Advances in Stochastic Inequalities and their Applications

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1 Overview of the Field

Stochastic inequalities play a crucial role in a wide variety of areas of mathematical science. Among these areas are learning theory, empirical processes, nonparametric function estimation, combinatorial optimization, high-dimensional geometry, random graphs, and Gaussian processes. Stochastic inequalities include concentration inequalities for functions of independent random variables, deviation inequalities for independent sums and their extension to functions of independent random variables such as U-Statistics, decoupling inequalities, as well as sharp moment inequalities for the norm of independent sums of Banach space valued random variables. Also partial extensions of these inequalities to dependent situations such as martingale and weakly dependent sequences have been accomplished and are important in applications.

2 Outcome of the Meeting

The objective of the workshop was to bring together a strong group of mathematicians who have made important contributions to stochastic inequalities and their applications. We witnessed a lively interdisciplinary exchange of ideas and methods that will surely lead to further progress in the particular research areas of the participants and will eventually lead to new developments.

The workshop featured 32 talks on a wide range of aspects of stochastic inequalities and their applications. Various speakers with different background presented their work in a way accessible to all participants which was important to help ideas penetrate across different areas and triggered interesting and fruitful discussions. In the next section we describe the some highlights of these talks.

3 Presentation Highlights

The 32 presentations considered different kinds of stochastic inequalities arising in different fields and various applications were developed. The topics included inequalities for Markov chains, inequalities for empirical processes, concentration inequalities, inequalities for regression, density estimation, and statistical learning theory, multivariate central limit theorems, U-statistics and chaoses, random matrix inequalities, inequalities for dependent random variables, applications for operations research, applications for high-dimensional geometry, and the Gaussian correlation conjecture. The titles and abstracts of the talks are listed below.

Tail inequalities for additive functionals and empirical processes of geometrically ergodic Markov chains

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I will present some Bernstein style tail inequalities for additive functionals and empirical processes of geometrically ergodic Markov chains. The bounds are expressed in terms of the asymptotic variance and and the L_∞ norm of the function defining the functional. The proofs are based on the classical regeneration method and some new inequalities for empirical processes of independent random variables with finite exponential Orlicz norms.

An invariance principle for set-valued M-estimators through the boundary empirical process.

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In current researches [1] with John Einmahl we combine several tools from the empirical process theory – all derived from concentration, symmetrization and moment inequalities – to describe the oscillation behavior of various kinds of set-valued M-estimators C_n in R^d. Among these empirical minimizers are excess mass sets, minimum volume sets, shorth sets – or maximum probability sets – selected in a class $\mathcal{C}_n$ by means of an i.i.d. sample having law P in R^d. When P has a density f these sets estimate a level set C of f. We do not study the volume $\lambda(C_n \Delta C)$ of the symetric difference between an empirical C_n and a target set C as in [8] but the set $C_n \Delta C$ itself, provided C is a convex body. To deal with weak convergence of random sets C_n we investigate a new kind of limit theorems. For this we use the cylinder description of the boundary empirical measure introduced in [6] and [7]. Also, the probability bounds for the strong gaussian approximation from [2,3] and for the stability of empirical minimizers from [4] both play a crucial role. It turns out that we can describe the joint limit law of the above error sets $C_n \Delta C$ in terms of an auxiliary Brownian motion indexed by functions describing the boundary ∂C, drifted by a deterministic process driven by the second order of P around ∂C. Extensions are on the way in clustering or quantization type problems such as the optimal k-balls covering, or in the trimmed k-means problem introduced in [5].

[1] Berthet, P. and J.H.J. Einmahl (2009). *Central limit theorems for level set estimators* and *Invariance principles for set valued M-estimators*.

[2] Berthet, P. and Mason, D.M. (2006). *Revisiting two strong approximation results of Dudley and Philipp*. IMS, Lecture Notes-Monograph Series, High Dimensional Probability, 51, pp 155-172.

[3] Berthet, P. and Mason, D.M. (2008). *Strong invariance principles for empirical processes indexed by functions*. Preprint.

[4] Berthet, P. and Saumard, A. (2009). *Stability of empirical minimizers through Gaussian approximation*. Preprint.

[5] Cuesta, J., Gordaliza, A., and Matrán, C. (1997). *Trimmed k-Means: An Attempt to Robustify Quantizers*. Ann. Statist., 25, 553–576.

[6] Einmahl, J.H.J. and Khmaladze, E.V. (2009). *Central limit theorems for local empirical processes near boundaries of sets*. To appear.

[7] Khmaladze, E.V. and Weil, W. (2008). *Local empirical processes near boundaries of convex bodies*. Ann. Inst. Statist. Math., 60, 813-842.

[8] Polonik, W. (1995). *Measuring mass concentrations and estimating density contour clusters - an excess mass approach*. Ann. Statist. 23, 855-881.

Self-bounding functions, Talagrand's convex distance inequality and related questions

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Using the (modular) entropy method, Boucheron, Lugosi and Massart (2003), Maurer (2006) obtained transparent proofs of parts of Talagrand's convex distance inequality (1995). The argument relied on a simple observation: the (random) Efron-Stein estimate of the variance of the convex distance is upper-bounded d by 1. This was not enough to handle the fluctuations of the convex distance to very small sets. By relating the squared convex distance to (general) weakly self-bounding functions, it is possible to recover the full power of Talagrand's convex distance inequality using a transparent and modular proof. This amounts to check that the squared convex distance (and many other weakly self-bounding functions) satsifies a Bernstein-like inequality.
Joint work with G. Lugosi and P. Massart.

Spectrum of large random Markov chains

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We consider the spectrum of random Markov chains with very large finite state space. The randomness of these chains, which appears as a random environment, is constructed by putting random weights on the edges of a finite graph. This approach raises stimulating open problems, lying at the interface between random matrix theory and random walks in random environment. Part of this work is in collaboration with Ch. Bordenave (Toulouse, France) and P. Caputo (Rome, Italy).

Exponential inequalities for self-normalized processes with applications

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We prove the following exponential inequality for a pair of random variables (A, B) with $B > 0$ satisfying the following *canonical assumption,* $E[\exp(\lambda A - \frac{\lambda^2 B^2}{2})] \leq 1$ for λ in *R*.

$$P\left(\frac{|A|}{\sqrt{\frac{2q-1}{q}(B^2 + (E(|A|^p))^{2/p})}} \geq x\right) \leq c_q x^{-\frac{q}{2q-1}} e^{-x^2/2}$$

where $C_q = (\frac{q}{2q-1})^{\frac{q}{2q-1}}$, $x > 0$ and $1/p + 1/q = 1$ for $p \geq 1$. Applying this inequality, we obtain subgaussian bounds for the tail probabilities for self-normalized martingale difference sequences. We propose a method of hypothesis testing for the L^p-norm ($p \geq 1$) of A (in particular, martingales) and some stopping times.

Multivariate Bahadur-Kiefer Representations

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The classical Bahadur-Kiefer representation (Bahadur (1967), Kiefer (1970)) gives a limit law (as $n \to \infty$) for the statistic $\|\alpha_n + \beta_n\|$, where α_n (resp. β_n) denotes a uniform empirical (resp. quantile) process based upon a sample of n uniformly distributed on $(0, 1)$ observations. Here, we set $\|f\| := \sup_{0 \leq t \leq 1} |f(t)|$ for the sup-norm of a bounded function f on $[0, 1]$. In this paper, we provide a multivariate extension of this result. We consider $d \geq 1$ pairs of empirical and quantile processes $\{\alpha_{n;j}, \beta_{n;j}\}$, $j = 1, \ldots, d$ and focus our interest in the case where these d pairs are mutually independent, corresponding to the situation where each individual pair is generated by an independent sequence of i.i.d. uniform $(0, 1)$ random variables. Setting $\mathbf{t} = (t_1, \ldots, t_d)$, we establish limit laws (as $n \to \infty$) for statistics of the form

$\sup_{\mathbf{t}\in[0,1]^d} \left|\sum_{j=1}^{d} \Psi_j(\mathbf{t})\{\alpha_{n;j}(t_j) + \beta_{n,j}(t_j)\}\right|$, where $\Psi_1, \ldots, \Psi_d$ are suitable continuous functions on $[0,1]^d$. Besides providing some extensions of the classical Bahadur-Kiefer representation, these results allow us to obtain optimal rates of strong approximation of empirical copula processes by sequences of Gaussian processes.

Markov Chain Coupling for Stochastic Domination of Order Statistics

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For the order statistics $(X(1:n), X(2:n), \cdots, X(n:n))$ of a collection of independent, not necessarily identically distributed random variables and for any $i \in [n]$, the conditional distribution $(X(i+1:n), \cdots, X(n:n) \mid X(i:n) > s)$ is shown to be stochastically increasing in s using a coupling of Markov chains. (Joint work with Olle Häggström.)

Uniform in bandwidth consistency of kernel regression estimators at a fixed point

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We show that the empirical process approach developed by Einmahl and Mason (2000, 2005) for proving uniform consistency results for kernel regression function estimators on compact sets can be adapted so as to also give optimal results for pointwise convergence. Our results are uniform in bandwidth and uniform over certain function classes. As in the previous work, we need good exponential deviation and moment inequalities for general empirical processes. As we are dealing with the pointwise convergence of such estimators it is sufficient to use a Bernstein type exponential inequality in terms of the strong second moments which is due to Yurinskii (1976) rather than the more elaborate inequality of Talagrand (1994) in terms of the weak second moments. The moment inequality we need seems to be new and might be of independent interest. Combining these tools we can obtain results with optimal convergence rates for function classes having an envelope function with finite moment generating function. This is different from the corresponding results on uniform convergence rates over compact sets where the function classes had to be bounded. One might wonder whether one can extend these results to the finite moment generating function case as well. We would be able to answer this question in the affirmative if we had a Bernstein type inequality for unbounded function classes in terms of the weak second moments. (This is joint work with Julia Dony, Free University of Brussels (VUB).)

A limit theorem for the distribution of the absolute deviation of linear wavelet density estimators

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The Smirnov-Bickel-Rosenblatt limit theorem for the sup-norm deviation of a convolution kernel density estimator is extended to some wavelet density estimators. This is joint work with Richard Nickl.

Optimal Rates in the Multivariate Central Limit Theorem for Balls

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We discuss the connections between asymptotic approximations of quadratic forms with generalized χ^2-limits in the multivariate central limit theorem with classical lattice point counting problems of Hardy and Landau. We describe recent optimal approximation bounds, which are valid starting at dimension 5 in the multivariate CLT obtained with A. Zaitsev. This is related to joint work with G. Margulis on the local equidistribution of values of indefinite quadratic forms on lattices.

Oracle Inequalities in Sparse Recovery Problems

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Numerous problems in Statistics and Learning Theory can be reduced to penalized empirical risk minimization over linear spans or convex hulls of large dictionaries of functions. The goal is to recover a sparse approximation of a target function (such as regression functions or Bayes classification rules) based on noisy observations at random locations. Convex complexity penalties are often used in empirical risk minimization to find such a sparse solution and sharp oracle inequalities with error terms that depend on the degree of sparsity of the problem have to be proved. The talk will deal with such inequalities in several problems including ℓ_1-norm penalized empirical risk minimization over linear spans and entropy penalized empirical risk minimization over convex hulls. Talagrand's concentration inequalities and other bounds for empirical and Rademacher processes are among the main tools in these problems.

Limit Theorems for High Dimensional Data

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We establish limit theorems for high dimensional data that is characterized by small sample sizes relative to the dimension of the data. In particular, we provide an infinite-dimensional framework to study statistical models that involve situations in which (i) the number of parameters increase with the sample size (that is allowed to be random) and (ii) there is a possibility of missing data. Under a variety of tail conditions on the components of the data, conditions for the law of large numbers, as well as various results concerning the rate of convergence in these models are obtained. We also present central limit theorems in this setting, some which involve data driven coordinate-wise normalizations.

Estimates of moments and tails for some multidimensional chaoses

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We present two sided estimates on moments and tails of random variables of the form

$$\sum a_{i_1,\ldots,i_d} X_{i_1} \cdots X_{i_d},$$

where X_i are independent symmetric random variables with logarithmically concave tails and $d \leq 3$. For $d > 3$ we are able so far to derive upper bounds only in the special cases (including exponential and Gaussian random variables). Estimates are exact up to constants depending on d only. As a tool we show a new bound for suprema of certain empirical processes.
The talk is based on joint work with Radoslaw Adamczak.

A Gaussian Inequality for Absolute Value of Products

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We will discuss the inequalities

$$E|X_1X_2\cdots X_n| \le \sqrt{\mathrm{perm}\Sigma} \le (EX_1^2X_2^2\cdots X_n^2)^{1/2}$$

for any centered Gaussian random variables $X_1, \cdots, X_n$ with the covariance matrix Σ. The first inequality is due to the speaker and the second inequality is due to Frenkel (2008). Various implications, examples, applications and conjectures will also be presented. This is a joint work with Ang Wei.

On the behavior of random matrices with independent columns

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The talk is based on joint works with R. Adamczak, O. Guédon, A. Pajor, and N. Tomczak-Jaegermann. We discuss behavior of several parameters of a random $n \times N$ matrix A, whose columns are independent random vectors in $\mathbf{R}^n$ satisfying some natural conditions. In particular, we obtain estimates for the spectral norm of A (i.e. the largest singular value of A or, equivalently, the operator norm $\|A : \ell_2^N \to \ell_2^n\|$); the smallest singular value; the norm of A on the set of all m-sparse vectors (i.e. vectors having at most m nonzero coordinates), which is denoted by A_m. Our estimates hold with overwhelming probability, that is, the probability tending to one as the dimension grows to infinity. In particular, we obtain that for isotropic log-concave i.i.d. random vectors X_i's

$$\mathrm{Prob}\left(\exists m \le N \,:\, A_m \ge C\left(\sqrt{n} + \sqrt{m}\log\frac{2N}{m}\right)\right) \le \exp\left(-c\sqrt{n}\right),$$

where c and C are absolute positive constants. Note here that $A_N = \|A\|$.

We apply our results to solve several problems. First, we provide asymptotically sharp answer to the question posed by R. Kannan, L. Lovász, M. Simonovits: *Let K be an isotropic convex body in $\mathbf{R}^n$. Given $\varepsilon > 0$, how many independent points X_i uniformly distributed on K are needed for the empirical covariance matrix to approximate the identity up to ε with overwhelming probability?* Namely, we show that it is enough to take $N \approx C(\varepsilon)n$ vectors. Then we turn to applications to compressed sensing and convex geometry. We investigate RIP (Restricted Isometry Property) of random matrices with independent columns and show that the matrix A, considered above, satisfies RIP. Thus, as was shown in works of E. Candes and T. Tao, and D. L. Donoho, such a matrix can be used to solve exact reconstruction process of m-sparse vectors via ℓ_1 minimization as well as to construct neighborly polytopes.

Concentration of measure and mixing for Markov chains.

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We discuss certain Markovian models on graphs with local dynamics. We show that, under suitable conditions, such Markov chains exhibit strong concentration of measure over long time intervals. Further, with additional assumptions, we also have both rapid convergence to equilibrium and strong concentration of measure in the stationary distribution.

A high dimensional Wilks phenomenon

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A theorem by Wilks asserts that in smooth parametric density estimation the difference between the maximum likelihood and the likelihood of the sampling distribution converges toward a chi-square distribution where the number of degrees of freedom coincides with the model dimension. This observation is at the core of some goodness-of-fit testing procedures and of some classical model selection methods. This paper describes a non-asymptotic version of the Wilks phenomenon in bounded contrast optimization procedures. Using concentration inequalities for general functions of independent random variables, it proves that in bounded contrast minimization (as for example in Statistical Learning Theory), the difference between the empirical risk of the minimizer of the true risk in the model and the minimum of the empirical risk (the excess empirical risk) satisfies a Bernstein-like inequality where the variance term reflects the dimension of the model and the scale term reflects the noise conditions. From a mathematical statistics viewpoint, the significance of this result comes from the recent observation that when using model selection via penalization, the excess empirical risk represents a minimum penalty if non-asymptotic guarantees concerning prediction error are to be provided. From the perspective of empirical process theory, this paper describes a concentration inequality for the supremum of a bounded non-centered (actually non-positive) empirical process. Combining the now classical analysis of M-estimation (building on Talagrand's inequality for suprema of empirical processes) and versatile moment inequalities for functions of independent random variables, this paper develops a genuine Bernstein-like inequality that seems beyond the reach of traditional tools.

Inequalities for Self-Bounding Random Variables

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The talk applies inequalities for self-bounding random variables to variance related objects. I give a result on the concentration of the empirical variance of a sample of independent, bounded variables, and show how it can be used to give tight empirical versions of Bernstein's inequality. A related result concerns the eigenvalues of the normalized Gramian generated by n independently drawn datapoints in a high-dimensional ball. Here the estimation error for the k-th largest eigenvalue can be bounded with high probability in terms of the largest eigenvalue and a remainder of order n^{-1}.

Quantitative asymptotics of graphical projection pursuit

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In 1984, Diaconis and Freedman proved a limit result stating roughly that, given a large number n of data points in a high dimension d, most one-dimensional projections of the data would look approximately Gaussian. In this talk, I will present a quantitative version of the theorem, in the form of a concentration inequality for the bounded-Lipschitz distance between the empirical distribution of a random projection of the data and a suitably scaled Gaussian distribution. I will also present a multivariate version, considering projections of the data onto subspaces of dimension k. In particular, I will discuss the issue of how k may grow with n and d for this normal-projections phenomenon to persist. The method of proof is by a combination of Stein's method, the concentration of measure phenomenon, and Dudley's entropy bound, and is likely to have many other applications.

Concentration of polynomial functions of random matrices

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In the spirit of results of Guionnet and Zeitouni and of free probability theory, we prove concentration inequalities for noncommutative polynomials of large independent random matrices. This is joint work with S. Szarek.

A Bernstein type inequality and moderate deviations for weakly dependent sequences

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In this talk I shall present a joint work with M. Peligrad and E. Rio, concerning a tail inequality for the maximum of partial sums of a weakly dependent sequence of random variables that is not necessarily bounded. The class considered includes geometrically and subgeometrically strongly mixing sequences. The result is then used to derive asymptotic moderate deviation results. Applications include classes of Markov chains, functions of linear processes with absolutely regular innovations and ARCH models.

Adaptive Confidence Bands in Density Estimation

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Given a sample from some unknown continuous density f $:\to$, we construct fully adaptive estimators for f and prove an exact Smirnov-Bickel-Rosenblatt type limit theorem for it. This allows to obtain adaptive confidence bands which are honest for all densities in a 'generic' subset of the union of t-Hölder balls, $0 < t \leq r$, where r is a fixed but arbitrary integer. The proofs are based on a precise analysis of the stochastic behaviour of certain linear wavelet or kernel density estimators, in particular exponential inequalities and extremal type limit theorems.

Weak vs. strong parameters for vector-valued Rademacher sums

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Weak and strong parameters (moments and tails) of vector-valued Rademacher sums are related by a deviation inequality. Estimates obtained imply asymptotic equality of the optimal constants in the Khinchine and Khinchine-Kahane inequalities. Aslo, they form a counterpart to the classical results of Talagrand about concentration on the discrete cube - the new bounds being of interest when weak parameter is rather large with respect to the strong one. The work is unpublished yet but not new and some parts of it were presented already back in 2005 and 2006; however, the presentation was restricted to the convex geometry circles and I think that both the results and their quite elementary proofs may be of some interest also for people working on stochastic inequalities.

Functional central limit theorem via martingale approximation

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Martingale approximation as a tool to obtain asymptotic results goes back to Gordin and the theory was developed by many mathematicians including Philipp, Kipnis, Varadhan, Hall, Heyde, Maxwell, Woodroofe, Zhao, Wu, Volny, Dedecker, Merlevède, Rio among others. We shall stress the characterization of stochastic processes that can be approximated by martingales for deriving the conditional functional central limit theorem. The results are easily applicable to a variety of examples, leading to a better understanding of the structure of several classes of stochastic processes and their asymptotic behavior. The approximation brings together many disparate examples in probability theory. It is valid for classes of variables defined by familiar projection conditions, various classes of mixing processes including the large class of strong mixing processes and to classes of reversible and normal Markov operators. The main tool in analyzing all these examples are maximal inequalities. Joint work with Mikhail Gordin.

On the Bennett-Hoeffding inequality

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The well-known Bennett-Hoeffding bound for sums of independent random variables is refined, by taking into account truncated third moments, and at that significantly improved by using, instead of the class of all increasing exponential functions, the much larger class of all generalized moment functions f such that f and f'' are increasing and convex. It is shown that the resulting bounds have certain optimality properties. Comparisons with related known bounds are given. The results can be extended in a standard manner to (the maximal functions of) (super)martingales. The proof of the main result is much more difficult than those of the previous results; it uses an apparently new method that may be referred to as infinitesimal spin-off.

Estimation of convex-transformed densities

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A convex-transformed density is a quasi-concave (or a quasi-convex) density which is a composition of monotone and convex functions. We consider nonparametric estimation in a scale of such families of densities on R^d indexed by a real parameter s. The value $s = 0$ corresponds to log-concave densities, while values of $s \neq 0$ correspond to heavier tailed densities or densities concentrated on particular subsets of R^d according as $s \leq 0$ or $s > 0$. Many parametric and non-parametric families of densities can be included in a suitable family of convex-transformed densities: normal, gamma, beta, Gumbel and other log-concave densities, multivariate Pareto, Burr, Student t, Snedecor etc. We study the properties of nonparametric estimation in these classes of convex-transformed densities, including existence and consistency of the maximum likelihood estimator, and asymptotic minimax lower bounds for estimation.

Stochastic Limit Theorems with Deterministic Analogs, Stationary Analogs, or No Analogs

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If you choose n points at random in the unit square, the classic theorem of Beardwood, Halton, and Hammersley tells you that the length of the shortest tour through these points is asymptotic to a constant times the square root of n. Now, consider the purely deterministic case where for each n we look at the worst case point configuration. Again, we get a sequence of lengths that are asymptotic to the square root of n. We consider several examples of such analog pairs, and also some cases where the analogy fails. In particular, we consider some instructive instances of failure where independent samples are replaced by sequences from a stationary ergodic process. Naturally, there are links with large deviation inequalities, Orlicz norm bounds, discrepancy theory, and empirical processes.

Vector-valued tangent sequences and decoupling

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The class of UMD spaces was extensively studied by Burkholder in the eighties. It is the right setting for vector-valued harmonic analysis and stochastic integration theory as developed in that same period. In more recent years progress on research in PDEs and harmonic analysis as well as progress in stochastic integration and SPDEs incited new interest in UMD spaces (see Kunstmann and Weis (2004) and references therein, and Neerven, Veraar and Weis (2007-now)). More precisely, decoupling inequalities related to UMD spaces proved to be useful. This talk focusses on such inequalities.
Hitczenko (1989) and McConnell (1989) proved decoupling inequalities for tangent martingale differences with values in a UMD space. Cox and Veraar (2007) considered a one sided version of the decoupling inequalities and showed that it also holds for L^1-spaces (which are not UMD). This inequality can be interpreted as

a probabilistic Banach space property, which we refer to as *the decoupling property*. In the talk we present some recent results such as p-independence of the property and constants, and we give examples of other spaces with the decoupling property.
We dicuss several open problems, explaining their importance to harmonic and stochastic analysis.

A Comparison of Three Methods for Bounding Moments of Sums

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Moment inequalities for sums of independent random vectors are important tools for statistical research. Nemirovski and coworkers (1983, 2000) and Pinelis (1994) derived one particular type of such inequalities: For certain Banach spaces $(\mathbb{B}, \|\cdot\|)$ there exists a constant $K = K(\mathbb{B}, \|\cdot\|)$ such that for arbitrary independent and centered random vectors $X_1, X_2, ..., X_n \in \mathbb{B}$, their sum S_n satisfies the inequality $E\|S_n\|^2 \leq K \sum_{i=1}^n E\|X_i\|^2$. We present and compare three different approaches to obtain such inequalities: The results of Nemirovski and Pinelis are based on deterministic inequalities for norms. Another possible vehicle are type and cotype inequalities, a tool from probability theory on Banach spaces. Finally, we use a truncation argument plus Bernstein's inequality to obtain another version of the moment inequality above. Interestingly, all three approaches have their own merits. (Talk based on joint work with Lutz Dümbgen, Sara van der Geer, and Mark Veraar.)

An Approach to the Gaussian Correlation Conjecture

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From the original question by Dunnett and Sobel (1955) to the present formulation given by Das Gupta, Eaton, Olkin, Perlman, Savage and Sobel (1970), many specific cases of the Gaussian Correlation Inequality have been proved. The methods of proof are quite varied. For example, Zbyněk ˇSidák (1967) used mainly algebraic and calculus methods to obtain the case when one of the sets is a symmetric strip. Loren Pitt (1977) used a geometric approach together with of a special case of what is often called the Gradient Conjecture to prove the conjecture in dimension 2. Gilles Hargé (1999) uses a semigroup and Dario Cordero-Erausquin (2002) uses a Mass Transport approach via a Theorem of Caffarelli (2000). Our approach is inductive.

Brunn-Minkowski type inequalities for Gaussian Measure.

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In this talk we will present a joint work with Richard Gardner. We will discuss the Brunn-Minkowski type inequalities for Gaussian Measure in R^n. The best-known of these are EhrhardÕs inequality, and the weaker logarithmic concavity inequality. We obtain some results concerning other inequalities of this type, as well as a best-possible dual Gaussian Brunn-Minkowski inequality (where the Minkowski sum is replaced by radial sum).