0,3)_{\mathrm{CY}_3} \end{array}$$

✓ Relative cohomology

- Relative forms:

$$\Omega^*(\mathrm{CY}_3, D) \;=\; \{\, \theta \in \Omega^*(\mathrm{CY}_3) \,|\, \iota^*\theta = 0 \,\}\;, \qquad \iota : D \hookrightarrow \mathrm{CY}_3$$

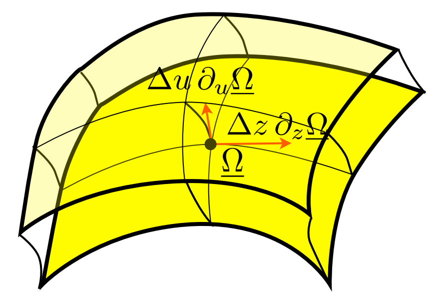
- Relative 3-form cohomology:

$$H^3(\mathrm{CY}_3, D) \;=\; \frac{\{d[\Omega^3(\mathrm{CY}_3, D)] = 0\}}{d[\Omega^2(\mathrm{CY}_3, D)]} \;\simeq\; H^3(\mathrm{CY}_3) \oplus H^2_{var}(D)$$

$$H^3(\mathrm{CY}_3, D) \;\ni\; \underline{\Theta} \;\simeq\; (\theta, \xi) \;\in\; H^3(\mathrm{CY}_3) \oplus H^2_{var}(D)$$

- Unique relative holomorphic *(3,0)*-form:

$$\underline{\Omega} \;\simeq\; (\Omega, 0) \;\in\; H^{3,0}(\mathrm{CY}_3, D)$$



B-branes on the Mirror Quintic

Walcher; Morrison, Walcher; Soroush, HJ; Alim, Hecht, Mayr, Mertens; Li, Lian, Yau

✓ Bulk and B-brane geometry

- Ambient space

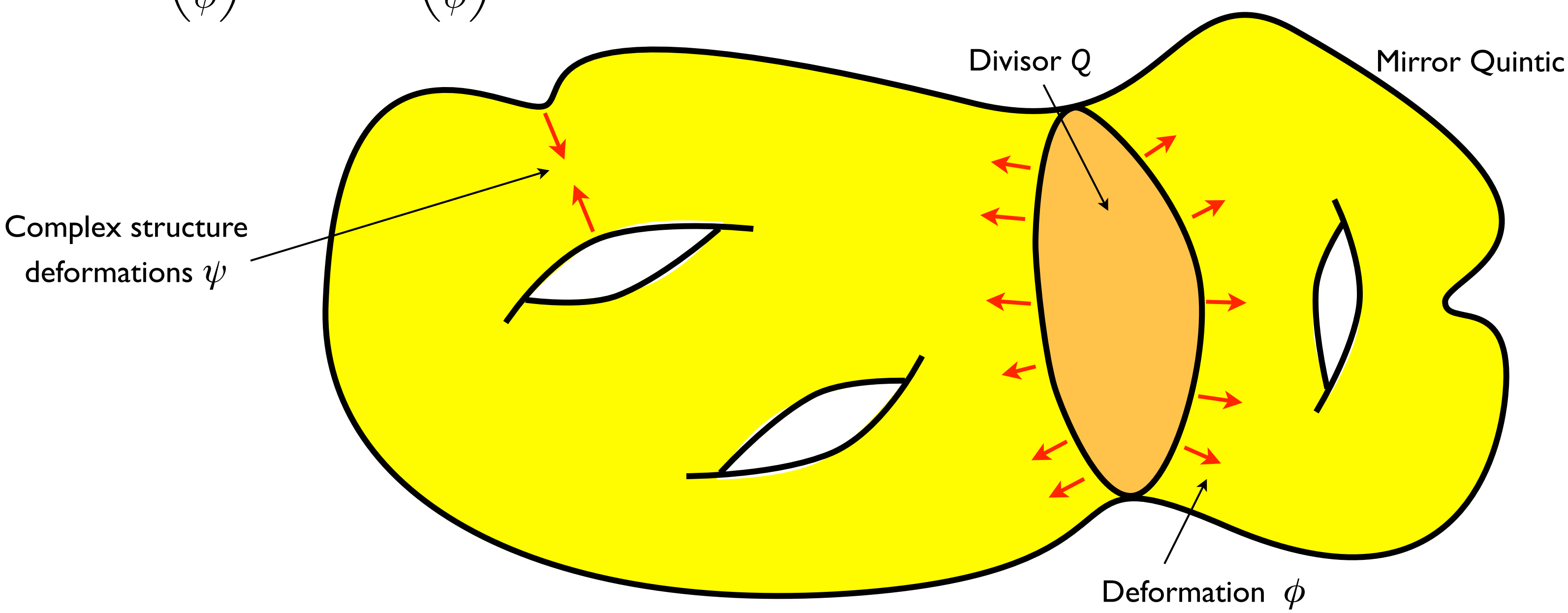
$$[x_1 : x_2 : x_3 : x_4 : x_5] \in \mathbb{CP}^4/(\mathbb{Z}_5)^3$$

- One complex structure modulus ψ, one deformation modulus ϕ

$$P(\psi) = x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 - 5\,\psi\, x_1x_2x_3x_4x_5 \qquad Q(\phi) = x_5^4 - \phi\, x_1x_2x_3x_4$$

✓ Discrete moduli spaces symmetries

$$\mathbb{Z}_5 \; : \; \begin{pmatrix}\psi \\ \phi\end{pmatrix} \mapsto e^{2\pi i/5} \begin{pmatrix}\psi \\ \phi\end{pmatrix}$$



Picard Fuchs differential equations

$$
\begin{array}{ccccccc}
 & & (2,0)_D & \xrightarrow{\delta_z} & (1,1)_D & \xrightarrow{\delta_z} & (0,2)_D \\
 & \nearrow^{\delta_u} & & \nearrow^{\delta_u} & & \nearrow^{\delta_u} & \\
(3,0)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (2,1)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (1,2)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (0,3)_{\mathrm{CY}_3}
\end{array}
$$

$$
\begin{aligned}
z &= -\frac{1}{(5\psi)^5} \\
u &= 5\psi\phi
\end{aligned}
$$

✓ Variation of mixed Hodge structure of the relative 3-forms

$$\mathcal{L}_\ell(z,u,\theta_z,\theta_u)\,\underline{\Omega}(z,u) \sim 0$$

$$
\begin{aligned}
\mathcal{L}_1 &= \mathcal{L}_1^{bdry}\theta_u = \left(\theta_z^3 + \frac{4\,z}{u}\prod_{k=1}^{3}(5\theta_z+\theta_u+k) + u\,\theta_z^3\right)\theta_u \\
\mathcal{L}_2 &= \mathcal{L}_2^{bdry}\theta_u = (\theta_z+\theta_u) + u\,\theta_u(4\theta_1+\theta_2+1) \\
\mathcal{L}_3 &= \mathcal{L}_3^{bulk} + \mathcal{L}_3^{bdry}\theta_u = (\theta_z+\theta_u)^4 + \left(\frac{4\,z}{u}\theta_u - 5z(5\theta_z+\theta_u+4)\right)\prod_{k=1}^{3}(5\theta_z+\theta_u+k)
\end{aligned}
$$

- The subsystem is governed by the Picard-Fuchs system of the K3 Geometry

$$\mathcal{L}^{sub} = \theta_w + 4\,w\prod_{k=1}^{4}(\theta_w+k) \qquad w = \frac{z}{u(1+u)^4}$$

- Remark: Derivation of the GKZ System via toric geometry techniques is more economical

Gauss-Manin System & Integrability

$$
\begin{array}{ccccccc}
 & & (2,0)_D & \xrightarrow{\delta_z} & (1,1)_D & \xrightarrow{\delta_z} & (0,2)_D \\
 & \nearrow^{\delta_u} & & \nearrow^{\delta_u} & & \nearrow^{\delta_u} & \\
(3,0)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (2,1)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (1,2)_{\mathrm{CY}_3} & \xrightarrow{\delta_z} & (0,3)_{\mathrm{CY}_3}
\end{array}
$$

✓ Relative cohomology basis from Griffiths transversality

$$\underline{\Theta} \;=\; \left(1, \theta_z, \theta_u, \theta_z^2, \theta_z\theta_u, \theta_z^3, \theta_z^2\theta_u\right) \underline{\Omega}$$

✓ Gauss-Manin system

$$\nabla_z \underline{\Theta} = (\partial_z - M_z)\underline{\Theta} = 0 \qquad \nabla_w \underline{\Theta} = (\partial_w - M_w)\underline{\Theta} = 0$$

✓ Integrability

$$\partial_z M_w - \partial_w M_z + [M_z, M_w] \;=\; 0$$

Relative periods & flat coordinates

✓ Relative periods

$$\mathcal{L}_\ell(z,u,\theta_z,\theta_u)\underline{\Pi}^\Sigma = 0\ , \qquad \underline{\Pi}^\Sigma = \int_{\underline{\Gamma}_\Sigma} \underline{\Omega}\ , \qquad \underline{\Gamma}_\Sigma \in H_3(\mathrm{CY}_3, D)$$

✓ Solutions in the vicinity of the point of maximal unipotent monodromy

$$\underline{\Pi}_2 = \log u + \ldots \qquad \underline{\Pi}_4 = \log^2 \cdot + \ldots \qquad \underline{\Pi}_6 = \log^3 \cdot + \ldots$$

$$\underline{\Pi}_0 = 1 + \ldots \qquad \underline{\Pi}_1 = \log z + \ldots \qquad \underline{\Pi}_3 = \log^2 \cdot + \ldots \qquad \underline{\Pi}_5 = \log^3 \cdot + \ldots$$

✓ Flat coordinates & relative periods

$$\vec{\underline{\Pi}}(t,\hat{t}) = \left(\frac{\underline{\Pi}_k}{\underline{\Pi}_0}\right) = \left(1\ ,\ t\ ,\ \hat{t}\ ,\ F_t(t)\ ,\ W(t,\hat{t})\ ,\ F_0(t)\ , T(t,\hat{t})\right)$$

$$F_t = \partial_t \mathcal{F} \qquad F_0 = 2\mathcal{F}(t) - t\,\partial_t F(t) \qquad T(t,\hat{t}) = \int d\hat{t}\left(\frac{a}{2}W_{\hat{t}}^2 + b\,W_{\hat{t}}\right) d\hat{t}$$

- N=1 special geometry is less constraining than the N=2 special geometry
- Double logarithmic periods encode generating function of open-string disk invariants
- Topological metric required to determine the integral linear combination of periods

Obstructions & 5-brane charges

✓ Deformation of the 7-brane divisor

$$P(\psi) \,=\, x_1^5 + x_2^5 + x_3^5 + x_4^5 + x_5^5 - 5\,\psi\, x_1 x_2 x_3 x_4 x_5 \qquad Q(\phi) \,=\, x_5^4 - \phi\, x_1 x_2 x_3 x_4$$

- By construction: bulk & boundary geometry are unobstructed
- Deformation parameters ψ and ϕ parametrize flat directions
- No effective superpotential for the deformations ψ and ϕ

Lüst, Mayr, Reffert, Stieberger; Louis, HJ; Gomis, Marchesano, Mateos; Martucci

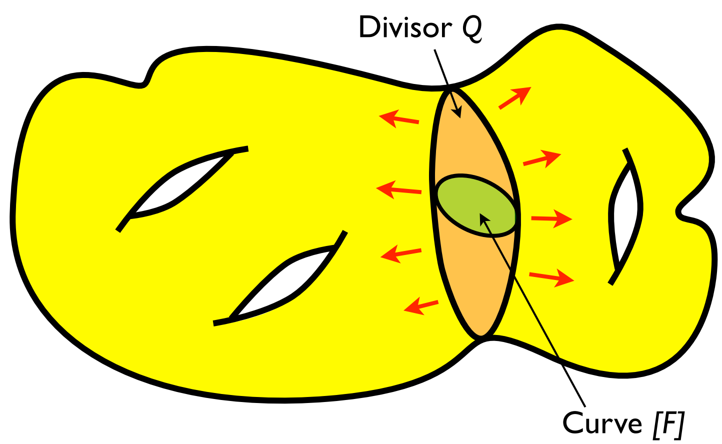
✓ Obstructions through lower-dimensional 5-brane charges

- Lower dimensional brane charges induce effective superpotential

$$W \;=\; \int_D F \wedge \omega_\zeta(\psi,\phi)$$

$$\omega_\zeta \;=\; \zeta^i(\phi)\,\Omega_{ijk}(\psi)\,dz^j \wedge dz^k$$

$$F \;\in\; H^2_{var}(D,\mathbb{Z})$$

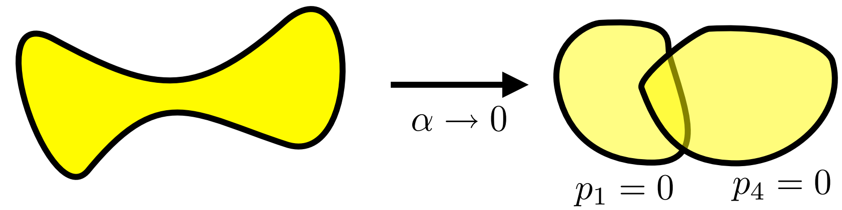


Large radius instanton expansion

Diaconescu, Florea; Alim, Hecht, Mayr, Mertens, Soroush, HJ

✓ Lagrangians in compact CY difficult

$$P_{Quintic}(\alpha) \;=\; p_1 \cdot p_4 + \alpha p_5$$



- Special point in the (complex structure) moduli space

Katz

✓ D4-brane tensions at the degeneration locus

- Idea: D4-brane tension splits in the presence of the Lagarangian 6-brane

$$\mathcal{T}_{D4}(t) \;=\; -\frac{5}{2}t^2 + \frac{1}{4\pi^2}\left(2875\sum_k \frac{e^{2\pi i k t}}{k^2} + \ldots\right) \xrightarrow{\alpha \to 0} \begin{array}{ll} \mathcal{T}^+_{D4}(t) & = -2t^2 + \frac{1}{4\pi^2}\left(1600\sum_k \frac{e^{2\pi i k t}}{k^2} + \ldots\right) + T_{open}(t,\hat{t}) \\ \mathcal{T}^-_{D4}(t) & = -\frac{1}{2}t^2 + \frac{1}{4\pi^2}\left(1275\sum_k \frac{e^{2\pi i k t}}{k^2} + \ldots\right) - T_{open}(t,\hat{t}) \end{array}$$

- The (double-logarithmic) split tensions solve the Picard-Fuchs equations

$$\mathcal{L}_\ell(t,\hat{t})\,\mathcal{T}^{\pm}_{D4}(t,\hat{t}) \;=\; 0$$

- Superpotential

$$W(t,\hat{t}) \;=\; \mathcal{T}^+_{D4}(t,\hat{t})$$

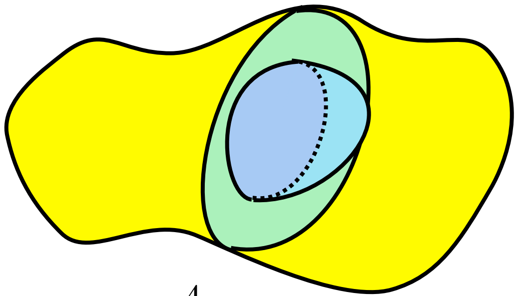
Ooguri Vafa invariants

Alim, Hecht, Mayr, Mertens, Soroush, HJ

✓ Disk partition function multi-covering formula

$$W(t,\hat{t}) \;=\; \sum_{\substack{d,\ell\\ k\geq 1}} \frac{n_{d,\ell}}{k^2}\left(e^{2\pi i t_1}\right)^{d\,k}\left(e^{2\pi i t_2}\right)^{\ell\,k} \qquad t = t_1 + t_2 \qquad \hat{t} = t_2$$

✓ Ooguri Vafa invariants



	$n_2 = 0$	1	2	3	4
$n_1 = 0$	0	20	0	0	0
1	−320	1600	2040	−1460	520
2	13280	−116560	679600	1064180	−1497840
3	−1088960	12805120	−85115360	530848000	887761280
4	119783040	−1766329640	13829775520	−83363259240	541074408000

c.f. disk invariants of involution branes: Walcher; Morrison, Walcher

Calabi-Yau 4-fold dualities & F-theory

Mayr; Lerche, Mayr; Alim, Hecht, Mayr, Mertens; Alim, Hecht, Mayr, Mertens, Soroush, HJ

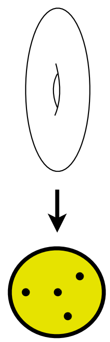
	Topological B-model		Topological A-model
compact Calabi-Yau 4-folds	X_B (F-theory)	⟷ 4-fold mirror symmetry	$Z_A \longrightarrow X_A$, $X_A \downarrow \mathbb{CP}^1$
	$g_S \to 0$ ↓ weak coupling limit		$\mathrm{Im}\ S \to +\infty$ ↓ large base limit
local Calabi-Yau 4-folds	X_B^{nc} (local F-theory)	⟷ local 4-fold mirror symmetry	$Z_A \longrightarrow X_A$, $X_A \downarrow \mathbb{C}$
	↕ open/closed duality		↕ A-model 4-fold duality
compact Calabi-Yau 3-folds with branes	(Z_B, D) (B-branes)	⟷ 3-fold open/closed mirror symmetry	(Z_A, L) (A-branes)

F-theory & heterotic duality

Morrison, Vafa; Friedman, Morgan, Witten; Aspinwall, Morrison; Berglund, Mayr

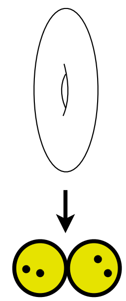
✓ Stable degeneration limit of K3

$$\begin{array}{ccc} T^2 & \longrightarrow & K3 \\ & & \downarrow \\ & & \mathbb{CP}^1 \end{array}$$



F-Theory on K3

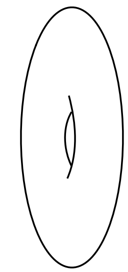
stable degeneration limit →



$K_1 \times_{T^2} K_2$

Stable degenerated K3

← F-Theory/Heterotic duality →



$E_8 \times E_8$

Heterotic String on T^2

✓ Fiberwise stable degeneration for Calabi-Yau fourfolds

$$\begin{array}{ccc} K3 & \longrightarrow & X_B \\ & & \downarrow \\ & & B \end{array}$$

F-Theory on X_B

stable degeneration limit →

$$\begin{array}{c} X_{B,1} \times_{Z_B} X_{B,2} \\ \cup \\ X_B^{\rm nc} \end{array}$$

Stable degenerated 4-fold

← F-Theory/Heterotic duality →

$$\begin{array}{ccc} T^2 & \longrightarrow & Z_B \\ & & \downarrow \\ & & B \end{array} \quad \subset \quad (Z_B, D)$$

$E_8 \times E_8$

Heterotic String on Z_B

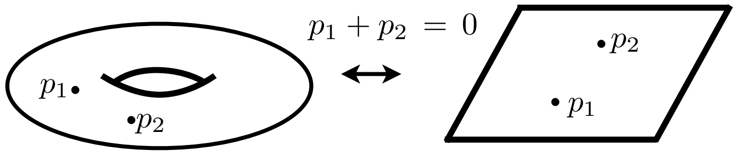
(large elliptic fiber limit)

Heterotic Bundles & spectral covers

Friedman, Morgan, Witten;   c.f. Donagi's talk

✓ stable SU(n)-bundles on the elliptic curve

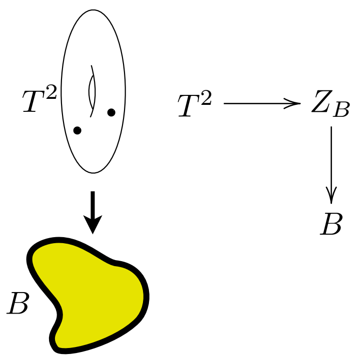
- *n* points on the elliptic curve: $p_1 + \ldots + p_n = 0$
- displacement of the *n* points ↔ *(n-1)* bundle moduli



e.g. SU(2) bundle of the elliptic curve

✓ Spectral covers for elliptically fibered Calabi-Yau threefolds

- *n*-fold spectral cover of the base B ↔ divisor *D* of the Calabi-Yau Z_B
- stable SU(n) bundle: Spectral Cover + holomorphic line bundle
- (normal) displacement of the divisor *D* moduli ↔ bundle moduli



✓ Example: SU(2) bundle on the mirror quintic

Mayr, Walcher, HJ

elliptic fiber coordinates (cubic torus)

$$\begin{aligned} P(X_B) &= p_0 + v^1 p_+ + v^{-1} p_- \\ p_0(Z_B) &= Y^3 + X^3 + XYZ(stu + s^3 + t^3) - \psi\, Z^3 (s^2 t^2 u^5) \\ p_+(D) &= X^3 - \phi\, YXZ(stu) \\ p_- &= \xi\, X^3 \end{aligned}$$

base coordinates

- ψ Calabi-Yau 3-fold modulus
- ϕ SU(2) bundle modulus
- ξ "stable degeneration modulus"

SU(2) bundle & Mirror Quintic

Curio, Donagi; Alim, Hecht, Mayr, Mertens, Soroush, HJ; Mayr, Walcher, HJ

✓ 4-fold geometry & Hodge structure

$$P(X_B) \;=\; Y^3 + X^3 + XYZ(stu + s^3 + t^3) - \psi\, Z^3(s^2t^2u^5) + v^1 X\left(X^2 - \phi\, YZ(stu)\right) + v^{-1}\xi\, X^3$$

$$h^{3,1} = 3\,, \qquad h^{1,1} = 299\,, \qquad h^{2,2} = 1252\,, \qquad h^{2,1} = 0\,,$$
$$\chi(X_B) = \int_{X_B} c_4(X_B) = 1860\,, \qquad \chi(X_B) \mod 24 = 12$$

$$\begin{array}{ccccccccc} H^{3,0}(Z_B) & \longrightarrow & H^{2,1}(Z_B) & \longrightarrow & H^{1,2}(Z_B) & \longrightarrow & H^{0,3}(Z_B) & & \\ \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\ H^{4,0}(X_B) & \longrightarrow & H^{3,1}(X_B) & \longrightarrow & H^{2,2}_{hor}(X_B) & \longrightarrow & H^{1,3}(X_B) & \longrightarrow & H^{0,4}(X_B) \\ & & \downarrow & & \downarrow & & \downarrow & & \\ & & H^{2,0}(D) & \longrightarrow & H^{1,1}_{var}(D) & \longrightarrow & H^{0,2}(D) & & \end{array}$$

✓ Gauge symmetry breaking pattern

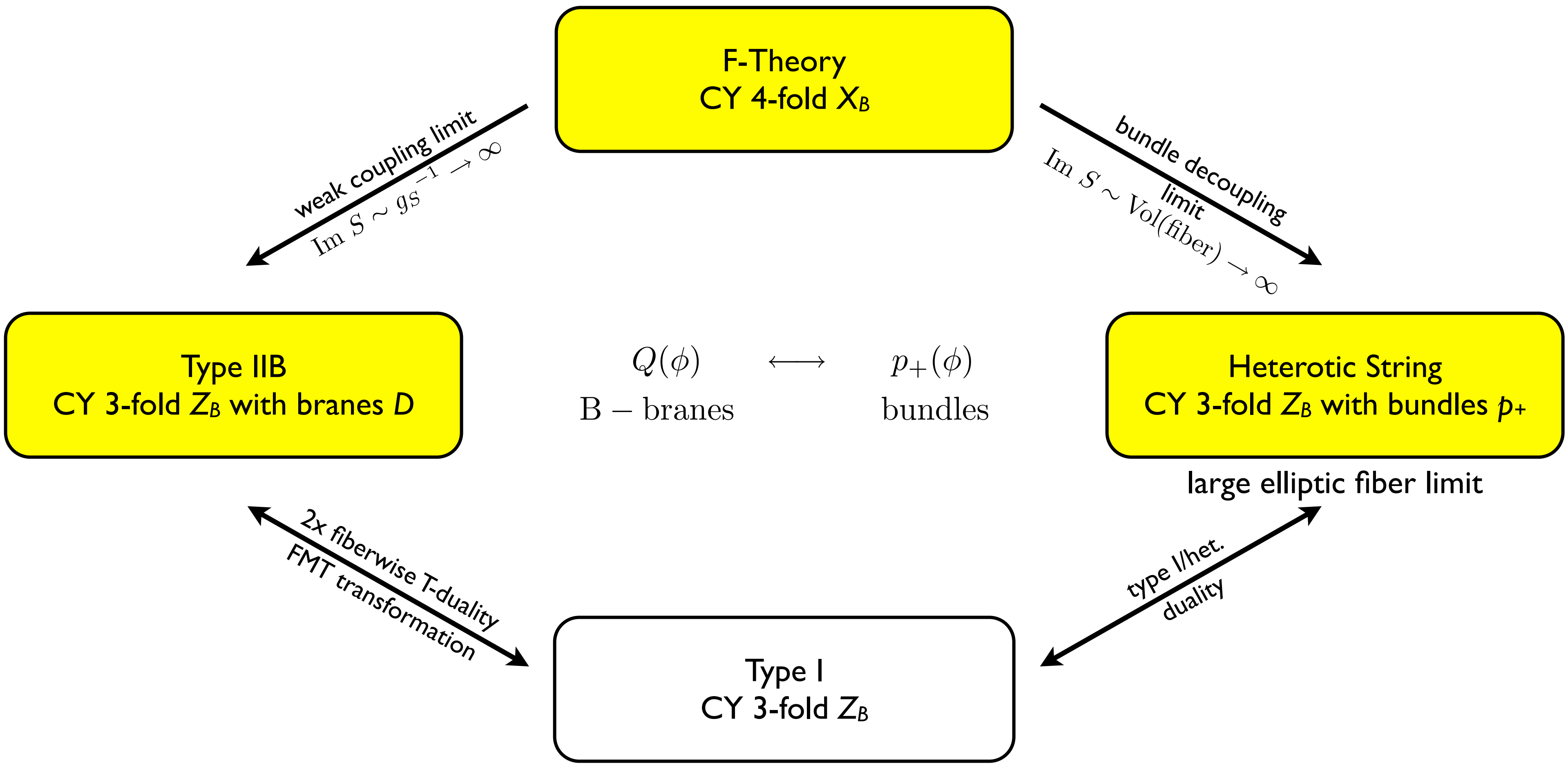
$$\frac{\chi}{24} = N + \frac{1}{2}\int \frac{G\wedge G}{(2\pi i)^2} \quad \Rightarrow \quad E_6\times E_6 \to SU(5)\times E_6 \quad \Rightarrow \quad SU(2) \text{ bundle structure group}$$

Alim, Hecht, Mayr, Mertens; Grimm, Ha, Klemm, Klevers; Mayr, Walcher, HJ

✓ Heterotic 5-brane & small instanton examples

- Divisors in the mirror Calabi-Yau threefold of $P^4(1,1,1,6,9)[18]$
- Divisor deformation is independent of elliptic fiber coordinates

Duality chains



Superpotentials

Mayr, Walcher, HJ

F-Theory flux superpotential

$$W(X_B) = \int_{X_B} \Omega^{(4,0)} \wedge F^{(4)} = \sum_{\Sigma} N_\Sigma \int_{\gamma_\Sigma} \Omega^{(4,0)} = N_\Sigma \Pi^\Sigma(X_B)(S,t,\hat{t})$$

$$\Pi(X_B)(S,t,\hat{t}) = \begin{cases} (1,S) \times \Pi(Z_B)(t) \\ \hat{t},\, W(t,\hat{t}),\, T(t,\hat{t}) \end{cases} + \mathcal{O}\left(e^{2\pi i S}\right)$$

$$\begin{array}{ccc} Z_A & \longrightarrow & X_A \\ & & \downarrow \\ & & \mathbb{CP}^1 \end{array}$$

Mirror 4-fold X_A

weak coupling limit

bundle decoupling limit

Type II orientifold flux/brane superpotential

$$W(Z_B, D) = \int_{Z_B} \Omega^{(3,0)} \wedge \left(F^{(3)} + S\, H^{(3)}\right) + \hat{N} \int_{\substack{\gamma \\ \partial\gamma\neq 0}} \Omega^{(3,0)} + \mathcal{O}(e^{\frac{1}{g_s}})$$

Gukov, Vafa, Witten; Giddings, Kachru, Polchinski

- GVW flux superpotential + brane superpotentials
- geometric transitions

Witten; Berglund, Mayr; Mayr, Walcher, HJ

- D-Instanton corrections for type II OF geometries

Heterotic superpotential flux/bundle superpotential

$$W(Z_B, p_+) = \int_{Z_B} \Omega^{(3,0)} \wedge \left(H^{(3)} + dJ\right) + \int_{Z_B} \Omega^{(3,0)} \wedge \mathrm{tr}\left(\frac{1}{2} A \wedge \bar{\partial} A + \frac{1}{3} A \wedge A \wedge A\right) + \mathcal{O}(e^{2\pi i S})$$

- NS-Flux + Geometric Flux superpotential

Strominger; Becker, Becker, Dasgupta, Green

- Heterotic strings in generalized geometry backgrounds

Desgupta, Rajesh, Sethi; Becker, Tseng, Yau; Fu, Yau; Mayr, Walcher, HJ

- Agreement with twisted K3 x T^2 generalized geometries

Witten; Morrison, Walcher

- Hol. Chern-Simons superpotential for bundle moduli

Thomas; Mayr, Walcher, HJ

- Stable deg. limit of appropriate F-theory flux quanta

Kähler potential

Alim, Hecht, Mayr, Mertens, Soroush, HJ; Mayr, Walcher, HJ

✓ Kähler potential of the 4-fold moduli space

$$K(X_B) = -\log \int_{X_B} \Omega^{4,0} \wedge \bar{\Omega}^{4,0} = -\log \sum_{\gamma_\Sigma, \gamma_\Lambda} \Pi^\Sigma(X_B) \eta_{\Sigma\Lambda} \bar{\Pi}^\Lambda(X_B)$$

✓ Kähler potential of the open-closed deformation space

$$K(X_B) \xrightarrow{\text{weak coupling limit}} K(Z_B, D) = -\log\left[-i \int_{Z_B} \Omega^{3,0} \wedge \bar{\Omega}^{3,0} + g_S \int_D \omega_\zeta \wedge \bar{\omega}_\zeta\right]$$

$$= -\log\left[-i \sum_{\underline{\gamma}_\Sigma, \underline{\gamma}_\Lambda \in H_3(Z_B, D)} \underline{\Pi}^\Sigma(Z_B, D) \underline{\eta}_{\Sigma\Lambda} \underline{\bar{\Pi}}^\Lambda(Z_B, D)\right]$$

$$\underline{\eta} = \begin{pmatrix} \eta(Z_B) & 0 \\ 0 & i g_S\, \eta(D) \end{pmatrix}$$

Louis, HJ

- Classical terms in agreement with dimensional reduction techniques

Mayr, Walcher, HJ

✓ Kähler potential of the bulk/geometric bundle deformation space

$$K(X_B) \xrightarrow{\text{bundle decoupling limit}}$$

$$K(Z_B, p_+) = -\log\left[-i \sum_{\underline{\gamma}_\Sigma, \underline{\gamma}_\Lambda \in H_3(Z_B, p_+)} \underline{\Pi}^\Sigma(Z_B, p_+) \underline{\eta}_{\Sigma\Lambda} \underline{\bar{\Pi}}^\Lambda(Z_B, p_+)\right]$$

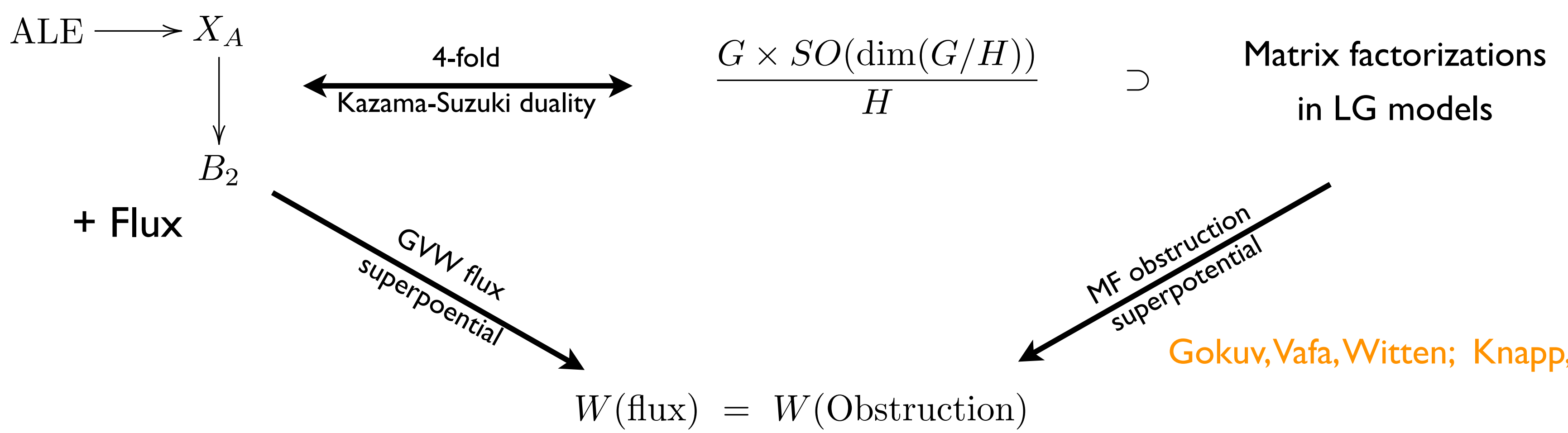
$$\underline{\eta} = \begin{pmatrix} \eta(Z_B) & 0 \\ 0 & i(\operatorname{Im} S)^{-1}\, \eta(D) \end{pmatrix}$$

- Classical terms are in (qualitative) agreement with dimensional reduction techniques

ALE Fibrations & Matrix factorizations

Eguchi, Warner, Yang

✓ Local 4-fold geometries



Gokuv, Vafa, Witten; Knapp, Omer

Mayr, Walcher, HJ

✓ Matrix factorizations in heterotic strings

Mayr

- Heterotic strings on 3-fold singularities ⇔ Moduli space of 2D field theories

Curto, Morrison

- Matrix factorizations describe bundles of ADE singularities

Conclusions

✓ **Techniques to compute effective couplings in N=1 theories:**

- Picard-Fuchs equations for 7-brane geometries & spectral covers for bundle moduli
- Effective superpotential and Kähler potential couplings
- Duality to 4-fold geometries & relation to N=1 F-theory compactifications
- Non-perturbative worldsheet and D-instanton corrections via 4-fold dualities

✓ **Open mirror symmetry & quantum corrections**

- Disk invariants via open/closed string mirror symmetry
- D-instanton/large fiber corrections computable via duality chains

✓ **Outlook**

- Independent check of proposed quantum corrections
- More general brane and bundle geometries
- The role of matrix factorizations for bundles and heterotic strings