Viability in Fishery Management

Michel De Lara[1] Pedro Gajardo[2] Héctor Ramírez C.[3]

[1]ENPC & CERMICS, Paris, France
[2]Depto. Matemáticas, Universidad Técnica Federico Sta. María, Valparaíso, Chile
[3]DIM & CMM, Universidad de Chile, Santiago, Chile

Mathematics and the Environment
May 8–13, 2007 - Banff, Canada

Outline

Outline

Discrete Time Control System

Let us consider a nonlinear control system described in discrete time by the difference equation

$$\begin{cases} x_{t+1} = g(x_t, u_t), & \forall t \in \mathbb{N}, \\ x_0 & \text{given}, \end{cases}$$

where

- The state variable x_t belongs to the state space $\mathbb{X} \subseteq \mathbb{R}^{n_{\mathbb{X}}}$.
- The control variable u_t is an element of the control set $\mathbb{U} \subseteq \mathbb{R}^{n_{\mathbb{U}}}$.
- The dynamics g maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

Discrete Time Control System

Let us consider a nonlinear control system described in discrete time by the difference equation

$$\begin{cases} x_{t+1} = g(x_t, u_t), & \forall t \in \mathbb{N}, \\ x_0 & \text{given}, \end{cases}$$

where

- The state variable x_t belongs to the state space $\mathbb{X} \subseteq \mathbb{R}^{n_{\mathbb{X}}}$.
- The control variable u_t is an element of the control set $\mathbb{U} \subseteq \mathbb{R}^{n_{\mathbb{U}}}$.
- The dynamics g maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

Discrete Time Control System

Let us consider a nonlinear control system described in discrete time by the difference equation

$$\begin{cases} x_{t+1} = g(x_t, u_t), & \forall t \in \mathbb{N}, \\ x_0 \quad \text{given}, \end{cases}$$

where

- The state variable x_t belongs to the state space $\mathbb{X} \subseteq \mathbb{R}^{n_{\mathbb{X}}}$.
- The control variable u_t is an element of the control set $\mathbb{U} \subseteq \mathbb{R}^{n_{\mathbb{U}}}$.
- The dynamics g maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

Discrete Time Control System

Let us consider a nonlinear control system described in discrete time by the difference equation

$$\begin{cases} x_{t+1} = g(x_t, u_t), & \forall t \in \mathbb{N}, \\ x_0 & \text{given}, \end{cases}$$

where

- The state variable x_t belongs to the state space $\mathbb{X} \subseteq \mathbb{R}^{n_{\mathbb{X}}}$.
- The control variable u_t is an element of the control set $\mathbb{U} \subseteq \mathbb{R}^{n_{\mathbb{U}}}$.
- The dynamics g maps $\mathbb{X} \times \mathbb{U}$ into $\mathbb{X}$.

Desirable Configurations

A decision maker describes “desirable configurations of the system” through a set $\mathbb{D} \subset \mathbb{X} \times \mathbb{U}$ termed the desirable set

$$(x_t, u_t) \in \mathbb{D}, \quad \forall t \in \mathbb{N},$$

where $\mathbb{D}$ includes both system states and controls constraints.

Example

- $\mathbb{D}_{ecol} := \{(x,u) \,:\, x > 0\}$ or $\mathbb{D}_{ecol} := \{(x,u) \,:\, x \geq \bar{x}\}$
- $\mathbb{D}_{econ} := \{(x,u) \,:\, Y(x,u) \geq y_{\min}\}$
- $\mathbb{D}_{ICES} := \{(x,u) \,:\, SSB(x) \geq B_{ref}, F(u) \leq F_{ref}\}$

Desirable Configurations

A decision maker describes “desirable configurations of the system” through a set $\mathbb{D} \subset \mathbb{X} \times \mathbb{U}$ termed the desirable set

$$(x_t, u_t) \in \mathbb{D}, \quad \forall t \in \mathbb{N},$$

where $\mathbb{D}$ includes both system states and controls constraints.

Example

- $\mathbb{D}_{ecol} := \{(x,u) \ : \ x > 0\}$ *or* $\mathbb{D}_{ecol} := \{(x,u) \ : \ x \geq \bar{x}\}$
- $\mathbb{D}_{econ} := \{(x,u) \ : \ Y(x,u) \geq y_{min}\}$
- $\mathbb{D}_{ICES} := \{(x,u) \ : \ SSB(x) \geq B_{ref}, F(u) \leq F_{ref}\}$

Desirable Configurations

A decision maker describes "desirable configurations of the system" through a set $\mathbb{D} \subset \mathbb{X} \times \mathbb{U}$ termed the desirable set

$$(x_t, u_t) \in \mathbb{D}, \quad \forall t \in \mathbb{N},$$

where $\mathbb{D}$ includes both system states and controls constraints.

Example

- $\mathbb{D}_{ecol} := \{(x,u) \ : \ x > 0\}$ *or* $\mathbb{D}_{ecol} := \{(x,u) \ : \ x \geq \bar{x}\}$
- $\mathbb{D}_{econ} := \{(x,u) \ : \ Y(x,u) \geq y_{min}\}$
- $\mathbb{D}_{ICES} := \{(x,u) \ : \ SSB(x) \geq B_{ref}, F(u) \leq F_{ref}\}$

Desirable Configurations

A decision maker describes “desirable configurations of the system” through a set $\mathbb{D} \subset \mathbb{X} \times \mathbb{U}$ termed the desirable set

$$(x_t, u_t) \in \mathbb{D}, \quad \forall t \in \mathbb{N},$$

where $\mathbb{D}$ includes both system states and controls constraints.

Example

- $\mathbb{D}_{ecol} := \{(x,u) \ : \ x > 0\}$ *or* $\mathbb{D}_{ecol} := \{(x,u) \ : \ x \geq \bar{x}\}$
- $\mathbb{D}_{econ} := \{(x,u) \ : \ Y(x,u) \geq y_{min}\}$
- $\mathbb{D}_{ICES} := \{(x,u) \ : \ SSB(x) \geq B_{ref}, F(u) \leq F_{ref}\}$

Viability Domains and Viability Kernel

Definition

- $\mathbb{V} \subset \mathbb{X}$ *is a Viability Domain if for all* $x \in \mathbb{V}$ *there exists* $u \in \mathbb{U}$ *such that* $(x,u) \in \mathbb{D}$ *and* $g(x,u) \in \mathbb{V}$.
- *Viability Kernel*

$$\mathbb{V}(g,\mathbb{D}) = \{x_0 \in \mathbb{X} \; : \; \textit{there exist} \;\; u_0, u_1, u_2, \ldots, \;\; x_1, x_2, \ldots \;\; \textit{such that } x_{t+1} = g(x_t,u_t) \;\; \textit{and} \;\; (x_t,u_t) \in \mathbb{D}\}.$$

Goals

Viability Domains and Viability Kernel

Definition

- $\mathbb{V} \subset \mathbb{X}$ *is a Viability Domain if for all* $x \in \mathbb{V}$ *there exists* $u \in \mathbb{U}$ *such that* $(x, u) \in \mathbb{D}$ *and* $g(x, u) \in \mathbb{V}$.
- *Viability Kernel*

$$\mathbb{V}(g, \mathbb{D}) = \{x_0 \in \mathbb{X} \ : \ \textit{there exist} \ \ u_0, u_1, u_2, \ldots, \ \ x_1, x_2, \ldots \\ \textit{such that } x_{t+1} = g(x_t, u_t) \ \textit{ and } \ (x_t, u_t) \in \mathbb{D}\}.$$

Goals

- *Determine or approximate the viability kernel* $\mathbb{V}(g, \mathbb{D})$ *for a given dynamics* g *and a given desirable set* $\mathbb{D}$.
- *Determine when a given set* $\mathbb{V}$ *is a viability domain.*

Viability Domains and Viability Kernel

Definition

- *$\mathbb{V} \subset \mathbb{X}$ is a Viability Domain if for all $x \in \mathbb{V}$ there exists $u \in \mathbb{U}$ such that $(x, u) \in \mathbb{D}$ and $g(x, u) \in \mathbb{V}$.*
- *Viability Kernel*

$$\mathbb{V}(g, \mathbb{D}) = \{x_0 \in \mathbb{X} \ : \ \textit{there exist} \ \ u_0, u_1, u_2, \ldots, \ \ x_1, x_2, \ldots \\ \textit{such that } x_{t+1} = g(x_t, u_t) \ \ \textit{and} \ \ (x_t, u_t) \in \mathbb{D}\}.$$

Goals

- *Determine or approximate the viability kernel $\mathbb{V}(g, \mathbb{D})$ for a given dynamics g and a given desirable set $\mathbb{D}$.*
- *Determine when a given set $\mathbb{V}$ is a viability domain.*

Viability Domains and Viability Kernel

Definition

- $\mathbb{V} \subset \mathbb{X}$ *is a Viability Domain if for all* $x \in \mathbb{V}$ *there exists* $u \in \mathbb{U}$ *such that* $(x, u) \in \mathbb{D}$ *and* $g(x, u) \in \mathbb{V}$.
- *Viability Kernel*

$$\mathbb{V}(g, \mathbb{D}) = \{x_0 \in \mathbb{X} \ : \ \text{there exist} \ \ u_0, u_1, u_2, \ldots, \ \ x_1, x_2, \ldots \\ \text{such that } x_{t+1} = g(x_t, u_t) \ \ \text{and} \ \ (x_t, u_t) \in \mathbb{D}\}.$$

Goals

- *Determine or approximate the viability kernel* $\mathbb{V}(g, \mathbb{D})$ *for a given dynamics* g *and a given desirable set* $\mathbb{D}$.
- *Determine when a given set* $\mathbb{V}$ *is a viability domain.*

Viability Domains and Viability Kernel

Definition

- $\mathbb{V} \subset \mathbb{X}$ *is a Viability Domain if for all* $x \in \mathbb{V}$ *there exists* $u \in \mathbb{U}$ *such that* $(x, u) \in \mathbb{D}$ *and* $g(x, u) \in \mathbb{V}$.
- *Viability Kernel*

$$\mathbb{V}(g, \mathbb{D}) = \{x_0 \in \mathbb{X} \ : \ \textit{there exist} \ \ u_0, u_1, u_2, \ldots, \ x_1, x_2, \ldots \\ \textit{such that } x_{t+1} = g(x_t, u_t) \ \ \textit{and} \ \ (x_t, u_t) \in \mathbb{D}\}.$$

Goals

- *Determine or approximate the viability kernel* $\mathbb{V}(g, \mathbb{D})$ *for a given dynamics* g *and a given desirable set* $\mathbb{D}$.
- *Determine when a given set* $\mathbb{V}$ *is a viability domain.*

Viability Domains and Viability Kernel

The state constraints set associated with $\mathbb{D}$ is obtained by projecting the desirable set $\mathbb{D}$ onto the state space $\mathbb{X}$:

$$\mathbb{V}^0 := \mathrm{Proj}_{\mathbb{X}}(\mathbb{D}) = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}\,,\ (x,u) \in \mathbb{D}\}.$$

By definition $\mathbb{V}(g,\mathbb{D}) \subset \mathbb{V}^0$.

Moreover, the viability kernel $\mathbb{V}(g,\mathbb{D})$ turns out to be the union of all viability domains, that is:

$$\mathbb{V}(g,\mathbb{D}) = \bigcup \left\{ \mathbb{V} \ :\ \mathbb{V} \subset \mathbb{V}^0,\ \mathbb{V} \text{ viability domain for } g \text{ in } \mathbb{D} \right\}.$$

Viability Domains and Viability Kernel

The state constraints set associated with $\mathbb{D}$ is obtained by projecting the desirable set $\mathbb{D}$ onto the state space $\mathbb{X}$:

$$\mathbb{V}^0 := \text{Proj}_{\mathbb{X}}(\mathbb{D}) = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}\,,\ (x,u) \in \mathbb{D}\}.$$

By definition $\mathbb{V}(g,\mathbb{D}) \subset \mathbb{V}^0$.

Moreover, the viability kernel $\mathbb{V}(g,\mathbb{D})$ turns out to be the union of all viability domains, that is:

$$\mathbb{V}(g,\mathbb{D}) = \bigcup \left\{ \mathbb{V} \ :\ \mathbb{V} \subset \mathbb{V}^0,\ \mathbb{V} \text{ viability domain for } g \text{ in } \mathbb{D} \right\}.$$

Viability Domains and Viability Kernel

The state constraints set associated with $\mathbb{D}$ is obtained by projecting the desirable set $\mathbb{D}$ onto the state space $\mathbb{X}$:

$$\mathbb{V}^0 := \mathrm{Proj}_{\mathbb{X}}(\mathbb{D}) = \{x \in \mathbb{X} \mid \exists u \in \mathbb{U}\,,\ (x,u) \in \mathbb{D}\}.$$

By definition $\mathbb{V}(g,\mathbb{D}) \subset \mathbb{V}^0$.

Moreover, the viability kernel $\mathbb{V}(g,\mathbb{D})$ turns out to be the union of all viability domains, that is:

$$\mathbb{V}(g,\mathbb{D}) = \bigcup \Big\{ \mathbb{V} \ :\ \mathbb{V} \subset \mathbb{V}^0,\ \mathbb{V} \text{ viability domain for } g \text{ in } \mathbb{D} \Big\}.$$

Monotonicity Properties on the Sets

Definition

- We say that a set $S \subset \mathbb{X}$ is increasing if it satisfies:

$$\forall x \in S\,, \quad \forall x' \in \mathbb{X}\,, \quad x' \geq x \Rightarrow x' \in S.$$

That is $S + \mathbb{R}_+^{n_\mathbb{X}} \subseteq S$.

- We say that a set $K \subset \mathbb{X} \times \mathbb{U}$ is increasing if it satisfies:

$$\forall (x,u) \in K\,, \quad \forall x' \in \mathbb{X}\,, \quad x' \geq x \Rightarrow (x',u) \in K.$$

That is $K + \mathbb{R}_+^{n_\mathbb{X}} \times \{0_{\mathbb{R}^{n_\mathbb{U}}}\} \subset K$.
(state and control do not play the same role)

Monotonicity Properties on the Sets

Definition

- We say that a set $S \subset \mathbb{X}$ is increasing if it satisfies:

$$\forall x \in S \,, \quad \forall x' \in \mathbb{X} \,, \quad x' \geq x \Rightarrow x' \in S .$$

  That is $S + \mathbb{R}_+^{n_\mathbb{X}} \subseteq S$.

- We say that a set $K \subset \mathbb{X} \times \mathbb{U}$ is increasing if it satisfies:

$$\forall (x,u) \in K \,, \quad \forall x' \in \mathbb{X} \,, \quad x' \geq x \Rightarrow (x',u) \in K .$$

  That is $K + \mathbb{R}_+^{n_\mathbb{X}} \times \{0_{\mathbb{R}^{n_\mathbb{U}}}\} \subset K$.
  (state and control do not play the same role)

Monotonicity Properties on the Dynamics

Definition

We say that the dynamics $g: \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is increasing with respect to the state if it satisfies

$$\forall (x, x', u) \in \mathbb{X} \times \mathbb{X} \times \mathbb{U}, \quad x' \geq x \Rightarrow g(x', u) \geq g(x, u),$$

and is decreasing with respect to the control if

$$\forall (x, u, u') \in \mathbb{X} \times \mathbb{U} \times \mathbb{U}, \quad u' \geq u \Rightarrow g(x, u') \leq g(x, u).$$

Monotonicity Properties on the Dynamics

Definition

We say that the dynamics $g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is increasing with respect to the state if it satisfies

$$\forall (x, x', u) \in \mathbb{X} \times \mathbb{X} \times \mathbb{U}\,, \quad x' \geq x \Rightarrow g(x', u) \geq g(x, u),$$

and is decreasing with respect to the control if

$$\forall (x, u, u') \in \mathbb{X} \times \mathbb{U} \times \mathbb{U}\,, \quad u' \geq u \Rightarrow g(x, u') \leq g(x, u).$$

Bioeconomics Dynamics

Definition

We say that $g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is a bioeconomics dynamics if g is increasing w.r.t the state and decreasing w.r.t. the control.

Definition

We say that $g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is a bioeconomics quasi-linear dynamics if

$$g(x, u) = G(u)x + H(u),$$

where $G(u)$ is a $n_{\mathbb{X}} \times n_{\mathbb{X}}$ matrix and $H(u) \in \mathbb{R}^{n_{\mathbb{X}}}$ for all $u \in \mathbb{U}$.

Bioeconomics Dynamics

Definition

We say that $g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is a bioeconomics dynamics if g is increasing w.r.t the state and decreasing w.r.t. the control.

Definition

We say that $g : \mathbb{X} \times \mathbb{U} \to \mathbb{X}$ is a bioeconomics quasi-linear dynamics if

$$g(x,u) = G(u)x + H(u),$$

where $G(u)$ is a $n_{\mathbb{X}} \times n_{\mathbb{X}}$ matrix and $H(u) \in \mathbb{R}^{n_{\mathbb{X}}}$ for all $u \in \mathbb{U}$.

Production and Preservation Desirable sets

Definition

A desirable set $\mathbb{D}$ is said to be a production desirable set if $\mathbb{D}$ is increasing w.r.t. both the state and to the control, that is

$$\forall\, u,\ u' \in \mathbb{U},\ x,\ x' \in \mathbb{X} \text{ s.t. } x' \geq x,\ u' \geq u$$
$$\text{if } (x,u) \in \mathbb{D} \text{ then } (x',u') \in \mathbb{D}.$$

Example

$$\mathbb{D}_{\text{yield}} = \{(x,u) \mid Y(x,u) \geq y_{\min}\},$$

where $Y : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ is increasing w.r.t. both variables (state and control).

Production and Preservation Desirable sets

Definition

A desirable set $\mathbb{D}$ is said to be a production desirable set if $\mathbb{D}$ is increasing w.r.t. both the state and to the control, that is

$$\forall u,\ u' \in \mathbb{U},\ x,\ x' \in \mathbb{X} \text{ s.t. } x' \geq x,\ u' \geq u$$
$$\text{if } (x,u) \in \mathbb{D} \text{ then } (x',u') \in \mathbb{D}.$$

Example

$$\mathbb{D}_{\text{yield}} = \{(x,u) \mid Y(x,u) \geq y_{\text{min}}\},$$

where $Y : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ is increasing w.r.t. both variables (state and control).

Production and Preservation Desirable Sets

Definition

A desirable set $\mathbb{D}$ *is said to be* ***a preservation desirable set*** *if* $\mathbb{D}$ *is increasing w.r.t. the state and decreasing w.r.t. the control:*

$$\forall u,\ u' \in \mathbb{U},\ x,\ x' \in \mathbb{X} \ s.t.\ x' \geq x,\ u' \leq u$$
$$if\ (x,u) \in \mathbb{D}\ then\ (x',u') \in \mathbb{D}.$$

Example

$$\mathbb{D}_{\text{protect}} = \{(x,u) \in \mathbb{X} \times \mathbb{U} \mid D(x,u) \geq d^\flat\},$$

where $D : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ *is increasing w.r.t. the state but decreasing w.r.t. the control.*

Production and Preservation Desirable Sets

Definition

A desirable set $\mathbb{D}$ is said to be a preservation desirable set if $\mathbb{D}$ is increasing w.r.t. the state and decreasing w.r.t. the control:

$$\forall u,\ u' \in \mathbb{U},\ x,\ x' \in \mathbb{X} \text{ s.t. } x' \geq x,\ u' \leq u$$
$$\text{if } (x,u) \in \mathbb{D} \text{ then } (x',u') \in \mathbb{D}.$$

Example

$$\mathbb{D}_{\text{protect}} = \{(x,u) \in \mathbb{X} \times \mathbb{U} \mid D(x,u) \geq d^\flat\},$$

where $D : \mathbb{X} \times \mathbb{U} \longrightarrow \mathbb{R}$ is increasing w.r.t. the state but decreasing w.r.t. the control.

Viability Kernels Estimates

Assuming they exist, denote $\mathbb{U}$, i.e. $u^\flat \leq u \leq u^\sharp$ for all $u \in \mathbb{U}$.

For every $t \geq 0$, define recursively the function $(g^\flat)^t : \mathbb{X} \longrightarrow \mathbb{X}$ by

$$\begin{cases} (g^\flat)^0(x) & := & x, \\ (g^\flat)^{t+1}(x) & := & g((g^\flat)^t(x), u^\flat), \quad t \in \mathbb{N}. \end{cases}$$

Proposition

Suppose that g is a bioeconomics dynamics, and consider the desirable sets $\mathbb{D}_{\text{yield}}$ and $\mathbb{D}_{\text{protect}}$. Then we have:

- *If $u^\sharp$ and $u^\flat$ exist, then* $\mathbb{V}(g, \mathbb{D}_{\text{yield}}) \subseteq \bigcap_{t \geq 0} \{x \in \mathbb{X} : Y((g^\flat)^t(x), u^\sharp) \geq y_{\min}\}$.
- *If $u^\flat$ exists, then* $\mathbb{V}(g, \mathbb{D}_{\text{protect}}) = \bigcap_{t \geq 0} \{x \in \mathbb{X} : D((g^\flat)^t(x), u^\flat) \geq d^\flat\}$.

Viability Kernels Estimates

Assuming they exist, denote $\mathbb{U}$, i.e. $u^\flat \leq u \leq u^\sharp$ for all $u \in \mathbb{U}$.

For every $t \geq 0$, define recursively the function $(g^\flat)^t : \mathbb{X} \longrightarrow \mathbb{X}$ by

$$\begin{cases} (g^\flat)^0(x) & := & x, \\ (g^\flat)^{t+1}(x) & := & g((g^\flat)^t(x), u^\flat), \quad t \in \mathbb{N}. \end{cases}$$

Proposition

Suppose that g is a bioeconomics dynamics, and consider the desirable sets $\mathbb{D}_{\text{yield}}$ and $\mathbb{D}_{\text{protect}}$. Then we have:

- *If $u^\sharp$ and $u^\flat$ exist, then*
  $\mathbb{V}(g, \mathbb{D}_{\text{yield}}) \subseteq \bigcap_{t \geq 0} \{x \in \mathbb{X} : Y((g^\flat)^t(x), u^\sharp) \geq y_{\min}\}.$
- *If $u^\flat$ exists, then*
  $\mathbb{V}(g, \mathbb{D}_{\text{protect}}) = \bigcap_{t \geq 0} \{x \in \mathbb{X} : D((g^\flat)^t(x), u^\flat) \geq d^\flat\}.$

Viability Kernels Estimates

Assuming they exist, denote $\mathbb{U}$, i.e. $u^\flat \leq u \leq u^\sharp$ for all $u \in \mathbb{U}$.

For every $t \geq 0$, define recursively the function $(g^\flat)^t : \mathbb{X} \longrightarrow \mathbb{X}$ by

$$\begin{cases} (g^\flat)^0(x) & := & x, \\ (g^\flat)^{t+1}(x) & := & g((g^\flat)^t(x), u^\flat), \quad t \in \mathbb{N}. \end{cases}$$

Proposition

Suppose that g is a bioeconomics dynamics, and consider the desirable sets $\mathbb{D}_{\text{yield}}$ and $\mathbb{D}_{\text{protect}}$. Then we have:

- *If $u^\sharp$ and $u^\flat$ exist, then*
  $\mathbb{V}(g, \mathbb{D}_{\text{yield}}) \subseteq \bigcap_{t \geq 0} \{x \in \mathbb{X} : Y((g^\flat)^t(x), u^\sharp) \geq y_{\min}\}$.
- *If $u^\flat$ exists, then*
  $\mathbb{V}(g, \mathbb{D}_{\text{protect}}) = \bigcap_{t \geq 0} \{x \in \mathbb{X} : D((g^\flat)^t(x), u^\flat) \geq d^\flat\}$.

Convexity of Viability Kernel

Proposition

If the dynamics g is bioeconomic quasi-linear and if $\mathbb{D}$ is a preservation desirable set which is convex w.r.t. the state, that is

$$\text{for all } u \in \mathbb{U},\ x,\ x' \in \mathbb{X} \text{ such that } (x,u),\ (x',u) \in \mathbb{D},$$
$$\text{it holds that } (\alpha x + (1-\alpha)x', u) \in \mathbb{D} \text{ for all } \alpha \in [0,1].$$

then the viability kernel $\mathbb{V}(g,\mathbb{D})$ is convex.

Polyhedral Viability Domains

Let g be a bioeconomic quasi-linear dynamics.

Let $\mathbb{D}_{\text{poly}}$ be a preservation desirable set given by

$$\mathbb{D}_{\text{poly}} = \{(x,u) \in \mathbb{X} \times \mathbb{U} \,:\, D(u)x \geq d^\flat\}.$$

Proposition

Let $\tilde{\mathcal{P}}$ be the polyhedron defined by

$$\tilde{\mathcal{P}} = \{x \in \mathbb{X} \,:\, (I - G(u^\flat))x \leq H(u^\flat) \text{ and } D(u^\flat)x \geq d^\flat\}.$$

Then, the set $\{x \geq \bar{x}\} = \bar{x} + \mathbb{R}^{n_X}_+$ is a viability domain iff $\bar{x} \in \tilde{\mathcal{P}}$.

Corollary

If $\bar{x}$ is a desirable equilibrium for $\mathbb{D}_{\text{poly}}$, then $\bar{x} \in \tilde{\mathcal{P}}$ and consequently $\{x \geq \bar{x}\}$ is a viability domain (Gajardo et al. 06).

Polyhedral Viability Domains

Let g be a bioeconomic quasi-linear dynamics.

Let $\mathbb{D}_{\mathsf{poly}}$ be a preservation desirable set given by

$$\mathbb{D}_{\mathsf{poly}} = \{(x, u) \in \mathbb{X} \times \mathbb{U} \,:\, D(u)x \geq d^\flat\}.$$

Proposition

Let $\tilde{\mathcal{P}}$ be the polyhedron defined by

$$\tilde{\mathcal{P}} = \{x \in \mathbb{X} \,:\, (I - G(u^\flat))x \leq H(u^\flat) \text{ and } D(u^\flat)x \geq d^\flat\}.$$

Then, the set $\{x \geq \overline{x}\} = \overline{x} + \mathbb{R}^{n_X}_+$ is a viability domain iff $\overline{x} \in \tilde{\mathcal{P}}$.

Corollary

If $\overline{x}$ is a desirable equilibrium for $\mathbb{D}_{\mathsf{poly}}$, then $\overline{x} \in \tilde{\mathcal{P}}$ and consequently $\{x \geq \overline{x}\}$ is a viability domain (Guilbaud et al.'06).

Polyhedral Viability Domains

Let g be a bioeconomic quasi-linear dynamics.

Let $\mathbb{D}_{\text{poly}}$ be a preservation desirable set given by

$$\mathbb{D}_{\text{poly}} = \{(x,u) \in \mathbb{X} \times \mathbb{U} \,:\, D(u)x \geq d^{\flat}\}.$$

Proposition

Let $\tilde{\mathcal{P}}$ be the polyhedron defined by

$$\tilde{\mathcal{P}} = \{x \in \mathbb{X} \,:\, (I - G(u^{\flat}))x \leq H(u^{\flat}) \text{ and } D(u^{\flat})x \geq d^{\flat}\}.$$

Then, the set $\{x \geq \bar{x}\} = \bar{x} + \mathbb{R}_{+}^{n_{\mathbb{X}}}$ is a viability domain iff $\bar{x} \in \tilde{\mathcal{P}}$.

Corollary

If $\bar{x}$ is a desirable equilibrium for $\mathbb{D}_{\text{poly}}$, then $\bar{x} \in \tilde{\mathcal{P}}$ and consequently $\{x \geq \bar{x}\}$ is a viability domain (Guilbaud et al.'06).

Polyhedral Viability Domains

Let g be a bioeconomic quasi-linear dynamics.

Let $\mathbb{D}_{\text{poly}}$ be a preservation desirable set given by

$$\mathbb{D}_{\text{poly}} = \{(x,u) \in \mathbb{X} \times \mathbb{U} \,:\, D(u)x \geq d^\flat\}.$$

Proposition

Let $\tilde{\mathcal{P}}$ be the polyhedron defined by

$$\tilde{\mathcal{P}} = \{x \in \mathbb{X} \,:\, (I - G(u^\flat))x \leq H(u^\flat) \text{ and } D(u^\flat)x \geq d^\flat\}.$$

Then, the set $\{x \geq \bar{x}\} = \bar{x} + \mathbb{R}_+^{n_\mathbb{X}}$ is a viability domain iff $\bar{x} \in \tilde{\mathcal{P}}$.

Corollary

If $\bar{x}$ is a desirable equilibrium for $\mathbb{D}_{\text{poly}}$, then $\bar{x} \in \tilde{\mathcal{P}}$ and consequently $\{x \geq \bar{x}\}$ is a viability domain (Guilbaud et al.'06).

Outline

Harvested Fish Population Age Structured Model

$$\begin{cases} N^1_{t+1} &= \varphi(SSB(N_t)), \\ N^a_{t+1} &= e^{-(M+\lambda F^{a-1})} N^{a-1}_t, \quad a \in \{2, \ldots, A-1\} \\ N^A_{t+1} &= e^{-(M+\lambda F^{A-1})} N^{A-1}_t + \pi \times e^{-(M+\lambda F^A)} N^A_t \end{cases}$$

- Time index t in years.
- State variable: $N = (N^a)_{a=1,..,A} \in \mathbb{X} = \mathbb{R}^A_+$, denotes the vector of abundances (biomass) at ages a.
- Control variable: $\lambda \in \mathbb{U} = \mathbb{R}_+$, fishing effort or exploitation pattern multiplier.
- φ describes the stock-recruitment relationship.
- The spanning stock biomass function is defined as

$$SSB(N) = \sum_{a=1}^{A} p_a w_a N_a.$$

- $\pi = 0$ ó 1 indicates if $a = A$ is a plus-group.

Harvested Fish Population Age Structured Model

$$\begin{cases} N^1_{t+1} &= \varphi(SSB(N_t)), \\ N^a_{t+1} &= e^{-(M+\lambda F^{a-1})} N^{a-1}_t, \quad a \in \{2, \ldots, A-1\} \\ N^A_{t+1} &= e^{-(M+\lambda F^{A-1})} N^{A-1}_t + \pi \times e^{-(M+\lambda F^A)} N^A_t \end{cases}$$

- Time index t in years.
- State variable: $N = (N^a)_{a=1,..,A} \in \mathbb{X} = \mathbb{R}^A_+$, denotes the vector of abundances (biomass) at ages a.
- Control variable: $\lambda \in \mathbb{U} = \mathbb{R}_+$, fishing effort or exploitation pattern multiplier.
- φ describes the stock-recruitment relationship.
- The spanning stock biomass function is defined as

$$SSB(N) = \sum_{a=1}^{A} p_a w_a N_a.$$

- $\pi = 0$ ó 1 indicates if $a = A$ is a plus-group.

Harvested Fish Population Age Structured Model

$$\left\{\begin{array}{lcl} N^1_{t+1} & = & \varphi(SSB(N_t)), \\ N^a_{t+1} & = & e^{-(M+\lambda F^{a-1})} N^{a-1}_t, \quad a \in \{2,\ldots,A-1\} \\ N^A_{t+1} & = & e^{-(M+\lambda F^{A-1})} N^{A-1}_t + \pi \times e^{-(M+\lambda F^A)} N^A_t \end{array}\right.$$

- Time index t in years.
- State variable: $N = (N^a)_{a=1,..,A} \in \mathbb{X} = \mathbb{R}^A_+$, denotes the vector of abundances (biomass) at ages a.
- Control variable: $\lambda \in \mathbb{U} = \mathbb{R}_+$, fishing effort or exploitation pattern multiplier.
- φ describes the stock-recruitment relationship.
- The spanning stock biomass function is defined as

$$SSB(N) = \sum_{a=1}^{A} p_a w_a N_a .$$

- $\pi = 0$ ó 1 indicates if $a = A$ is a plus-group.

Harvested Fish Population Age Structured Model

$$\begin{cases} N^1_{t+1} &= \varphi(SSB(N_t)), \\ N^a_{t+1} &= e^{-(M+\lambda F^{a-1})} N^{a-1}_t, \quad a \in \{2, \dots, A-1\} \\ N^A_{t+1} &= e^{-(M+\lambda F^{A-1})} N^{A-1}_t + \pi \times e^{-(M+\lambda F^{A})} N^A_t \end{cases}$$

- Time index t in years.
- State variable: $N = (N^a)_{a=1,..,A} \in \mathbb{X} = \mathbb{R}^A_+$, denotes the vector of abundances (biomass) at ages a.
- Control variable: $\lambda \in \mathbb{U} = \mathbb{R}_+$, fishing effort or exploitation pattern multiplier.
- φ describes the stock-recruitment relationship.
- The spanning stock biomass function is defined as

$$SSB(N) = \sum_{a=1}^{A} p_a w_a N_a .$$

- $\pi = 0$ ó 1 indicates if $a = A$ is a plus-group.

Harvested Fish Population Age Structured Model

$$\begin{cases} N_{t+1}^1 & = & \varphi(SSB(N_t)), \\ N_{t+1}^a & = & e^{-(M+\lambda F^{a-1})} N_t^{a-1}, \quad a \in \{2,\ldots,A-1\} \\ N_{t+1}^A & = & e^{-(M+\lambda F^{A-1})} N_t^{A-1} + \pi \times e^{-(M+\lambda F^A)} N_t^A \end{cases}$$

- Time index t in years.
- State variable: $N = (N^a)_{a=1,..,A} \in \mathbb{X} = \mathbb{R}_+^A$, denotes the vector of abundances (biomass) at ages a.
- Control variable: $\lambda \in \mathbb{U} = \mathbb{R}_+$, fishing effort or exploitation pattern multiplier.
- φ describes the stock-recruitment relationship.
- The spanning stock biomass function is defined as

$$SSB(N) = \sum_{a=1}^{A} p_a w_a N_a .$$

- $\pi = 0$ ó 1 indicates if $a = A$ is a plus-group.

Harvested Fish Population Age Structured Model

$$\begin{cases} N^1_{t+1} & = & \varphi(SSB(N_t)), \\ N^a_{t+1} & = & e^{-(M+\lambda F^{a-1})} N^{a-1}_t, \quad a \in \{2, \ldots, A-1\} \\ N^A_{t+1} & = & e^{-(M+\lambda F^{A-1})} N^{A-1}_t + \pi \times e^{-(M+\lambda F^{A})} N^A_t \end{cases}$$

- Time index t in years.
- State variable: $N = (N^a)_{a=1,..,A} \in \mathbb{X} = \mathbb{R}^A_+$, denotes the vector of abundances (biomass) at ages a.
- Control variable: $\lambda \in \mathbb{U} = \mathbb{R}_+$, fishing effort or exploitation pattern multiplier.
- φ describes the stock-recruitment relationship.
- The spanning stock biomass function is defined as

$$SSB(N) = \sum_{a=1}^{A} p_a w_a N_a .$$

- $\pi = 0$ ó 1 indicates if $a = A$ is a plus-group.

Harvested Fish Population Age Structured Model

Example

Possibles stock-recruitment relationship:

- *Constant:* $\varphi(B) = \alpha$.
- *Linear:* $\varphi(B) = \alpha B$.
- *Beverton-Holt:* $\varphi(B) = \frac{B}{\alpha + \beta B}$.
- *Ricker:* $\varphi(B) = \alpha B e^{-\beta B}$.

Harvested Fish Population Age Structured Model

Example

Possibles stock-recruitment relationship:

- *Constant:* $\varphi(B) = \alpha$.
- *Linear:* $\varphi(B) = \alpha B$.
- *Beverton-Holt:* $\varphi(B) = \frac{B}{\alpha+\beta B}$.
- *Ricker:* $\varphi(B) = \alpha B e^{-\beta B}$.

Harvested Fish Population Age Structured Model

Example

Possibles stock-recruitment relationship:

- *Constant:* $\varphi(B) = \alpha$.
- *Linear:* $\varphi(B) = \alpha B$.
- *Beverton-Holt:* $\varphi(B) = \frac{B}{\alpha + \beta B}$.
- *Ricker:* $\varphi(B) = \alpha B e^{-\beta B}$.

Harvested Fish Population Age Structured Model

Example

Possibles stock-recruitment relationship:

- *Constant:* $\varphi(B) = \alpha$.
- *Linear:* $\varphi(B) = \alpha B$.
- *Beverton-Holt:* $\varphi(B) = \frac{B}{\alpha+\beta B}$.
- *Ricker:* $\varphi(B) = \alpha B e^{-\beta B}$.

Harvested Fish Population Age Structured Model

Definition

- *The catch at age a over the period $[t-1,t)$ is:*

$$C_{t,a} = \frac{\lambda_t F_a}{\lambda_t F_a + M}\left(1 - e^{-(M+\lambda_t F_a)}\right) N_{t,a}.$$

- *The yield (in terms of biomass) at time t is:*

$$Y_t = \sum_{a=1}^{A} w_a \, C_{t,a}.$$

- *The mean fishing mortality function is defined to be:*

$$F(\lambda) = \frac{\lambda}{A_r - a_r + 1}\sum_{a=a_r}^{A_r} F^a.$$

Definition

- *The catch at age a over the period $[t-1, t)$ is:*

$$C_{t,a} = \frac{\lambda_t F_a}{\lambda_t F_a + M}\left(1 - e^{-(M+\lambda_t F_a)}\right) N_{t,a}.$$

- *The* *yield* *(in terms of biomass) at time t is:*

$$Y_t = \sum_{a=1}^{A} w_a \, C_{t,a}.$$

- *The mean fishing mortality function is defined to be:*

$$F(\lambda) = \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{A_r} F^a.$$

Harvested Fish Population Age Structured Model

Definition

- *The catch at age a over the period $[t-1, t)$ is:*

$$C_{t,a} = \frac{\lambda_t F_a}{\lambda_t F_a + M} \left(1 - e^{-(M+\lambda_t F_a)}\right) N_{t,a}.$$

- *The yield (in terms of biomass) at time t is:*

$$Y_t = \sum_{a=1}^{A} w_a \, C_{t,a}.$$

- *The mean fishing mortality function is defined to be:*

$$F(\lambda) = \frac{\lambda}{A_r - a_r + 1} \sum_{a=a_r}^{A_r} F^a.$$

Current ICES Advices for Fisheries Management

- Indicators and their associated reference points are key elements of current fisheries management advice of the International Council for the Exploration of the Sea (ICES):
  - *Keeping spawning stock biomass* SSB above a threshold reference value B_{ref}.
  - *Restricting mean fishing mortality* F below a threshold reference value F_{ref}
- At the same time, ICES uses this first condition as a policy to be checked each year.

Current ICES Advices for Fisheries Management

- Indicators and their associated reference points are key elements of current fisheries management advice of the International Council for the Exploration of the Sea (ICES):
  - *Keeping spawning stock biomass* SSB above a threshold reference value B_{ref}.
  - *Restricting mean fishing mortality* F below a threshold reference value F_{ref}
- At the same time, ICES uses this first condition as a policy to be checked each year.

Current ICES Advices for Fisheries Management

- Indicators and their associated reference points are key elements of current fisheries management advice of the International Council for the Exploration of the Sea (ICES):
  - *Keeping spawning stock biomass* SSB above a threshold reference value B_{ref}.
  - *Restricting mean fishing mortality* F below a threshold reference value F_{ref}
- At the same time, ICES uses this first condition as a policy to be checked each year.

Current ICES Advices for Fisheries Management

- Indicators and their associated reference points are key elements of current fisheries management advice of the International Council for the Exploration of the Sea (ICES):
  - *Keeping spawning stock biomass* SSB above a threshold reference value B_{ref}.
  - *Restricting mean fishing mortality* F below a threshold reference value F_{ref}
- At the same time, ICES uses this first condition as a policy to be checked each year.

Are the ICES recommandations "sustainable"?

$$\begin{cases} N_{t+1} = g(N_t, \lambda_t), & t \geq 0 \\ N_0 & \text{given} \end{cases}$$

ICES Recommandation: Given a stock N such that $SSB(N) \geq B_{ref}$, use the maximal fishing effort λ such that $SSB(f(N,\lambda)) \geq B_{ref}$ and $F(\lambda) \leq F_{ref}$

- $\mathbb{D}_{ICES} = \{(N,\lambda) \ : \ SSB(N) \geq B_{ref}, F(\lambda) \leq F_{ref}\}$
- $\mathbb{V}_{ICES} = \{N \ : \ SSB(N) \geq B_{ref}\}$

Proposition (Guilbaud et al. 2006)

$\mathbb{V}_{ICES}$ is not always a viability domain for $\mathbb{D}_{ICES}$

Are the ICES recommandations "sustainable"?

$$\begin{cases} N_{t+1} = g(N_t, \lambda_t), & t \geq 0 \\ N_0 & \text{given} \end{cases}$$

ICES Recommandation: Given a stock N such that $SSB(N) \geq B_{ref}$, use the maximal fishing effort λ such that $SSB(f(N, \lambda)) \geq B_{ref}$ and $F(\lambda) \leq F_{ref}$

- $\mathbb{D}_{ICES} = \{(N, \lambda) \ : \ SSB(N) \geq B_{ref}, F(\lambda) \leq F_{ref}\}$
- $\mathbb{V}_{ICES} = \{N \ : \ SSB(N) \geq B_{ref}\}$

Proposition (Guilbaud et al. 2006)

$\mathbb{V}_{ICES}$ is not always a viability domain for $\mathbb{D}_{ICES}$

Are the ICES recommandations "sustainable"?

$$\begin{cases} N_{t+1} = g(N_t, \lambda_t), & t \geq 0 \\ N_0 & \text{given} \end{cases}$$

ICES Recommandation: Given a stock N such that $SSB(N) \geq B_{ref}$, use the maximal fishing effort λ such that $SSB(f(N,\lambda)) \geq B_{ref}$ and $F(\lambda) \leq F_{ref}$

- $\mathbb{D}_{ICES} = \{(N,\lambda) \ : \ SSB(N) \geq B_{ref}, F(\lambda) \leq F_{ref}\}$
- $\mathbb{V}_{ICES} = \{N \ : \ SSB(N) \geq B_{ref}\}$

Proposition (Guilbaud et al. 2006)

$\mathbb{V}_{ICES}$ is not always a viability domain for $\mathbb{D}_{ICES}$

Obtention of Bioeconomics Indicators

- If the recruitment function φ is increasing, then g is a bioeconomics dynamics.
- Since $g(N,\lambda) = G(\lambda)N + \begin{pmatrix} \varphi(SSB(N)) \\ \vec{0} \end{pmatrix}$, if φ is constant or lineal, then g is also quasi-linear.
- If $\mathbb{D} = \mathbb{D}_{\text{poly}}$ then $\mathbb{V}(g, \mathbb{D})$ is convex.

Proposition (De Lara, Gajardo & Ramírez 2007)

Consider $\mathbb{D}_{\text{yield}} = \{(N,\lambda) \ : \ Y(N,\lambda) \geq y_{\min}\}$, and suppose that φ is increasing and $\varphi \leq R$. If N belongs to the associated viability kernel, then $SSB(N) \geq B_{ref}$ for some reference value $B_{ref} > 0$. That is

$$N \in \mathbb{V}(g, \mathbb{D}_{\text{yield}}) \Rightarrow SSB(N) \geq B_{ref}.$$

Obtention of Bioeconomics Indicators

- If the recruitment function φ is increasing, then g is a bioeconomics dynamics.
- Since $g(N,\lambda) = G(\lambda)N + \begin{pmatrix} \varphi(SSB(N)) \\ \vec{0} \end{pmatrix}$, if φ is constant or lineal, then g is also quasi-linear.
- If $\mathbb{D} = \mathbb{D}_{\text{poly}}$ then $\mathbb{V}(g,\mathbb{D})$ is convex.

Proposition (De Lara, Gajardo & Ramírez 2007)

Consider $\mathbb{D}_{\text{yield}} = \{(N,\lambda) \ : \ Y(N,\lambda) \geq y_{min}\}$, and suppose that φ is increasing and $\varphi \leq R$. If N belongs to the associated viability kernel, then $SSB(N) \geq B_{ref}$ for some reference value $B_{ref} > 0$. That is

$$N \in \mathbb{V}(g,\mathbb{D}_{\text{yield}}) \Rightarrow SSB(N) \geq B_{ref}.$$

Obtention of Bioeconomics Indicators

- If the recruitment function φ is increasing, then g is a bioeconomics dynamics.
- Since $g(N,\lambda) = G(\lambda)N + \begin{pmatrix} \varphi(SSB(N)) \\ \vec{0} \end{pmatrix}$, if φ is constant or lineal, then g is also quasi-linear.
- If $\mathbb{D} = \mathbb{D}_{\text{poly}}$ then $\mathbb{V}(g,\mathbb{D})$ is convex.

Proposition (De Lara, Gajardo & Ramírez 2007)

Consider $\mathbb{D}_{\text{yield}} = \{(N,\lambda) \;:\; Y(N,\lambda) \geq y_{min}\}$, and suppose that φ is increasing and $\varphi \leq R$. If N belongs to the associated viability kernel, then $SSB(N) \geq B_{ref}$ for some reference value $B_{ref} > 0$. That is

$$N \in \mathbb{V}(g,\mathbb{D}_{\text{yield}}) \Rightarrow SSB(N) \geq B_{ref}.$$

Obtention of Bioeconomics Indicators

- If the recruitment function φ is increasing, then g is a bioeconomics dynamics.
- Since $g(N, \lambda) = G(\lambda)N + \begin{pmatrix} \varphi(SSB(N)) \\ \vec{0} \end{pmatrix}$, if φ is constant or lineal, then g is also quasi-linear.
- If $\mathbb{D} = \mathbb{D}_{\mathsf{poly}}$ then $\mathbb{V}(g, \mathbb{D})$ is convex.

Proposition (De Lara, Gajardo & Ramírez 2007)

Consider $\mathbb{D}_{\mathsf{yield}} = \{(N, \lambda) \; : \; Y(N, \lambda) \geq y_{\mathrm{mkn}}\}$, *and suppose that* φ *is increasing and* $\varphi \leq R$. *If* N *belongs to the associated viability kernel, then* $SSB(N) \geq B_{ref}$ *for some reference value* $B_{ref} > 0$. *That is*

$$N \in \mathbb{V}(g, \mathbb{D}_{\mathsf{yield}}) \Rightarrow SSB(N) \geq B_{ref}\,.$$

Conclusions and Perspectives

- We have presented viability issues in a general framework of discrete time control system.
- Some theoretical properties (such as convexity) and some estimations or approximations of the viability kernel have been proved for particular cases.
- This has led to new viability indicators in the fishery management problem.

Perspectives:

- We expect to exploit more some properties of the viability kernel (such as convexity, polyhedral).
- We expect to obtain new indicators for viability in the fishery management problem.
- We would like to extend this approach to models with "two zones" or two interacting species.

Conclusions and Perspectives

- We have presented viability issues in a general framework of discrete time control system.
- Some theoretical properties (such as convexity) and some estimations or approximations of the viability kernel have been proved for particular cases.
- This has led to new viability indicators in the fishery management problem.

Perspectives:

- We expect to exploit more some properties of the viability kernel (such as convexity, polyhedral).
- We expect to obtain new indicators for viability in the fishery management problem.
- We would like to extend this approach to models with “two zones” or two interacting species.

Conclusions and Perspectives

- We have presented viability issues in a general framework of discrete time control system.
- Some theoretical properties (such as convexity) and some estimations or approximations of the viability kernel have been proved for particular cases.
- This has led to new viability indicators in the fishery management problem.

Perspectives:

- We expect to exploit more some properties of the viability kernel (such as convexity, polyhedral).
- We expect to obtain new indicators for viability in the fishery management problem.
- We would like to extend this approach to models with "two zones" or two interacting species.

Conclusions and Perspectives

- We have presented viability issues in a general framework of discrete time control system.
- Some theoretical properties (such as convexity) and some estimations or approximations of the viability kernel have been proved for particular cases.
- This has led to new viability indicators in the fishery management problem.

Perspectives:

- We expect to exploit more some properties of the viability kernel (such as convexity, polyhedral).
- We expect to obtain new indicators for viability in the fishery management problem.
- We would like to extend this approach to models with "two zones" or two interacting species.

Conclusions and Perspectives

- We have presented viability issues in a general framework of discrete time control system.
- Some theoretical properties (such as convexity) and some estimations or approximations of the viability kernel have been proved for particular cases.
- This has led to new viability indicators in the fishery management problem.

Perspectives:

- We expect to exploit more some properties of the viability kernel (such as convexity, polyhedral).
- We expect to obtain new indicators for viability in the fishery management problem.
- We would like to extend this approach to models with "two zones" or two interacting species.

Conclusions and Perspectives

- We have presented viability issues in a general framework of discrete time control system.
- Some theoretical properties (such as convexity) and some estimations or approximations of the viability kernel have been proved for particular cases.
- This has led to new viability indicators in the fishery management problem.

Perspectives:

- We expect to exploit more some properties of the viability kernel (such as convexity, polyhedral).
- We expect to obtain new indicators for viability in the fishery management problem.
- We would like to extend this approach to models with "two zones" or two interacting species.

Bibliography

C.W. Clark
Bioeconomics modelling and fisheries management
Wiley Interscience publication (1985).

M. De Lara, P. Gajardo & H. Ramírez C.
Estimates of viability domains and kernel for bioconomics problems
Working paper.

T. Guilbaud, M.J. Rochet, L. Doyen & M. De Lara
Reconciling managment objectives and operational reference points through a viability approach
To appear in ICES Journal of Marine Science.

T. J. Quinn & R. B. Deriso
Quantitative Fish Dynamics
Oxford University Press (1999).

Thanks!!


280195
Grilled mackerel
©Fotos mitgeschmack / Stockfood
Viewed by Guest on 9/7/2006