TALK
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- NC-SYM / NC-QSYM
- SUPER CHAR THEORY OF UPPER TRIANG. MATRICES (OVER F_2)
- SC $\xrightarrow{\sim}$ NC-SYM (HOPF ALG.)
- ?? $\xrightarrow{\sim}$ NC-QSYM.

NC-SYM

SYM. FUNCT. IN NON-COM. VARIABLES [WOLF – SAGAN · ROSAS – ...] MANY OF US

S_∞ ACTS ON $\mathbb{Q}\langle x_1, x_2, \dots\rangle$

NC-SYM $= \mathbb{Q}\langle x_1, x_2, \dots\rangle^{S_\infty}$

- ORBITS INDEXED BY SET PARTITION [BASES]
- MULT : CONCAT
- CO-MULT: $f[X+Y]$
- HOPF ALGEBRA COMBINATORIAL

NC-QSYM [HIVERT — B~~K~~Z (?)] (*) FIND

S_∞ HIVERT-ACTION ON $\mathbb{Q}\langle x_1, x_2, \dots\rangle$

$$s_i * w = \begin{cases} s_i . w & \text{IF } x_i, x_{i+1} \in \text{SUPP}(w) \\ w & \text{OTHERWISE} \end{cases}$$

NC-QSYM $= \mathbb{Q}\langle x_1, x_2, \dots\rangle^{*S_\infty}$

- ORBITS INDEXED BY SET-COMPOSITION
  ⋮
- COMBINATORIAL HOPF ALGEBRA

GROTHENDICK RING $\longleftrightarrow$ CHA

"REPRESENTATION OF SYMMETRIC GROUPS" $\longleftrightarrow$ SYMMETRIC FUNCTIONS (COMMUTATIVE)

$\boxed{\text{THM [BLL]} \quad r^n n!}$? ? ? $\longrightarrow$ NC-SYM

IN A RECENT AIM WORKSHOP WE SOLVED THIS [ARXIVE].

Solution:

CONSIDER UPPER TRIANGULAR $n \times n$ MATRICES OVER $\mathbb{F}_2$

"U_n" $n \geq 0$

- REPRESENTATION THEORY IS WILD. !!!

- C. ANDRÉ HAD THE IDEA OF LUMPING CONJ. CLASSES AND CHARACTER TO DEVELOP A THEORY RICH ENOUGH TO WORK WITH BUT TAMED.

$A \in U_n$ $\qquad R(A - Id)T = B - I$

$A \sim B$

- INDEXED BY SET PARTITION!

$\mathrm{Res}^{U_n}_{U_k \times U_{n-1}}$

(NOT IND.) $\mathrm{INF}^{U_n}_{U_k \times U_{n-1}}$

$\Bigg\}$ INDUCES MULT. & CO-MULT. OF S.C.

- $\underbrace{\text{SC}}$ $\xleftrightarrow{\sim}$ NCSYM.

|| INTERESTING SUBALGEBRAS

NEXT STEP

$\longrightarrow$ NC-QSYM

- QUESTION
- CHA (ABS)
  - TRIVIAL CHARACTERS
- DGG (BLL)
  - WHAT IS $r^n h$! Here?

is $r = 2$???