Western Canada Linear Algebra Meeting
Banff International Research Station
May 7–9, 2010

Invited Speakers

Shmuel Friedland, University of Illinois, USA
Ilse Ipsen, University of North Carolina, USA
Francoise Tisseur, University of Manchester, UK

Organizing Committee

Shaun Fallat, University of Regina
Hadi Kharaghani, University of Lethbridge
Steve Kirkland, National University of Ireland
Peter Lancaster, University of Calgary, (Chair)
Michael Tsatsomeros, Washington Sate University
Pauline van den Driessche, University of Victoria
Paul Binding, University of Calgary, (local arrangements)
Peter Zizler, Mount Royal University, (local arrangements)
Dali Zhang, University of Calgary, (local arrangements)

Acknowledgements of Financial Support

Banff International Research Station
Mount Royal University
Pacific Institute for the Mathematical Sciences
University of Calgary

WCLAM-2010 Workshop Information

MEALS

Coffee Breaks: As per schedule on page 2, 2nd floor lounge, Corbett Hall (*included in workshop*).
For *meal options* at the Banff Centre, there are buffets (breakfast: 7:00-9:30am; lunch: 11:30am-1:30pm; dinner: 5:30-7:30pm). Gooseberry's Deli, located in the Sally Borden Building, and The Kiln Cafe, located beside Donald Cameron Hall. There are also plenty of restaurants and cafes in the town of Banff, a 10-15 minute walk from Corbett Hall.

MEETING ROOMS

All lectures are held in Max Bell 159. LCD projector, overhead projectors and blackboards are available for presentations *Note that the meeting space designated for BIRS is the lower level of Max Bell, Rooms 155-159. Please respect that all other space has been contracted to other Banff Centre guests, including any Food and Beverage in those areas.*

Poster displays will be in Max Bell by the lecture room.

Friday	
16:00	Check-in begins (Front Desk - Professional Development Centre - open 24 hours). Meeting/Lecture rooms available after 16:00 in Max Bell, lower level.
19:30	Informal gathering in 2nd floor lounge, Corbett Hall. Beverages and a small assortment of snacks are available in the lounge on a cash honour system.
Saturday	
7:00-9:00	Breakfast
9:00	Lectures
	Coffee Break, 2nd floor lounge, Corbett Hall
	Lectures
	Lunch
13:40	Lectures
	Coffee Break, 2nd floor lounge, Corbett Hall
	Lectures, Posters
19:00	Dinner (Space is reserved at the "ELK and OARSMAN" upstairs at 119 Banff Ave. Pay your own way.)
Sunday	
7:00-9:00	Breakfast
9:00	Lectures
	Coffee break, 2nd floor lounge, Corbett Hall
	Lectures
	Checkout from bedroom by 12 noon.

** 2-day workshops are welcome to use BIRS facilities (2nd Floor Lounge, Max Bell Meeting Rooms, Reading Room) until 15:00 on Sunday, although participants are still required to checkout of the guest rooms by 12 noon. There is no coffee break service on Sunday afternoon, but self-serve coffee and tea are always available in the 2nd floor lounge of Corbett Hall. **

Schedule

Saturday, May 8th, 2010

Date	Time	Speaker
May 8	8:50 – 9:00	Opening
	9:00 – 9:50	Francoise Tisseur
	9:50 – 10:20	Ion Zaballa
	10:20 – 10:40	*Coffee Break*
	10:40 – 11:10	Uwe Prells
	11:10 – 11:40	Kevin Vander Meulen
	11:40 – 12:10	Rajesh Pereira

Lunch

Date	Time	Speaker
May 8	13:40 – 14:30	Shmuel Friedland
	14:30 – 15:00	Anne Greenbaum
	15:00 – 15:20	*Coffee Break*
	15:20 – 15:50	Michael Cavers
	15:50 – 16:20	Leslie Hogben
	16:20 – 16:45	Poster Session (Garvey, Grundy, Zinchenko)
	16:45 – 17:15	Louis Deaett
	17:15 – 17:45	Elizabeth Bodine

Dinner**

Sunday, May 9th, 2010

Date	Time	Speaker
May 9	9:00 – 9:50	Ilse Ipsen
	9:50 – 10:20	Chun-Hua Guo
	10:20 – 10:50	*Coffee Break**
	10:50 – 11:20	Judi McDonald
	11:20 – 11:50	Minerva Catral
	11:50 – 12:20	Shaun Fallat
	12:20	Closing

*Participants may wish to use this break to check-out from their rooms.

**Space is reserved at the “Elk and Oarsman” pub, 119 Banff Ave. for 7:00pm. (West side of Banff Ave. up a flight of stairs.)

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Title and Abstracts

Spectrally arbitrary patterns

Elizabeth J. Bodine *Washington State University.*

An $n \times n$ zero-nozero pattern $\mathcal{A}_n$ is spectrally arbitrary over a field $\mathbb{F}$ provided that for each monic polynomial $r(x)$ of degree n with coefficients in $\mathbb{F}$, there is a matrix with entries in $\mathbb{F}$ having zero-nonzero pattern $\mathcal{A}_n$ and characteristic polynomial $r(x)$. In this talk, we will note some of the significant results in the study of spectrally arbitrary patterns over $\mathbb{R}$, $\mathbb{C}$ and finite fields, particularly noting the varying techniques used depending on the specified field. We will examine patterns that demonstrate fundamental differences in the algebraic structure of different fields.

(with Judith J. McDonald)

Eventually nonnegative matrices and related classes

Minerva Catral *Iowa State University.*

A matrix A is eventually positive (respectively, nonnegative) if all sufficiently large powers of A are positive (respectively, nonnegative). The concepts of strong eventual nonnegativity, the semi-strong Perron-Frobenius property and the subsequent classes of matrices that possess these properties are introduced. In this poster, a Venn diagram that illustrates the relationship among the following eight classes of matrices is presented: eventually positive matrices (EP), strongly eventually nonnegative matrices (SEN), matrices A such that both A and A^T have the semi-strong Perron-Frobenius property (SSPF), irreducible matrices, r-cyclic matrices, eventually nonnegative matrices (EN), nilpotent matrices and nonnegative matrices.

(with Craig Erickson, Leslie Hogben, D. D. Olesky, P. van den Driessche)

The normalized Laplacian energy of graphs

Michael Scott Cavers *University of Regina.*

The concept of graph energy was defined by Ivan Gutman in 1978 and originates from theoretical chemistry. To determine the energy of a graph, we essentially add up the eigenvalues (in absolute value) of the adjacency matrix of a graph. Recently, a few analogous quantities of energy have been defined, including the Laplacian energy and distance energy. In this talk, we analyze the normalized Laplacian energy of a graph. We highlight some key results, show its connection to standard energy, and relate it to a parameter called the general Randić index of a graph.

The magical pattern $\mathcal{L}$

Louis Deaett *University of Victoria.*

The combinatorial placement of the nonzero entries in a real matrix has an interesting relationship with its spectral properties. This relationship is reflected, for instance, in the refined inertia of the matrix, a quadruple specifying the inertia as well as the number of zero eigenvalues.

We introduce a zero-nonzero pattern called $\mathcal{L}$ to show that it is possible for a pattern to allow complete freedom in the refined inertia even while restricting the characteristic polynomial. Moreover, the characteristic polynomials allowed by $\mathcal{L}$ have a surprisingly succinct characterization.

Finally, we use the aforementioned characterization to show that the direct sum of $\mathcal{L}$ with another well-studied zero-nonzero pattern yields a direct sum that is spectrally arbitrary (that is, allows any characteristic polynomial) despite the fact that neither of its summands is spectrally arbitrary.

(with Dale Olesky, Pauline van den Driessche)

Rank deficiency and shadows in totally nonnegative matrices

Shaun Fallat *University of Regina.*

An $m \times n$ matrix is called totally nonnegative (TN) if all of its minors are nonnegative. It is a simple consequence of this definition to deduce that if $A = [a_{ij}]$ is TN with no zero rows or columns, and if $a_{kl} = 0$, then A will contain a block of zeros determined by the (k, l) position. In this talk, I will present a generalization of this phenomenon to larger sized rank deficient blocks and discuss some related results on row and column inclusion for TN matrices.

Tensors and matrices

Shmuel Friedland *University of Illinois at Chicago.*

In recent years the study of tensors, i.e. multiarrays, became a topic of an extensive research in applied and pure mathematics. Many problems in tensors are variations of problems in matrices, and many numerical and theoretical approaches to solve these problems using tools and results from matrices. In this talk I will survey some results and open problems mostly for 3-tensors. We will discuss the following topics.

(1) Rank and border of tensors.
(2) Low rank approximation of tensors.
(3) Scaling of nonnegative tensors.

References

[1] S. Friedland. Results and problems for 3-tensors. *Slides of lecture in NIU Linear Algebra Meeting, August 12-14, DeKalb, Illinois, USA, http://www.math.niu.edu/LA09/slides/friedland.pdf*, 2009.

Parameterising structure preserving transformations connecting quadratic matrix polynomials

Seamus D Garvey *University of Nottingham (Poster Presentation).*

The structure-preserving transformations (SPTs) of interest here are mappings which connect strictly-isospectral quadratic matrix polynomials of the same dimension. Given any one quadratic matrix polynomial, all other strictly-isospectral quadratic matrix polynomials can be generated by applying SPTs to the original system [**2**]

If all eigenvalues are finite, SPTs can be implemented via similarity transformations acting on the companion matrix to form a new companion matrix. Any one structure-preserving similarity (SPS) is parameterised by two $(n \times n)$ matrices. The new mass matrix is arbitrary (though non-singular) - giving a further $(n \times n)$ matrix in a complete SPT parameterisation. Various symmetries in the original system can be preserved [**3**]

If one or more system eigenvalues are infinite, applying SPSs directly to the companion matrix is not possible. This paper presents a parameterisation for SPTs applicable to all regular (in the sense of [**1**]) quadratic matrix polynomials. Given $(n \times n)$ system matrices $\{K_0, D_0, M_0\}$ defining a regular matrix quadratic, the structure-preserving relationships

$$\begin{bmatrix} W_L & X_L \\ Y_L & Z_L \end{bmatrix}^\top \begin{bmatrix} 0 & K_0 \\ K_0 & D_0 \end{bmatrix} \begin{bmatrix} W_R & X_R \\ Y_R & Z_R \end{bmatrix} = \begin{bmatrix} 0 & K_1 \\ K_1 & D_1 \end{bmatrix} \tag{1}$$

$$\begin{bmatrix} W_L & X_L \\ Y_L & Z_L \end{bmatrix}^\top \begin{bmatrix} K_0 & 0 \\ 0 & -M_0 \end{bmatrix} \begin{bmatrix} W_R & X_R \\ Y_R & Z_R \end{bmatrix} = \begin{bmatrix} K_1 & 0 \\ 0 & -M_1 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} W_L & X_L \\ Y_L & Z_L \end{bmatrix}^\top \begin{bmatrix} -D_0 & -M_0 \\ -M_0 & 0 \end{bmatrix} \begin{bmatrix} W_R & X_R \\ Y_R & Z_R \end{bmatrix} = \begin{bmatrix} -D_1 & -M_1 \\ -M_1 & 0 \end{bmatrix} \tag{3}$$

are satisfied if and only if the following hold for four $(n \times n)$ parameter matrices, $\{F_L, G_L, F_R, G_R\}$

$$\begin{bmatrix} W_L & X_L \\ Y_L & Z_L \end{bmatrix} = \begin{bmatrix} (F_L - \frac{1}{2} G_L D_0^\top) & (-G_L M_0^\top) \\ (G_L K_0^\top) & (F_L + \frac{1}{2} G_L D_0^\top) \end{bmatrix}^{-1} \tag{4}$$

$$\begin{bmatrix} W_R & X_R \\ Y_R & Z_R \end{bmatrix} = \begin{bmatrix} (F_R - \frac{1}{2} G_R D_0) & (-G_R M_0) \\ (G_R K_0) & (F_R + \frac{1}{2} G_R D_0) \end{bmatrix}^{-1} \tag{5}$$

subject to the matrix quadratic constraint

$$F_R G_L^\top + G_R F_L^\top = 0. \tag{6}$$

With this, it is straightforward to preserve all possible symmetries of the original system.

(with Atanas A Popov)

References

[1] I. Gohberg, P. Lancaster, and L. Rodman. *Matrix Polynomials*. Academic Press, 1982. SIAM (Philadelphia), 2009.
[2] P. Lancaster and U. Prells. This one incomplete.
[3] U. Prells and P. Lancaster. Isospectral vibrating systems. part 2: Structure preserving transformations. *Operator Theory: Advances and Applications*, 63:275–298, 2005.

GMRES residual norm bounds using the minimal norm interpolating function

Anne Greenbaum *University of Washington.*

Knowing the eigenvalues $\lambda_1, \ldots, \lambda_n$ and the field of values $W(A)$ of an n by n matrix A, what can be said about the quantity

$$\|P_k(A)\| \equiv \min_{c_1,\ldots,c_k} \|I + \sum_{j=1}^{k} c_j A^j\|,$$

where $\|\cdot\|$ denotes the operator 2-norm? This quantity gives an upper bound on the residual norm reduction after k steps of the GMRES algorithm for solving a linear system $A\mathbf{x} = \mathbf{b}$. Clearly $\|P_k(A)\| \leq 1$, and if the inequality is strict, then this implies that the GMRES algorithm restarted every k steps converges to the solution of the linear system.

It is known that $\|P_1(A)\| < 1$ if and only if $0 \notin W(A)$, but if $0 \in W(A)$, then little is known about whether $\|P_k(A)\| < 1$ for some k between 2 and $r-1$, where r is the degree of the minimal polynomial of A.. We derive bounds on $\|P_k(A)\|$ that hold even when $0 \in W(A)$ by using the fact that if f is any analytic function that matches P_k (and perhaps some of its derivatives) at the eigenvalues of A, then $f(A) = P_k(A)$. By using bounds on $\|f(A)\|$ based on the ∞-norm of f on $W(A)$ [**2**] or on a disk containing $W(A)$ [**1**] or on a disk of radius $\|A\|$ about the origin [**3**], and by choosing f to be the analytic function of smallest ∞-norm on the appropriate set that satisfies $f(A) = p_k(A)$ for some kth degree polynomial p_k with $p_k(0) = 1$, we thereby obtain bounds on $\|P_k(A)\|$.

References

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Potential stability of sign pattern matrices

David A. Grundy *University of Victoria (Poster Presentation).*

Let $\mathcal{A} = [\alpha_{ij}]$ be an $n \times n$ sign pattern matrix where $\alpha_{ij} \in \{+, 0, -\}$ and $Q(\mathcal{A}) = \{B = [b_{ij}] :$ sign $b_{ij} = \alpha_{ij}$ for all $i, j\}$ be the set of all $n \times n$ real matrices with that sign pattern. A sign pattern $\mathcal{A}$ is potentially stable if there exists a stable matrix $A \in Q(\mathcal{A})$, i.e., a matrix with each of its eigenvalues having negative real part. A number of operations are identified that can be applied to certain potentially stable sign patterns to give classes of potentially stable sign patterns of higher order. Proof techniques include identifying a nested sequence of principal minors of alternating sign, using a bordering technique based on similarity transformations, and analysis of characteristic polynomials.

On a nonlinear matrix equation arising in nano research

Chun-Hua Guo *University of Regina.*

The matrix equation $X + A^T X^{-1} A = Q$ arises in Green's function calculations in nano research, where A is a real square matrix and Q is a real symmetric matrix dependent on a parameter and is usually indefinite. In practice one is only interested in those values of the parameter for which the matrix equation has no stabilizing solutions. The solution of interest in this case is a special weakly stabilizing complex symmetric solution X_*, which is the limit of the unique stabilizing solution X_η of the perturbed equation $X + A^T X^{-1} A = Q + i\eta I$, as $\eta \to 0^+$. We show that a doubling algorithm can be used to compute X_η efficiently even for very small values of η, thus providing good approximations to X_*. It has been observed by nano scientists that a modified fixed-point method can sometimes be very efficient, particularly for computing X_η for many different values of the parameter. We provide a rigorous analysis of this modified fixed-point method and its variant, and of their generalizations.

(with Y. Kuo, W. Lin)

Sign patterns that require or allow eventual positivity, eventual nonnegativity, or power-positivity

Leslie Hogben *Iowa State University and American Institute of Mathematics.*

A real square matrix A is eventually positive (respectively, eventually nonnegative) if there exists a positive integer k_0 such that for all $k \geq k_0$, $A^k > 0$ (respectively, $A^k \geq 0$), where these inequalities are entrywise. A real square matrix A is called power-positive if there is a positive integer k such that $A^k > 0$. It is known that A is power positive if and only if A or $-A$ is eventually positive. In some applications, the data are not known precisely but the signs of the entries are known. In such a case, the problem can be studied through sign patterns. A sign pattern requires property P if every real matrix described by the sign pattern has P and allows P if some real matrix described by the sign pattern has P. This talk will summarize recent results characterizing sign patterns that require eventual positivity, sign patterns that require eventual nonnegativity, and sign patterns that require power-positivity, and will present several necessary or sufficient conditions for a sign pattern to allow eventual positivity.

(with Abraham Berman, Minerva Catral, Luz M. DeAlba, Abed Elhashash, Elisabeth Ellison, Frank J. Hall, In-Jae Kim, D. D. Olesky, Pablo Tarazaga, Michael J. Tsatsomeros, P. van den Driessche)

Numerical issues in randomized algorithms

Ilse C.F. Ipsen *North Carolina State University.*

Randomized algorithms are starting to find their way into a wide variety of applications that give rise to enormously large matrices. Among these applications are medical imaging, information retrieval (e.g. analysis of large term document matrices), genetics (e.g. analysis of DNA micro arrays and DNA single nucleotide polymorphisms), and machine learning. Randomized algorithms downsize the enormous matrices by picking and choosing only particular columns or rows, thereby producing potentially huge savings in storage and computing speed.

Although randomized algorithms can be fast and efficient, not much is known about their numerical properties. We will discuss how accurate and reliable these randomized algorithms are in comparison to their deterministic counterparts. Algorithms under consideration include randomized versions of matrix multiplications, QR decomposition, and subset selection.

Constructing non-negative matrices with prescribed structure

Judi McDonald *Washington State University.*

As part of attempting to solve the inverse eigenvalue problem for nonnegative, nonnegative symmetric, and nonnegative normal matrices, we have developed a variety of construction techniques with interesting properties.

Multiplier sequences, majorization and the products of normal matrices

Rajesh Pereira *University of Guelph.*

In 1914, Polya and Schur studied real sequences $\{\gamma_k\}_{k=0}^{\infty}$ which have the property that if $\sum_{k=0}^{n} a_k x^k$ is a polynomial all of whose roots are real so is $\sum_{k=0}^{n} \gamma_k a_k x^k$; they called such sequences multiplier sequences. We show how matrix techniques can be used to prove some classical results on multiplier sequences as well as some new connections between multiplier sequences and majorization. Some natural conjectures on the eigenvalues of products of normal matrices which arise in this work are also discussed.

(with A. Church, D. Kribs)

On the isospectral class of regular quadratic matrix polynomials

Uwe Prells *University of Nottingham, United Kingdom.*

Two matrix polynomials are called *isospectral* if they share the same Jordan matrix. For monic matrix polynomials the associated isospectral class can be generated from a given monic matrix polynomial by applying *structure preserving similarities* to the corresponding *companion polynomial*. Structure preserving similarities are **strict** equivalence transformation of a particular linearization and correspond to a **non-strict** equivalence transformation of the matrix polynomial. Those non-strict equivalence transformations are sometimes called *filters*.

The problem of generating the isospectral class becomes more subtle if we allow matrix polynomials with a singular leading coefficient because of the eigenstructure at infinity. This presentation attempts to extent the concept of *structure preserving transformations* to such regular matrix polynomials. The main result is a parameterization of the *structure preserving transformations* of the *companion polynomial* for regular quadratic matrix polynomials. The corresponding *filters* are derived and several illustrative examples are given. Difficulties that arise for matrix polynomials of higher degree are emphasized.

(with S.D. Garvey, P. Lancaster, A.A. Popov, I. Zaballa)

References

[1] S. Garvey, P. Lancaster, A.A Popov, U. Prells, and I. Zaballa. Filters connecting quadratic systems. part 2. *in preparation.*

[2] S.D. Garvey, U. Prells, M.I. Friswell, and C. Zheng. General isospectral flows for linear dynamic system. *LAA*, 87:335–368, 2004.

[3] L. Gohberg, P. Lancaster, and L. Rodman. Matrix polynomials. *SIAM Classics in Applied Mathematics*, 2009.

[4] P. Lancaster and U. Prells. Isospectral families of high-order systems. *ZAMM*, 87:219–234, 2007.

[5] P. Lancaster, U. Prells, and L. Rodman. Canonical structures for palindromic matrix polynomials. *OaM*, 1(3):469–489, 2007.

[6] U. Prells and S.D. Garvey. On the class of strictly isospectral systems. *MSSP*, 23(6):2000–2007, 2009.

Hermitian matrix polynomials with real eigenvalues of definite type

Francoise Tisseur *The University of Manchester.*

Eigenvalue problems $Ax = \lambda x$, with Hermitian A have many desirable properties which lead to a variety of special algorithms. Here we consider what can be regarded as the closest analogues of this class of problems for the generalized eigenvalue problem $L(\lambda)x = 0$, with $L(\lambda) = \lambda A - B$, $A = A^*$, $B = B^*$, and for the polynomial eigenvalue problem $P(\lambda)x = 0$, with

$$P(\lambda) = \sum_{j=0}^{m} \lambda^j A_j, \quad A_j = A_j^*,\ j = 0\colon m, \tag{7}$$

namely, the classes of definite, definitizable, hyperbolic, quasihyperbolic, overdamped and gyroscopically stabilized eigenproblems [**1**], [**2**], [**5**], [**3**], [**4**]. A property common to all these problems is that the eigenvalues are all real and of definite type, that is, $x^*P'(\lambda_0)x \neq 0$ for all nonzero $x \in \ker P(\lambda_0)$ and for all eigenvalues λ_0. We give a unified treatment of these classes that uses the eigenvalue type (or sign characteristic) as a common thread. Equivalent conditions are given for each class in a consistent format. We show that these classes form a hierarchy, all of which are contained in the new class of quasidefinite matrix polynomials. By analyzing their effect on eigenvalue type, we show that homogeneous rotations allow results for matrix polynomials with nonsingular or definite leading coefficient to be translated into results with no such requirement on the leading coefficient. We also give a necessary and sufficient condition for a quasidefinite matrix polynomial to be diagonalizable by structure preserving congruence, and show that this condition is always satisfied in the quadratic case and for any hyperbolic matrix polynomial, thereby identifying an important new class of diagonalizable matrix polynomials.

(with Maha Al-Ammari)

References

[1] M. Al-Ammari and F. Tisseur. Hermitian matrix polynomials with real eigenvalues of definite type. part i: Classification. *MIMS EPrint, The University of Manchester, UK*, 2010.

[2] R.J. Duffin. A minimax theory for overdamped networks. *J. Rat. Mech. Anal.*, 4:0221–233, 1955.

[3] P. Lancaster and Q. Ye. Definitizable hermitian matrix pencils. *Aequationes Mathematicae*, 46:44–55, 1993.

[4] A.S. Markus. *Introduction to the Spectral Theory of Polynomial Operator Pencils*. American Mathematical Society, Providence, RI, USA, ISBN 0-8218-4523-3, iv+250 pp., 1988.

[5] D.S. Mackey N.J. Higham and F. Tisseur. Definite matrix polynomials and their linearization by definite pencils. *SIAM J. Matrix Anal. Appl.*, 31:478–502, 2009.

Structure of nilpotent patterns

Kevin N. Vander Meulen *Redeemer University College.*

A zero-nonzero pattern $\mathcal{A}$ is said to be potentially nilpotent over field $\mathbb{F}$ if there exists a nilpotent matrix with entries in $\mathbb{F}$ having zero-nonzero pattern $\mathcal{A}$. We explore the construction of potentially nilpotent patterns over a field. We will describe order-four patterns which are potentially nilpotent over various fields, highlighting those without two-cycles. We present classes of patterns which are potentially nilpotent over a field $\mathbb{F}$ if and only if the field $\mathbb{F}$ contains certain roots of unity.

(with Natalie Campbell, Adam van Tuyl)

On classical modal control of quadratic systems

Ion Zaballa *Universidad del País Vasco.*

Classical modal control refers to the possibility of driving a quadratic system to quadratic diagonal form by strict equivalence. This amounts to finding non-singular square matrices P and Q such that $PL(\lambda) = \widetilde{L}(\lambda)Q$ where $L(\lambda) = M\lambda^2 + D\lambda + K$ is the given system and $\widetilde{L}(\lambda) = \widetilde{M}\lambda^2 + \widetilde{D}\lambda + \widetilde{K}$ is a diagonal quadratic matrix polynomial.

The main (an by now classical) result by Caughey and O'Kelly ([**1**]) gives a necessary and sufficient condition when $L(\lambda)$ is symmetric and M is positive definite. Ma and Caughey ([**5**]) studied this problem for general systems and Lancaster and Zaballa ([**4**]) provided a solution for symmetric systems when the pencil $\lambda M + K$ is semisimple and its eigenvalues are of definite type, and for general systems when $\lambda M + K$ has simple eigenvalues. In all these cases, the necessary and sufficient condition for reducing a given system to a diagonal one by strict equivalence has the following commutative form: $KM^{-1}D = DM^{-1}K$.

Recently, the notion of Filters connecting two isospectral quadratic systems has been developed ([**3, 2**]). Based on this concept new and more general necessary and sufficient conditions in terms of the spectral data of the given system can be provided. The aim of this talk is to present these new conditions.

(with S. Garvey, P. Lancaster, A. Popov, U. Prells)

References

[1] T.K. Caughey and M.B.J. O'Kelly. Classical normal modes in damped linear dynamic systems. *ASME J.of Applied Mechanics*, 32:583–588, 1965.

[2] S. Garvey, P. Lancaster, A. Popov, U. Prells, and I. Zaballa. Filters connecting quadratic systems . part 2. *Preprint.*

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Real proof of Gårding's convexity of hyperbolicity cones

Yuriy Zinchenko *University of Calgary (Poster Presentation).*

A homogeneous polynomial $p : \Re^n \to \Re$ of degree m is *hyperbolic* [**4**] with respect to $d \in \Re^n$, if the univariate polynomial $t \mapsto p(x + td)$ has m real roots for all x. Hyperbolic polynomials were first carefully studied in 1950's by Gårding in the context of PDEs; also, these polynomials give rise to an important class of convex optimization problems and in many ways mimic the behavior of matrix determinants [**1**].

We review some elementary properties of hyperbolic polynomials, reproving Gårding's key result on convexity of the so-called hyperbolicity cones using much simpler approach, and discuss a possible relationship of these polynomials with a cone of positive definite matrices, sometimes referred to as (generalized) Lax conjecture [**2, 3**].

References

[1] H. Bauschke, O. Guler, A.S. Lewis, and H.S. Sendov. Hyperbolic polynomials and convex analysis. *Canadian Journal of Mathematics*, 53:470–488, 2001.

[2] J. W. Helton and V. Vinnikov. Linear matrix inequality representation of sets. *Communications on Pure and Applied Mathematics*, 60:654–674, 2006.

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[4] L. Gårding. An inequality for hyperbolic polynomials. *Journal of Mathematcis and Mechanics*, 8:957–965, 1959.

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